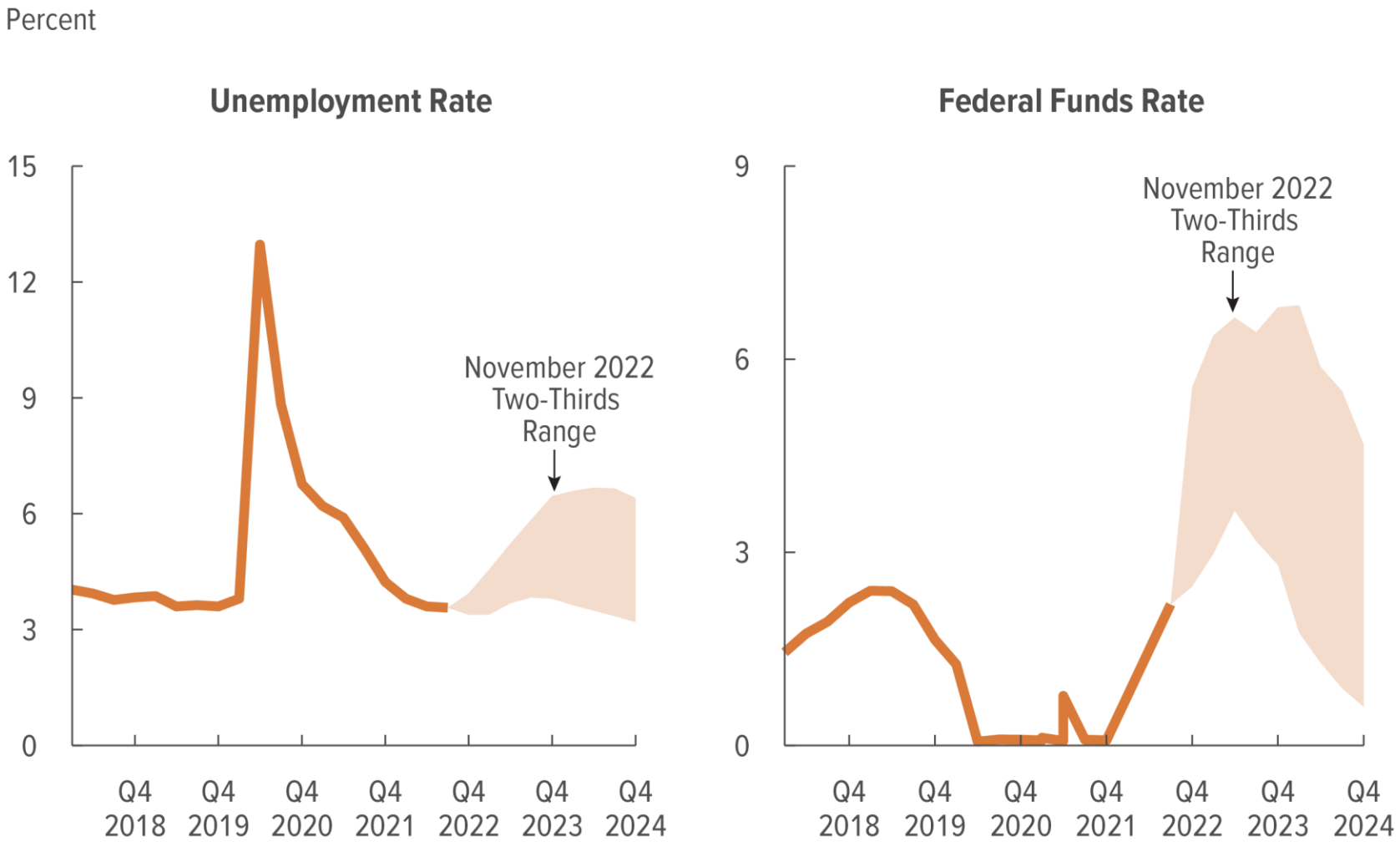
Congressional Budget Office
Nonpartisan Analysis for the U.S. Congress

Estimating the Uncertainty of the Economic Forecast Using CBO's Expanded Markov-Switching Model

January 2023

CBO's View of the Economy as of November 2022



In November 2022, the Congressional Budget Office was asked about its view of the economy. For the fourth quarter of 2023, CBO estimated that there is a two-thirds chance that:

- The unemployment rate will be between 3.8 percent and 6.4 percent, and
- The federal funds rate—the rate that financial institutions charge each other for overnight loans—will be between 2.7 percent and 6.8 percent.

For details, see Congressional Budget Office, *CBO's Current View of the Economy in 2023 and 2024 and the Budgetary Implications* (November 2022), www.cbo.gov/publication/58757. Q = quarter.

CBO's Analytic Method for Estimating Uncertainty

The analysis of economic uncertainty was conducted in three main steps:

- Preliminary economic projections provided central estimates for each variable;
- 100 simulations of the rates of unemployment, inflation, and interest were jointly estimated around the central estimates, reflecting asymmetric dynamics and relating the variables through an expectations-augmented Phillips curve and an inertial Taylor rule; and
- Forecasts conditional on those rates were estimated using symmetric distributions in which economic output and other variables were synchronized with the simulations of unemployment, inflation, and interest rates.

This document focuses on the second step, which used CBO's expanded Markov-switching model.

For discussion of the first step, see Robert W. Arnold, *How CBO Produces Its 10-Year Economic Forecast*, Working Paper 2018-02 (Congressional Budget Office, February 2018), www.cbo.gov/publication/53537. For discussion of the third step, see Congressional Budget Office, "Estimating the Uncertainty of the Economic Forecast Using CBO's Bayesian Vector Autoregression Model" (January 2023), www.cbo.gov/publication/58883. For related discussion, see Mark Lasky, *The Congressional Budget Office's Small-Scale Policy Model*, Working Paper 2022-08 (Congressional Budget Office, September 2022), www.cbo.gov/publication/57254.

Historical Dynamics Reflected in the Modeling

Key asymmetries include:

- The unemployment rate rising rapidly in recessions and falling gradually in economic expansions, and
- Interest rates not falling below zero.

Lower unemployment increases inflation; higher unemployment decreases inflation.

Lower unemployment and higher inflation increase interest rates; higher unemployment and lower inflation decrease interest rates.

Higher real (inflation-adjusted) interest rates increase the unemployment rate; lower real interest rates decrease the unemployment rate.

A smaller positive (or more negative) spread between long-term and short-term interest rates increases the probability of entering and the probability of staying in a recession.

How the Expanded Markov-Switching Model Works

Inputs include central forecasts and key parameters from CBO's large-scale macroeconometric model as well as recent data.

For each calendar quarter in the projection period, CBO uses information about the previous quarter to estimate:

- The unemployment rate,
- Inflation rates, and
- Interest rates.

After a set of simulations for a given quarter have been produced, the values are calibrated so that their average equals CBO's central forecast before the projection for the next quarter.

The parameters of the model are estimated using data from 1961 through 2019.

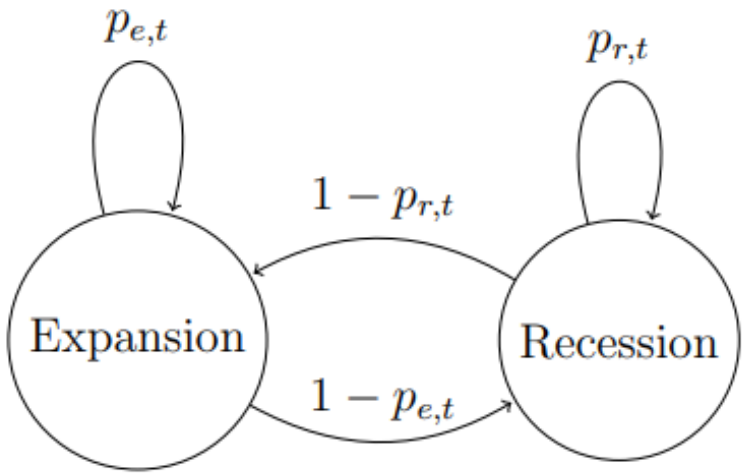
How the Model Switches Between Expansions and Recessions

Define t as an index for calendar quarters.

Define p_{et} as the probability of remaining in the expansion state (e) in quarter t conditional on being in that state in the previous quarter.

Define p_{rt} as the probability of remaining in the recession state (r) in quarter t conditional on being in that state in the previous quarter.

Define i as an index for two states of the model: expansion (e) and recession (r).



The model is referred to as a Markov-switching model because it uses a sequence of expansion and recession states known as a Markov chain.

For additional details, see Michael McGrane, *A Markov-Switching Model of the Unemployment Rate*, Working Paper 2022-05 (Congressional Budget Office, March 2022), www.cbo.gov/publication/57582.

How Simulations Differ in Expansions and Recessions

The simulation of a variable in each quarter is the sum of a predicted value from an equation and a shock. This approach is used for the unemployment, inflation, and interest rate variables.

The parameter values in the unemployment gap equation differ depending on whether a simulated economy is in an expansion or recession state.

In simulations, shocks are drawn from the historical distributions of residuals formed by the difference between actual and predicted values. The sampling probability in a simulation during an expansion or recession state is proportional to the estimated probability that a historical residual occurred in that state.

How Key Unemployment and Inflation Variables Are Defined

Define $\tilde{u}$ as the unemployment gap—the actual unemployment rate minus the noncyclical rate in the central forecast.

Define π^* as the perceived target rate of inflation.

For parameter estimation, those values are consistent with the forecast of inflation over 10 years in the Survey of Professional Forecasters. In simulations, the model uses an equation from the Federal Reserve with weight on the value in the previous period, inflation in the previous period (π_{t-1}), and the Federal Reserve's target rate of inflation (π_F^*), which is fixed at 2 percent.

$$\pi_t^* = 0.9\pi_{t-1}^* + 0.05\pi_{t-1} + 0.05\pi_F^*$$

For information about the Federal Reserve's model, see Federal Reserve Board, "FRB/US Project" (updated August 16, 2022), www.federalreserve.gov/econres/us-models-about.htm.

How Key Interest Rate Variables Are Defined

Define r^* as the real natural rate of interest in the central forecast.

Define $\tilde{f}$ as the real federal funds rate gap. That gap is the nominal federal funds rate minus r^* minus the expected rate of inflation. For parameter estimation, the expected rate of inflation is an estimated historical series of expected inflation. In simulations, the value is the perceived target rate of inflation (π_t^*).

Define $\tilde{s}$ as the spread in interest rates. That spread is the 10-year Treasury note rate minus the federal funds rate.

How the Unemployment Rate Is Projected

The unemployment gap, $\tilde{u}_t$, depends on previous values of the unemployment rate gap, on previous values of the real federal funds rate gap, and on whether the economy is in an expansion or recession state.

$$\tilde{u}_t = \alpha_i + \beta_{1i}\tilde{u}_{t-1} + \beta_{2i}\tilde{u}_{t-2} + \gamma_i\tilde{f}_{t-1} + \sigma_i\varepsilon_t$$

The model's coefficients depend on being in an expansion or recession (i).

The probabilities of remaining in an expansion or recession state depend on $\tilde{s}$.

$$p_{it} = \exp(\delta_i + \varphi_i\tilde{s}_{t-1})/(1 + \exp(\delta_i + \varphi_i\tilde{s}_{t-1}))$$

The initial probability of being in the recession state is calibrated so that the mean of the simulations is close to CBO's central forecast of the unemployment rate.

Define σ_i as the standard deviation of the errors in expansion and recession states, and define ε_t as the error term from a standard normal distribution.

How the Rate of Inflation Is Projected

The inflation rate, π_t, follows an expectations-augmented Phillips curve that depends on previous values of inflation, the perceived target rate of inflation, and the current unemployment rate gap.

$$\pi_t = \rho_1^{\pi}\pi_{t-1} + \rho_2^{\pi}\pi_{t-2} + (1 - \rho_1^{\pi} - \rho_2^{\pi})\pi_t^* + \theta^{\pi}\tilde{u}_t + \nu_t$$

Define π_t as the percentage change in the personal consumption expenditures price index (PCEPI) excluding food and energy (core PCEPI). PCEPI is a function of core PCEPI. The two indices are modeled as being cointegrated.

Define ν as the error whose variance depends on the variance of inflation in the previous quarter.

How the Federal Funds Rate Is Projected

The federal funds rate, f_t, follows a modified inertial Taylor rule that depends on a weighted average of the previous value of the federal funds rate and a combination of the real natural rate of interest, the current value of inflation, the target rate of inflation, and the current and previous value of the unemployment rate gap. The modified rule imposes a lower bound on the rate.

$$\tau_t = \omega f_{t-1} + (1-\omega)[r_t^* + \pi_t + \rho^f(\pi_t - \pi_F^*) + \theta^f(\pi_t, \tilde{u}_t - \tilde{u}_{t-1})\tilde{u}_t] + \eta_t$$
$$f_t = \mathrm{g}(\tau_t, f_{t-1}, \pi_t, \tilde{u}_t)$$

Define $\theta^f()$ as a coefficient that has a larger negative value if inflation is below a threshold value and unemployment is rising rapidly—corresponding to a larger potential cut in the federal funds rate. Define η_t as a normal error term.

Define $\mathrm{g}()$ as a function that sets the federal funds rate to τ_t except when the federal funds rate would otherwise be less than 0.125 percent or when the federal funds rate in the previous period was 0.125 percent, the inflation rate is below a threshold value, and the unemployment gap is above a threshold value. In those cases, the federal funds rate is set to 0.125 percent.

How the 10-Year Treasury Note Rate Is Projected

The 10-year Treasury note rate, κ_t, depends on the expected path of the federal funds rate and the term premium.

$$\kappa_t = \lambda_t(f_t, \pi_t, \pi_{t-1}, \tilde{u}_t, \tilde{u}_{t-1}) + m_t(r_t^* + \pi_t^*, \tilde{u}_t) + v_t$$

Define $\lambda_t()$ as the average of the federal funds rate for the current quarter and its projection for the next 39 quarters using a linear version of the Markov model. In this linear model, there are no separate states of the economy, which greatly simplifies simulation. CBO uses the linear model and values of the federal funds rate, the inflation rate, and the unemployment gap from the current quarter and the previous quarter to project their values for the next quarter.

Define $m_t()$ as a term premium that is an increasing function of the nominal equilibrium federal funds rate ($r_t^* + \pi_t^*$) and of the unemployment rate gap. In the first projection period, the function is calibrated so that the average term premium in the simulations equals the term premium in the central forecast.

Define v_t as a serially correlated normal error term.

About This Document

This document was prepared to enhance the transparency of CBO's work and to encourage external review of that work. In keeping with CBO's mandate to provide objective, impartial analysis, the document makes no recommendations.

Michael McGrane prepared the document with guidance from Sebastien Gay. Robert Arnold, Jaeger Nelson, and Byoung Hark Yoo provided comments.

Mark Hadley and Jeffrey Kling reviewed the document. Christine Browne edited it and R. L. Rebach created the graphics. The document is available at www.cbo.gov/publication/58884.

CBO seeks feedback to make its work as useful as possible. Please send comments to communications@cbo.gov.