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The Welfare Effects of Debt: Crowding Out and Risk Shifting

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Abstract

Government debt affects people's welfare through two distinct channels: It crowds out capital, and it shifts risk from current to future generations. This study extends Olivier Blanchard's 2019 analysis of the welfare effects of debt by decomposing his estimates into those two categories. Blanchard estimated the change in average utility under simulations of an overlapping generations model with and without a transfer of wealth from the younger to the older generation. This study decomposes those estimated welfare effects into crowding-out and risk-shifting components and estimates total effects and separate effects of the two components under alternative assumptions about technology and preferences. Even though crowding out can increase welfare under some conditions in overlapping generations models by reducing the overaccumulation of capital, I estimate the crowding-out effect of debt on welfare to be consistently negative. Government debt shifts risk to future generations by giving certainty to current generations that are saving for retirement at the expense of greater risk to future generations. I find the risk-shifting effect of debt on welfare to be positive under Blanchard's assumptions about technology and preferences, and that positive effect partially or fully offsets the cost of crowding out on the total welfare effect of risk-free transfers. Under alternative assumptions—persistent technology shocks and a greater aversion to intergenerational risk—risk-free debt is much more costly than under Blanchard's assumptions, partly because the crowding-out effect is larger and partly because risk shifting has negative rather than positive effects.

Keywords: interest rates, fiscal policy, crowding out, government debt, risk shifting

JEL Classification: E22, E23, E43, E62, H50, H63

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Introduction

Persistently low government interest rates have led researchers and policymakers to suggest that the welfare effects of rising government debt in the United States and other high-income countries might be smaller than previously thought (see Blanchard 2019 and Furman and Summers 2019). U.S. federal debt is highly liquid and widely viewed as offering a risk-free return. The average nominal interest rate on federal government debt (“ r ”) in the United States has declined from 6.6 percent in the 1990s to 1.9 percent from 2010 to 2021 (see Figure 1). The growth rate of the economy (“ g ”) has fallen as well, but not as much as “ r ” has declined; since 2010, “ r ” has averaged 2.1 percent less than “ g .” In a situation in which $r < g$ and the federal budget is in primary balance (that is, governmental noninterest spending is equal to revenues), the ratio of debt to gross domestic product (GDP) will shrink.¹ The U.S. debt-to-GDP ratio has risen in the past two decades even though $r < g$, mainly because of large primary deficits (which result when noninterest spending is higher than revenues). The Congressional Budget Office projects that primary deficits will continue through the next 30 years (see Congressional Budget Office 2022). Even so, spending on interest has remained historically low because of historically low levels of “ r .”

Although government interest rates have declined since 2010, the marginal rate of return on capital (“ R ”) has remained steady over the same period (see Council of Economic Advisers 2017, Figure 5). The marginal rate of return on capital is relevant to the welfare effect of higher government debt because one of the main effects of government debt is to crowd out private investment in capital. The marginal rate of return measures the benefits of the capital that is crowded out and thus the welfare cost of reduced investment. Given a steady marginal rate of return coupled with declining growth and risk-free interest rates, the difference between the marginal product of capital and the growth rate of the economy has increased, as has the risk premium ($R - r$).

The gap between r and R leads to the question, which rate matters for the welfare effects of debt? Debt might finance investment in public capital or human capital, and its welfare effects might depend partially on the return on those investments. However, under the overlapping generations theory developed by Peter Diamond, government debt’s main role is to finance transfer payments to older generations. The original model of overlapping generations developed by Diamond (1965) was deterministic and thus did not distinguish between the risk-free rate and the marginal product of capital—in other words, between r and R . Thus, it has unclear implications in a situation in which $r < g < R$. To sort out whether r or R is more important, Blanchard (2019, hereafter just “Blanchard”) analyzed the welfare effects of transfers and debt rollovers in a

¹ In CBO’s projections, a higher ratio of debt to GDP results in increases in interest rates that ultimately propel r above g by 2042.

stochastic overlapping generations (OLG) model in which r and R differ. Under his model, a representative household lives two periods—one in which it is young and working and the other in which it is retired and living off investments made when it was young—each equal in length to a generation. Blanchard found that in his model the effect of a debt-financed transfer program on households' lifetime utility depended on both r and R . He also conducted simulations of a steady-state economy under different levels of transfers and debt rollovers, finding that both r and R are associated with the welfare consequences of debt and transfers. He concluded that additional debt either has low welfare costs or might produce small welfare benefits under a situation in which $r < g$.

In this study, I extend Blanchard's analysis to estimate two distinct effects on welfare, which I term crowding out and risk shifting. Crowding out occurs because government debt absorbs private savings that could otherwise be used for private capital investment. Government debt also shifts risk from current to future generations. It takes risk away from current generations by giving them a certain return in their retirement. It places the obligation associated with that certain return on future generations, although their resources to shoulder that burden are uncertain. Shifting risk from current to future generations can be Pareto-improving (making some generations better off without making other generations worse off), or it can have the opposite effects, depending on the persistence of technology shocks and preferences (see Falkenheim 2021).

The distinction between crowding out and risk shifting is important for understanding the policy implications of Blanchard's results. Blanchard considers an increase in risk-free transfers and public debt as a main policy alternative to a situation with no increase in debt. An increase in debt results in both crowding out and risk shifting. Other policies can produce one outcome without the other, though. For example, the government could shift risk without crowding out capital if it issued debt and invested the proceeds in risky assets. Under that policy, debt held by the public would be higher, but debt net of government-owned assets would not. Alternatively, the government could achieve crowding out with less risk shifting if it took on obligations that did not shift risk to future generations. Bohn (2008) suggests that a debt obligation whose payments were tied to GDP would shift the burden of repayment to future generations without shifting risk. Although that form of debt might exist only in theory, types of obligations differ in how much risk they shift from current to future generations (see Fischer 1983 and Bohn 1990). As an example, unfunded wage-indexed obligations (such as Social Security benefits) place a burden on future generations just like federal debt but leave a considerable amount of market risk with the beneficiary. They thus do not pass as much risk to future generations as federal debt does (see Geanakoplos and Zeldes 2010 and Bohn 2009).

This paper extends Blanchard's analysis by separating the total estimated welfare effects in his simulations into crowding-out and risk-shifting components. My main strategy for decomposition is to treat a risk-free transfer as a combination of a risky transfer (which achieves

crowding out) and a swap (which shifts risk). Noting that intergenerational transfers have similar outcomes as debt, Blanchard analyzed a transfer of a fixed amount (“D”) that a younger generation would give to an older generation in one period and then, in the next period, would receive from the younger generation when they were older. In this paper, I divide that transfer of D into two components: a risky transfer that averages D but depends on the state of the economy in the next period through the rate of technological progress, and a swap of that risky return for the certain amount of D. I also replicate Blanchard’s simulations of economies before and after transfers are introduced and decompose their welfare effects into crowding-out and risk-shifting components. Under stochastic simulations, I measure welfare with and without transfers by comparing the average lifetime utility of the households in the steady state. My findings under Blanchard’s parameters are that crowding out is costly for welfare if $R > g$ and beneficial if $R < g$. The effects of risk shifting on welfare are usually, but not always, positive.

The welfare effects are larger when I exclude a risk-free endowment that Blanchard introduced explicitly to avoid the possibility of a default from the transfer. Without that endowment, I find that the effect of crowding out on welfare is more negative than under Blanchard’s assumptions, the risk-shifting effect is more positive, and the total effect on welfare is more negative.

Because risk shifting accounts for much of the welfare effect of debt, that effect is likely to be sensitive to the same factors that determine the effects of risk shifting. In Falkenheim (2021), I found that assumptions about technology and preferences determined the likely welfare effects of government policies that shift risk to future generations. For example, shifting risk to future generations was likely to be Pareto-efficient under temporary technology shocks (such as those assumed in Blanchard) and the opposite under permanent shocks. I also found that the welfare effects of risk shifting were highly sensitive to the relative values of two parameters in the utility function—one representing the risk of uneven consumption over a lifetime and the other representing the risk of unequal lifetime consumption between generations.

With those findings in mind, I vary assumptions related to risk aversion and the persistence of technology shocks. Blanchard’s utility function weighted generations against each other on the basis of a log function, in which aversion to intergenerational inequality was low relative to aversion to risk within the lifetime of a single generation. I consider higher levels of aversion to inequality between generations, so if a generation yet to be born suffers losses because of bad draws to the rate of technological progress, that generation will be relatively worse off than under the baseline assumptions. Blanchard simulated the effect of transfers on the steady state of an economy with temporary technology shocks. In this paper, I also consider persistent technology shocks that are autocorrelated across periods.

I find that welfare effects are very sensitive to those assumptions, especially to the assumptions about risk aversion. Under a specification with no risk-free endowment, high aversion to lifetime consumption risk, and persistent technology shocks, the welfare cost of debt is about four times

as large as under Blanchard's assumptions. Although the results validate the finding that the welfare cost of debt is smaller when $r < g$ than it is otherwise, they also suggest that those costs might be large even in that case.

Those findings are consistent with previous research suggesting that debt might have larger costs than estimated in Blanchard. Evans (2020) shows that Blanchard's positive welfare effects of increased debt in periods of low interest rates depend entirely on Blanchard's assumption of a large risk-free endowment—an assumption also relaxed in this paper. Evans also shows that persistence in technology increases the welfare cost of debt. Hasanhodzic (2020) forgoes the risk-free endowment in a 10-period OLG model with two types of technology shocks. She finds that a pay-as-you-go Social Security program funded by a 15 percent payroll tax makes future generations much worse off, equivalent to a 20 percent loss of consumption.

Crowding Out and Risk Shifting in Overlapping Generations Models

The Congressional Budget Office (2022) describes crowding out as follows: “When the federal government borrows in financial markets, it competes with other participants for funds. That competition can crowd out private investment, reducing economic output and income in the long term.” That description does not specify whether crowding out has a negative or positive effect on welfare.

Although crowding out of capital might seem to be a negative consequence of government debt, it can theoretically have a positive effect on consumption and welfare. Capital investment can be too high under *laissez faire* in the classic overlapping generations analysis of Diamond (1965). Under that model, young people have no other savings vehicle besides productive capital and may invest in capital beyond the level that maximizes aggregate consumption to smooth consumption between their youth and old age. Given that overaccumulation of capital, government policies that crowd out capital can make all generations better off, by raising consumption and smoothing it across generations. Those government policies can take the form of debt-funded pensions—which absorb savings from the young and transfer it to the old—or a simple transfer from younger people to older people. Either policy raises utility by giving the younger generation a source of income in their old age that lessens their need to invest in private capital when it has accumulated to the point of having a low marginal return.

Given that potential for capital to be too high, Abel and others (1989) studied whether the U.S. economy had excess capital. They found no evidence of overinvestment in capital, based on the relationship between capital's share of GDP and investment. (That relationship has not changed substantially since publication of their study.) Blanchard came to a somewhat different conclusion. He analyzed the welfare effects of transfers in a stochastic overlapping generations model and found that when $r < g$, either the welfare cost of debt might be low or, in some cases, debt could result in welfare benefits. As mentioned in the introduction, one possible reason for

those differing results is that the studies differ in whether they incorporate the risk-shifting effects of debt. The measures in Abel and others (1989) are designed to detect whether the quantity of capital is too high but not whether there is an efficient level of risk shifting. Blanchard’s analysis incorporates risk shifting largely because its focus is on prices instead of quantities. It calibrates a measure of risk aversion to the risk premium—the difference between the marginal product of capital and the risk-free rate.

Decomposing the welfare effects into crowding out and risk shifting might shed light on the limitations of large-scale heterogeneous OLG models, which tend not to incorporate aggregate uncertainty, and show how to use such models and stochastic OLG models as complements. Large-scale heterogeneous OLG models (for instance, Congressional Budget Office 2019) generally do not assume any uncertainty at the aggregate level and thus are not well-suited to estimating the effects of intergenerational risk sharing. In general, they incorporate many more than two overlapping generations and multiple categories of households within each generation. Those features make large-scale OLG models suitable for studying the distributional effects of fiscal policies, both within and across generations. That same richness in detail makes it difficult for them to handle the computational burden of aggregate uncertainty, though. To limit that computational burden, most large-scale OLG models portray individual households as facing uncertainty in their own income but not in the total income of all households, such that all risk is idiosyncratic. Thus, risk shifting is entirely within generations and not between them. In contrast, the model in this study features only one representative household per generation that faces systematic risk.

Decomposing the Effect of Intergenerational Transfers on Lifetime Utility

In section II of his analysis, Blanchard assumes that each generation has a lifetime utility that is a weighted average of utility between their youth and old age:

$$U = \beta U(C_{1t}) + (1 - \beta) E_t[U(C_{2t+1})] \quad (1.)$$

Where β is a parameter between 0 and 1 determining the weight on utility when the generation is young in period t relative to when they are old in period $t + 1$. Consumption for generation t is C_{1t} in their youth and C_{2t+1} in their old age. Those levels of consumption are given by the following expressions.

$$C_{1t} = W_t - K_{t+1} - D_t \quad (2.)$$

$$C_{2t} = R_{t+1}K_{t+1} + D_{t+1} \quad (3.)$$

Where W_t equals the wages that the generation earns in time t , K_{t+1} represents the savings that they invest in capital for time $t + 1$, and R_{t+1} is one plus the return on that capital at time $t + 1$.

The young make a transfer D_t to the previous generation, which is old in time t , and then in their old age receive a transfer D_{t+1} from the next generation that is young at time $t + 1$.

Blanchard considers a fixed transfer such that $D_t = D$. He solves for the derivative of lifetime utility with respect to D and finds that it has two components representing a direct effect (dU_{at}) and an indirect effect (dU_{bt}) that come about through changes in the level of capital that induce changes in the relative prices of capital and labor.²

The first term measuring the direct effect is a function of the risk-free rate.

$$dU_{at} = \beta(1 - r)E_t[U'(C_{2t+1})]dD \quad (4.)$$

Where r represents one plus the risk-free interest rate. Given that Blanchard looks at the case of zero growth, the condition $r < 1$ can be interpreted as a situation in which $r < g$. Under that condition, $(1 - r)$ is positive and thus the direct welfare effect of higher transfers is as well.

I assume that D_t is a function of two parameters: D_V , the size of the variable (risky) transfer, and D_S , the size of the swap of a risky transfer for a fixed transfer.

$$D_t = D_V \frac{A_t}{\bar{A}} + D_S \left(1 - \frac{A_t}{\bar{A}}\right) \quad (5.)$$

The amount of the risky transfer varies directly with the level of total factor productivity A_t , and the swap exchanges that transfer for a fixed transfer. If I set $D_V = D_S = D$, then I replicate the fixed transfer $D_t = D$ for all t studied in Blanchard. The direct effect captured in equation (4) can be decomposed as $dU_{at} = dU_{Vt} + dU_{St}$, where dU_{Vt} is the partial derivative of lifetime utility with respect to the size of the risky transfer, and dU_{St} is the partial derivative of lifetime utility with respect to the size of the swap.

The first partial derivative (see the appendix for the computation), measuring the effect of the risky transfer, is:

$$dU_{Vt} = \beta E_t[R_{t+1}U'(C_{2t+1})] \left(\frac{1}{\bar{R}} - \frac{A_t}{\bar{A}}\right) dD_V \quad (6.)$$

The direct effect of the risky transfer in that expression depends on the marginal product of capital and not on the risk-free rate. That partial derivative is negative if $\frac{1}{\bar{R}} < \frac{A_t}{\bar{A}}$. Because $\frac{A_t}{\bar{A}}$ averages 1, and the marginal rate of return R is generally higher than 1, $\frac{1}{\bar{R}} < \frac{A_t}{\bar{A}}$ will hold true, on average.

² For more details, including an expression for dU_{bt} , see Blanchard section II.

The partial derivative of utility with respect to the size of the swap dU_{St} is given by the following expression:

$$dU_{St} = \beta E_t[U'(C_{2t+1})] \left\{ 1 + \frac{A_t}{\bar{A}} - r \left(\frac{\bar{R}+1}{\bar{R}} \right) \right\} dD_S \quad (7.)$$

That partial derivative is smaller the lower the value of r and the higher the value of R . Thus, the direct benefit of a swap increases with the risk premium (the difference between R and r).

Simulations With Risky and Risk-Free Transfers

In this section, I decompose the welfare effects of intergenerational transfers into crowding-out and risk-shifting components by separately simulating the effect of risky and risk-free transfers. Whereas Blanchard looked only at fixed transfers, I also simulate risky transfers that vary with the realization of the level of technological productivity A_{t+1} and thus with ex post returns on capital.

I extend Blanchard's simulations in other ways as well. For instance, I develop more general versions of the technology shocks and preferences in Blanchard, allowing for technology shocks that persist across generations and preferences with greater aversion to intergenerational inequality.

Notation

Each generation lives two periods, one in which they are young and working and one in which they are old and retired. In retirement, the older generation lives off the returns from their investment in capital and potentially a government transfer. One period in this analysis represents a generation, set to 25 years in the model. The generation born at time t consumes C_{1t} when they are young (in period t) and C_{2t+1} when they are retired (in period $t + 1$).

Production. The production function is stochastic with A_t as the level of productivity. I focus mainly on Cobb-Douglas production but also consider the linear case.

Cobb-Douglas:
$$f(K_t, N_t) = A_t K_t^b N_t^{1-b}$$

Linear output:
$$f(K_t, N_t) = A_t [bK_t + (1 - b)N_t]$$

Where K_t is the level of capital and b is capital's share of output. The amount of effective labor, N_t , is the product of the number of workers and the units of effective labor per worker. The model captures both population and productivity growth in g , the growth rate of N_t , such that $N_{t+1} = (1 + g)N_t$. I define k_t as the level of capital per unit of effective labor $k_t \equiv \frac{K_t}{N_t}$, and under that definition output equals $N_t A_t k_t^b$. In the case where $g = 0$, as studied in Blanchard, I

set N_t to 1 and output simplifies to $A_t k_t^b$ in the Cobb-Douglas case and $A_t[bK_t + (1 - b)]$ under the linear production function.

In competitive equilibrium, the wage rate W_t and return on capital R_t are as follows.

$$\text{Cobb-Douglas:} \quad W_t = (1 - b)A_t k_t^b, \quad R_t = bN_t A_t k_t^{b-1}$$

$$\text{Linear output:} \quad W_t = (1 - b)A_t, \quad R_t = bA_t$$

Technology. In addition to the growth expressed in g , the path of technology follows an autoregressive process such that $\log A_{t+1} = \mu + \rho(\log A_t - \mu) + \varepsilon_t$, where ε_t is normally distributed with mean 0 and variance σ^2 . The parameter μ is the long-term mean of $\log A_t$, and ρ determines the degree of persistence of shocks away from that mean. The expectation of A_{t+1} at time t is given by the following expression:

$$E_t[A_{t+1}] = e^{\mu + \rho(\log A_t - \mu) + \sigma^2/2} \quad (8.)$$

Under Blanchard, $\rho = 0$ and thus A_t is white noise and $E_t[A_{t+1}]$ becomes constant at $e^{\mu + \sigma^2/2}$. In this study, $\rho = 0$ is treated as a special case.

Endowment. Following Blanchard, I assume that the young receive a risk-free endowment, X_t , in addition to their wages. Blanchard introduced that assumption “to make sure that the deterministic transfer from the young to the old is always feasible.” I include that endowment in the initial set of simulations for comparison with Blanchard’s results, but I draw policy implications from the set of simulations without the transfer.

I set the endowment X_t to mean wages in period $t + 1$ ($\overline{W_{t+1}}$) conditional on the expected value of A_{t+1} . In expectation, conditional on a marginal product of capital, R , and the expected value of productivity, $E_t[A_{t+1}]$, the endowment (and mean wages) are given by the following expression.³

$$X_t = \overline{W_{t+1}} = (1 - b)N_t \left(\frac{R}{b}\right)^{\frac{b}{b-1}} (E_t[A_{t+1}])^{\frac{1}{1-b}} \quad (9.)$$

That endowment scales according to $N_t E_t[A_{t+1}]^{\frac{1}{1-b}}$. $E_t[A_{t+1}]^{\frac{1}{1-b}}$ is constant if $\rho = 0$. I include that endowment in my initial projections to produce results that are comparable with Blanchard’s, but then forgo it in the remainder of the analysis because it does not prove

³ Blanchard estimated mean wages initially on the basis of that formula and then developed a more precise estimate based on the average from steady-state simulations. I forgo that second step here because I examine cases where ρ is not equal to 0 and that more precise estimate is less easy to calculate.

necessary to make the transfer feasible. Evans, Kotlikoff, and Phillips (2013) found some risk of default on a transfer like the one that Blanchard analyzed and concluded that the possibility that it might not be paid could lead to a risk premium in the valuation of the promised transfer. In contrast, my simulations without the transfer did not produce any cases in which the young's wages were insufficient to cover the transfer, perhaps because the transfer was small compared with average wages. I analyze a transfer of 20 percent of average capital in the steady state without a transfer, which is equal to the larger of the two transfers considered by Blanchard. Without a transfer, the young save a share of their wages equal to β , which has a calibrated value of approximately 0.5; thus, the transfer is set to approximately one-tenth ($0.5 * 0.2$) of steady-state wages. For the transfer to become infeasible without the endowment, actual wages would need to drop below one-tenth of steady-state wages, which does not occur in the simulations—even following a sequence of low draws of the random component in A_t .

Transfers. Blanchard examined the effect of a fixed transfer, D , on utility. I simulate the model under three scenarios: 1) without a transfer, 2) with a fixed transfer (similar to Blanchard's), and 3) with a transfer amount that is permitted to vary with persistent shocks in technology. In cases where $\rho = 0$, the transfer per unit of labor becomes fixed just like X_t . I define the “risk-free” transfer as a function of \overline{W}_{t+1} .

$$D_{t+1,risk\ free} = \alpha \overline{W}_{t+1} \quad (10.)$$

Where α is a constant that is calibrated to make the transfer equal to the targeted 20 percent share of capital, on average. I decompose $D_{t+1,risk\ free}$ into a risky transfer $D_{t+1,risky}$

$$D_{t+1,risky} = D_{t+1,risk\ free} \frac{A_{t+1}}{E[A_{t+1}]} \quad (11.)$$

and a swap of $D_{t+1,risky}$ for $D_{t+1,risk\ free}$.

If $\rho = 0$, as under Blanchard, \overline{W}_{t+1} becomes constant and the fixed transfer becomes constant as well, such that $D_{t+1,risk\ free} = D$. Otherwise, $D_{t+1,risk\ free}$ becomes known with certainty at time t , before the young generation decides how much to consume and save. Making $D_{t+1,risk\ free}$ a function of the expected level of technology keeps it relatively constant as a share of capital. I focus on the case in Blanchard where the transfer is set to 20 percent of steady-state capital, setting the parameter α to a level that produces a transfer that is 20 percent of capital, on average.

Utility. I assume a generalized form of the Epstein-Weil-Zin⁴ utility function used by Blanchard:

$$U_t(C_{1t}, E_t[C_{2t+1}^{1-\gamma} | \beta]) = C_{1t}^{1-\beta} E_t \left[\frac{C_{2t+1}^{1-\gamma}}{1-\gamma} \right]^\beta \quad (12.)$$

The parameter γ represents the aversion of a young generation to risk affecting its consumption in old age. Thus, the parameter γ represents the aversion of the generation that is currently in its youth to the possibility that consumption in its old age will not measure up to the consumption that it is already enjoying. To measure the welfare effects of debt I use a social welfare function to aggregate utility across generations.

$$Welfare(\chi) = \sum_{t=1}^T \frac{1}{1-\chi} \left(U_t(C_{1t}, E_t[C_{2t+1}^{1-\gamma} | \beta]) \right)^{1-\chi} \quad (13.)$$

The parameter χ measures aversion to the risk that a generation will be born into a lifetime of low consumption but not the risk that the generation will face uncertainty within its lifetime. The parameter χ affects the trade-off between utility of different generations under a social planner's decisions. The higher the values of χ , the more costly variance between the lifetime consumption of different generations will be, because it relates to a risk that a generation will be born to an inferior set of choices, as opposed to the risk that its choices will turn out badly. If $\chi = 1$ then the social welfare function becomes logarithmic and identical to the process used in Blanchard to aggregate utility across generations.

$$Welfare(1) = \sum_{t=1}^T (1-\beta) \log C_{1t} + \frac{\beta}{1-\gamma} \log E_t[C_{2t+1}^{1-\gamma}] \quad (14.)$$

Savings and Consumption. Under the utility function (12) the following Euler equation determines how much the younger generation will save and invest in capital along with the budget constraints.

$$(1-\beta) \frac{1}{C_{1t}} = \beta \frac{E_t[R_{t+1}(C_{2,t+1})^{-\gamma}]}{E_t[C_{2,t+1}^{1-\gamma}]} \quad (15.)$$

Consumption for the young is equal to their income ($N_t W_t + X_t$) minus the transfer that they make to the older generation (D_t) and their investment in capital K_{t+1} .

$$C_{1t} = N_t W_t + X_t - D_t - K_{t+1} \quad (16.)$$

⁴ See Epstein and Zin (1989) and Weil (1990).

That generation's consumption in the next period (when they are retired) is equal to the transfer they receive from the young (D_{t+1}) plus capital earnings, which is equal to the marginal product of capital in time $t+1$.

$$C_{2t+1} = D_{t+1} + bN_{t+1}A_{t+1}k_{t+1}^b \quad (17.)$$

Equations (15), (16), and (17) yield a solution for C_{1t} and k_{t+1} , which determines the value of $E_t[C_{2t+1}^{1-\gamma}]$.

Calibration

Following the general process of Blanchard, I calibrate the parameters of that system to different target values of the annual risk-free interest rate r and the annual marginal product of capital R (see Table 1). Because this study considers a generation to last 25 years (following Blanchard), it raises those annual rates of return to the power of 25. The mean value of $\log A_t$, μ , is set without loss of generality to 3, following Blanchard for the Cobb-Douglas production function, and calibrated to R in the linear version.⁵ I set the growth rate g to 0 for all but one set of simulations, to be consistent with Blanchard such that $N_t = 1$ for all t . The share of capital in the production function is set to 1/3, consistent with both Blanchard and commonly used estimates in macroeconomics. The standard deviation of technological change across a generation, σ , is set to 0.2 for all sets of r and R (following Blanchard), who noted that value is between the observed volatility of productivity growth and the value implied by the equity risk premium. (The fact that the two differ substantially is the essence of the equity risk premium puzzle in Mehra and Prescott 1985.) The risk aversion parameter, γ , is a direct function of σ , R , and r .

$$\gamma = \frac{\log R - \log r}{\sigma^2} \quad (18.)$$

For the Cobb-Douglas model, I calibrate β by simulating the stochastic steady state of the economy under different values and iterating toward the one that produces an average marginal product of capital equal to the target value of R . I set β to 0.325, following Blanchard in the linear model.

Simulation of Steady State Without and With Transfers

Following calibration to each pair of values r and R , I simulate the steady-state economy under three conditions—no transfer, a risky transfer, and a risk-free transfer—while holding all the parameters fixed. I compare average lifetime utility under those three simulations.

⁵ I rely on the computer code available for Blanchard, which is publicly available at <http://doi.org/10.3886/E112856V1>, in addition to the article itself for details about Blanchard's assumptions. I adapted that code and translated it from MATLAB into Python for this study. A link to that code is posted along with this paper at www.cbo.gov/publication/58849.

1. A baseline without any transfer;
2. One with the risky transfer $D_{t+1,risky}$, as defined in equation (11); and
3. One with the risk-free transfer $D_{t+1,risk\ free}$ that is known with certainty at time t .

The estimated effect of crowding out on welfare is equal to the difference between utility under the second simulation and the first. The estimated effect of risk shifting on welfare is equal to the difference in utility under the third simulation and the second. The total welfare effect incorporating both components (and comparable to the estimates in Blanchard) is estimated as the difference between the third and first simulation.

Simulation Results

The simulations produce estimates of the total welfare effects that are reported in Figure 9 of Blanchard (see Figure 2). In this analysis (as in Blanchard's), both r and R are important in determining the welfare effect of a transfer. The effect of crowding out on welfare is a function of only R and is negative as long as R is greater than 1. The risk-shifting effect is more heavily influenced by r and is positive in all cases, offsetting the negative effect of crowding out in both Cobb-Douglas and linear production (see Figure 3 and Figure 4, respectively).

The welfare effects have the same sign and are larger without the risk-free endowment. The effects of crowding out on welfare are more negative. The risk-shifting effects are more positive, partially offsetting the increased cost of crowding out. The total effect is more negative (see Figure 5). The endowment might make the transfer less costly because it increases the income of the representative household and thus the amount that it needs to save to smooth consumption in its old age. In doing so, it raises the potential for overaccumulation of capital and lowers the welfare cost (or raises the welfare benefits) of policies that counter that overaccumulation.

The analysis suggests that the crowding out associated with debt is costly and that the risk-shifting properties of debt tend to partially offset that cost. Under that finding, high structural deficits, as projected in the United States, come at a significant cost, at least relative to a policy that issues an equal amount of debt but invests it in financial assets.⁶ The findings also confirm that risk shifting accounts for the difference between Blanchard's finding that additional debt has low cost and Abel and others' (1989) finding that the level of capital is not excessive.

⁶ Those conclusions do not apply to cyclical (demand-related) deficits. The model implicitly assumes that demand is always sufficient to reach maximal employment and thus has no implication for fiscal policy's role in supporting aggregate demand.

Sensitivity Analysis

In addition to excluding the assumption of a risk-free endowment for each generation, I study three variations on Blanchard's assumptions: alternative preferences that make intergenerational inequality more costly, persistent technology shocks, and positive economic growth.

Shifting risk to future generations can be Pareto-efficient if those generations are either less exposed to the risks facing current generations or less averse to them. If technology shocks are permanent, as discussed in Falkenheim (2021), future generations will be as exposed to them as current generations. Whether future generations are averse to those risks depends on how unhappy they are likely to be if born into a relatively unfavorable economy.

Alternative Preferences

The preferences in Blanchard represent a special case of the preferences in equation (13), one in which the parameter $\chi = 1$ (and as a result, each generation's utility) is logarithmic. As discussed in Falkenheim (2021), the parameter χ is in the optimization problem governing Pareto-efficient policy but does not enter into the decisions of each generation as it evaluates its own choices. The parameter represents how much happier each generation might have been if it had been born with a different opportunity set than the one it actually has and does not affect how that generation chooses from within the opportunity set that it does have.

The relative values of γ and χ determine whether the risk within a generation's lifetime or the inequality between generations is more important. Under a situation in which $\chi > \gamma$, inequality between generations is more costly than the risk within the lifetime of a generation. The value of γ is calibrated in this analysis to the risk premium—the difference between the marginal return on capital and the risk-free rate—and is greater than 1 in most cases. Thus, the simulations in Blanchard and those earlier in this study generally assume that risk within a generation's lifetime is more important than fluctuations in consumption between generations. In this sensitivity analysis, I examine a high value of χ where the opposite is true.

Persistent Technology Shocks

Under Blanchard, technology shocks are supposed to be temporary. Under Falkenheim (2021), the question of whether shocks are temporary or permanent is highlighted as a key determinant of which direction risk transfer would go under optimal policy. Under temporary technology shocks, Falkenheim (2021) found that Pareto-efficient policy would transfer risk from current to future generations; permanent shocks would do the opposite. In this analysis, I consider the possibility that $\rho = 0.9$ and thus that technology shocks are nearly permanent.

Results

The estimated welfare effects are sensitive to assumptions about preferences and technology. In particular, the welfare cost of transfers is larger in a context with high aversion to risk to lifetime consumption, as measured by the parameter χ . Under a high value of χ , the welfare effect of debt

is uniformly more negative than under a logarithmic ($\chi = 1$) specification (see Figure 6). Under that logarithmic specification, the welfare effects of transfers are no different with persistent technology shocks than they are with temporary ones (see Figure 7). However, the welfare cost of debt is higher under a combination of high values of χ and high values of ρ than under either alone (see Figure 8). In other words, debt is the costliest under a combination of persistent technology shocks that affect future generations more heavily and preferences that make future generations more sensitive to those risks. In sum, excluding the endowment, raising aversion to lifetime consumption risk (raising χ) and making technology shocks persistent (setting $\rho = 0.9$) raises the welfare cost of debt by a factor of about four from the levels in Blanchard (see Figure 9). Under those assumptions, shifting risk to future generations is almost always costly (see Figure 10). That is, instead of offsetting the negative effects on welfare of crowding out, risk shifting adds to them.

Positive Economic Growth

Following Blanchard, the preceding analysis set the growth rate to zero and used the terms R and r to represent excesses of the marginal product of capital and risk-free rate over the growth rate, that is $(R - g)$ and $(r - g)$. Blanchard omitted the growth rate entirely from his equations. Blanchard (2022) presents a version of the same equations with population and technological growth to demonstrate that many of the principles in the 2019 paper held with positive growth. The steady-state welfare effects for any given levels of $(R - g)$ and $(r - g)$ are slightly more negative when g is set to a positive level than when it is set to zero (see Figure 11). To calibrate to target values of $(R - g)$ and $(r - g)$, the parameter β rises with g , raising savings and offsetting the effect of growth in effective labor on the capital-to-labor ratio.

The analysis in this study, following Blanchard, compares the utility in the steady state after transition has taken place and does not consider effects on the utility to current generations or other generations in the relatively near future on the transition path to a new steady state. Although different growth rates might not produce much difference in steady-state welfare effects (holding $(R - g)$ and $(r - g)$ constant), different growth rates might have different implications for the trade-off between higher consumption in the near term and lower consumption in the long run. Under the model in this study, higher growth rates can come from increases in population growth or in the growth of the level of effective labor per worker, which represents the trend of growth in technological productivity.

Under a higher growth rate of technological productivity, the per capita consumption of generations in the future steady state will be relatively high. As a result, their marginal utility of consumption will be relatively low, as long as they are not saddled with so much debt from current generations so as to make their consumption lower. Saddling those generations with more debt will have a smaller negative effect on their utility if growth is positive than if it is zero. More growth thus implies that an increase in debt will have lower costs to future generations to offset its benefits to current generations.

Figures

Figure 1. Nominal Growth Rate of the U.S. Economy and Effective Interest Rate on Federal Debt (Percent)

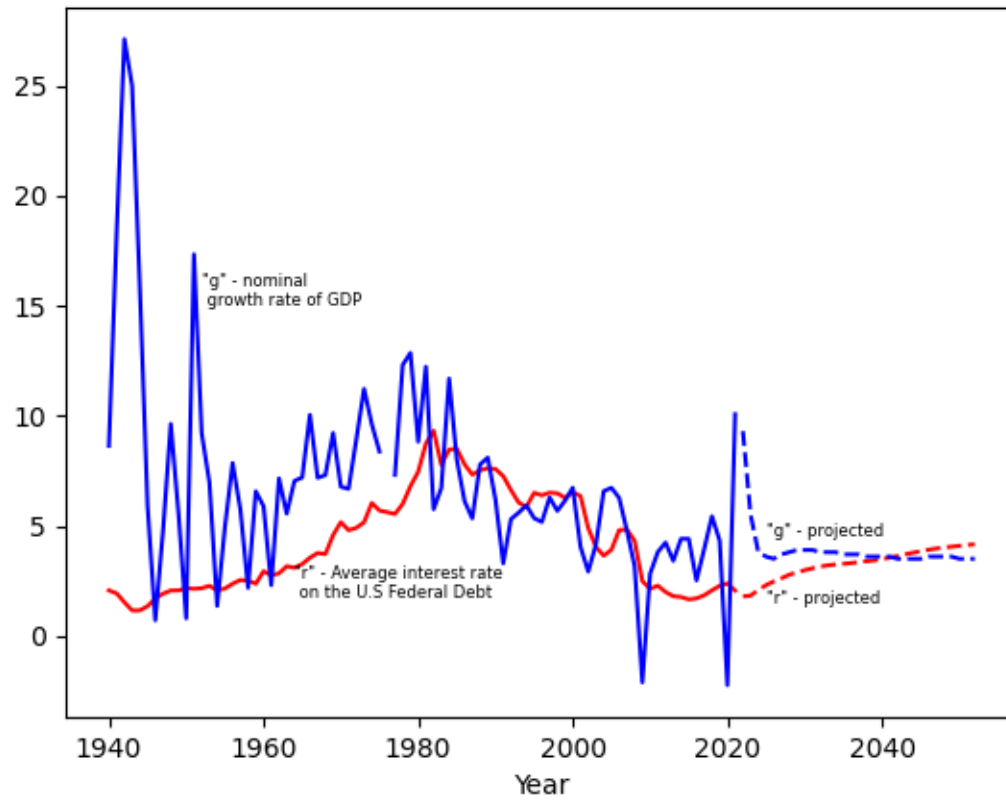
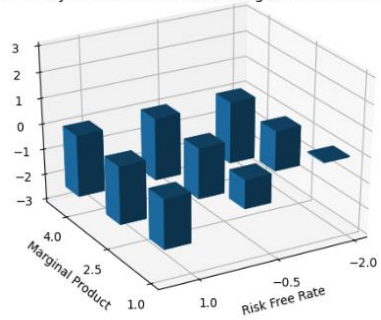
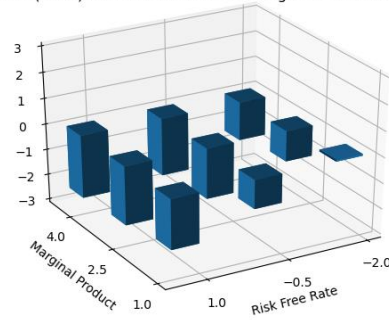


Figure 2. Effects of a 20 Percent Transfer on Lifetime Utility: Blanchard (2019) Versus Current Study
 (Percentage change)

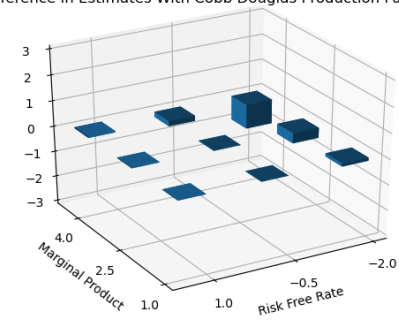
Current Study Estimates With Cobb Douglas Production Function



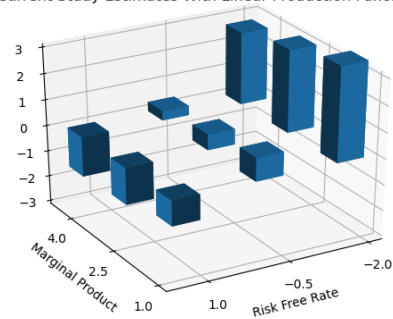
Blanchard (2019) Estimates With Cobb Douglas Production Function



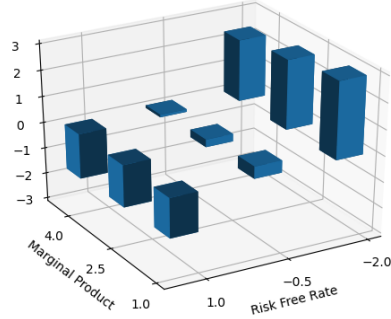
Difference in Estimates With Cobb Douglas Production Function



Current Study Estimates With Linear Production Function



Blanchard (2019) Estimates With Linear Production Function



Difference in Estimates With Linear Production Function

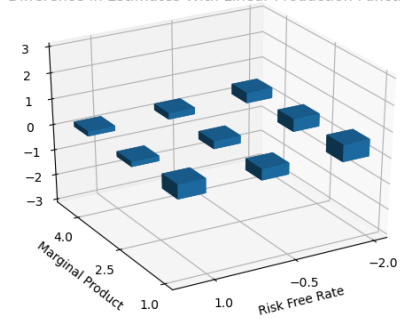


Figure 3. Effects of a 20 Percent Transfer on Lifetime Utility Under a Cobb-Douglas Production Function, Without Endowment
 (Percentage change)

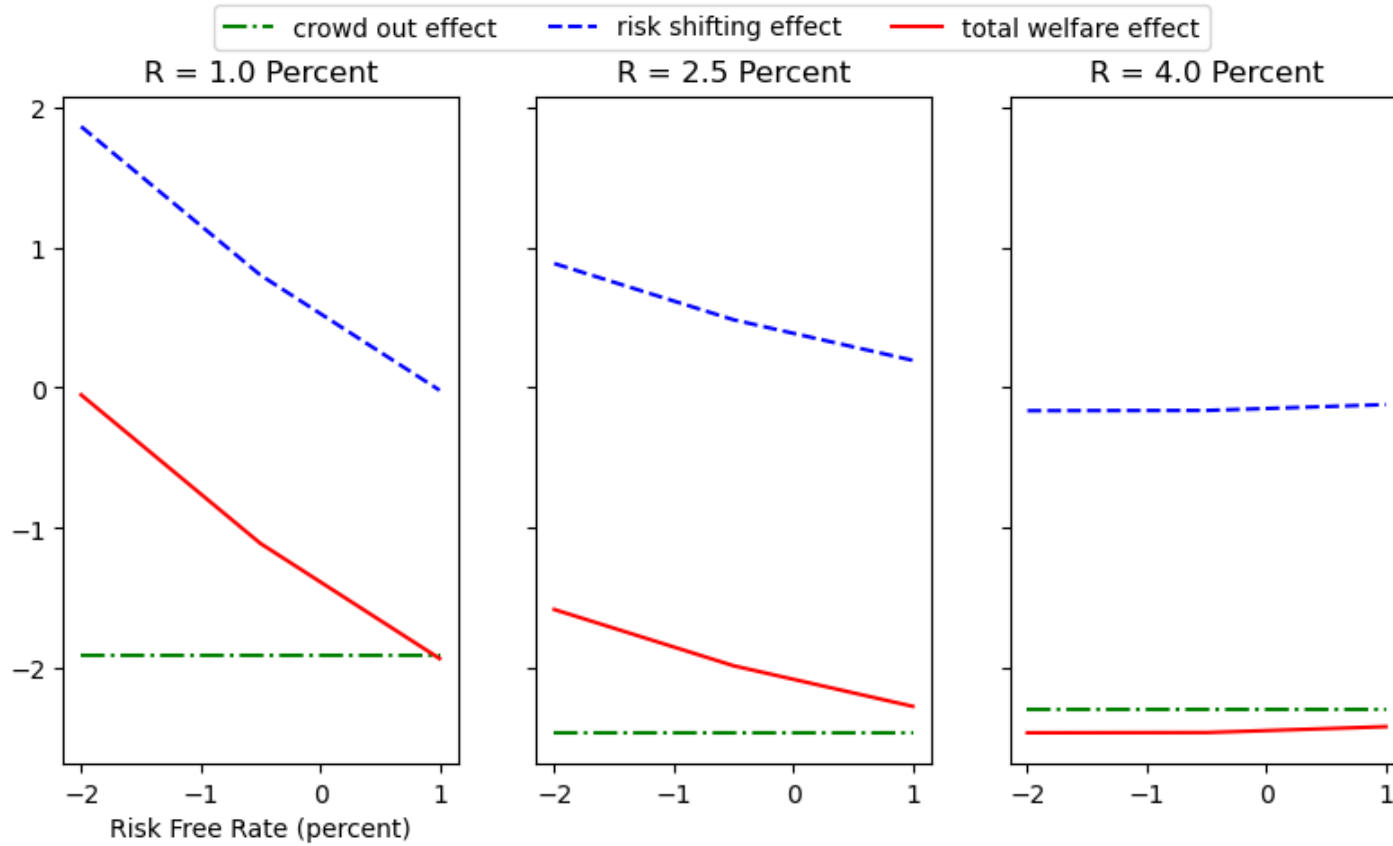


Figure 4. Effects of a 20 Percent Transfer on Lifetime Utility Under a Linear Production Function, Without Endowment
(Percentage change)

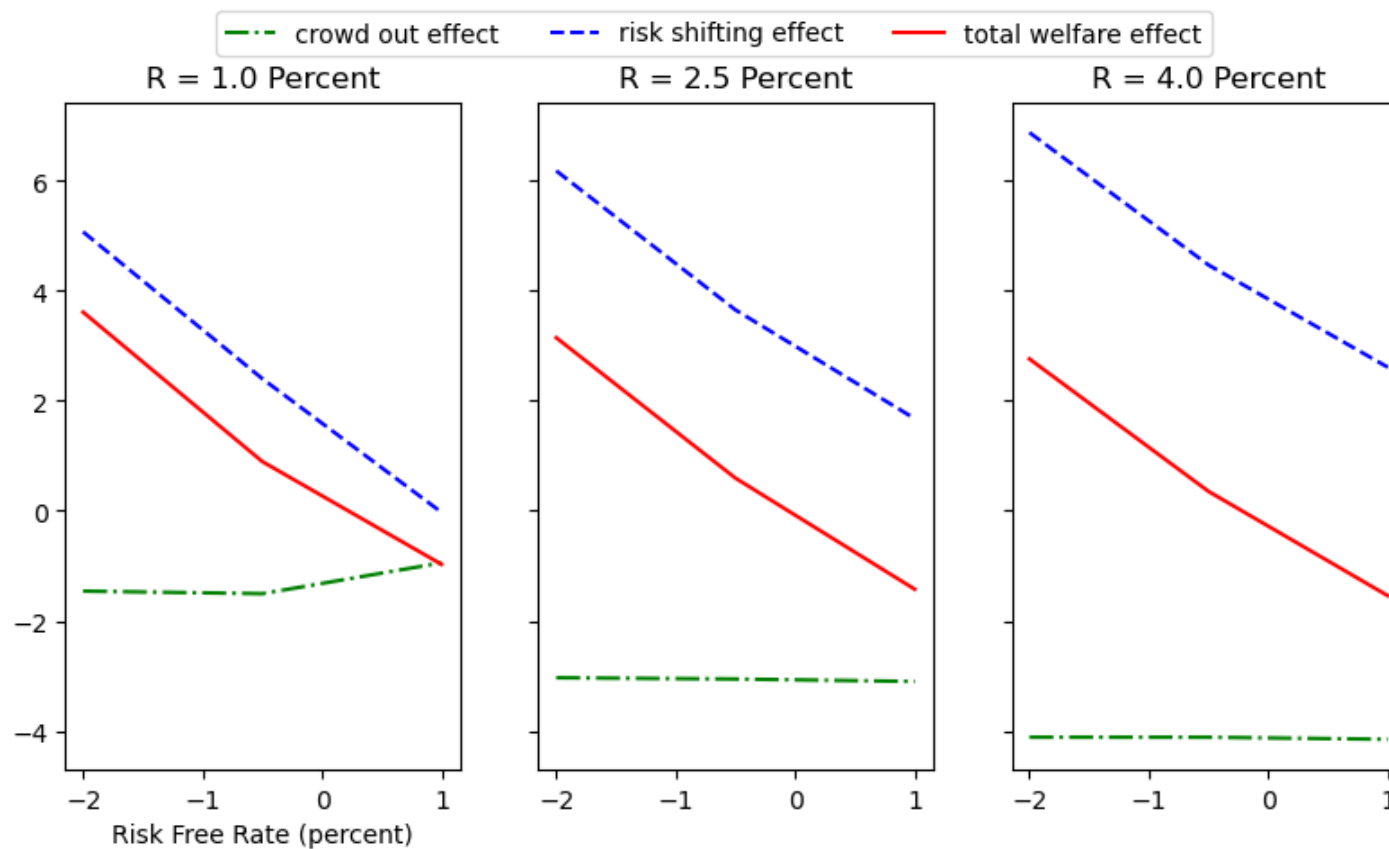


Figure 5. Effects of a 20 Percent Transfer on Lifetime Utility, With and Without Risk-Free Endowment (Percentage change)

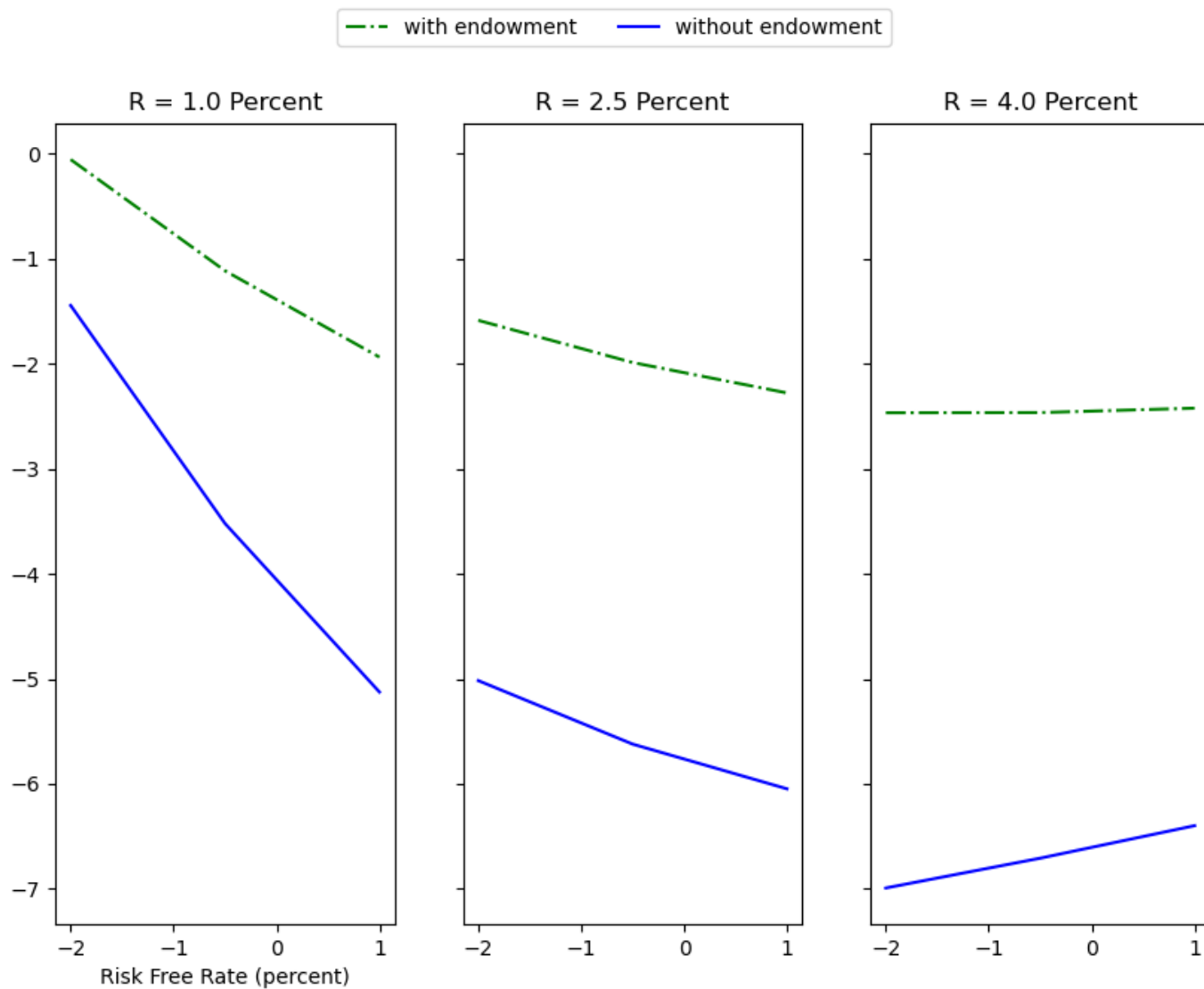


Figure 6. Effects of a 20 Percent Transfer on Lifetime Utility by Level of Aversion to Intergenerational Risk

(Percentage change)

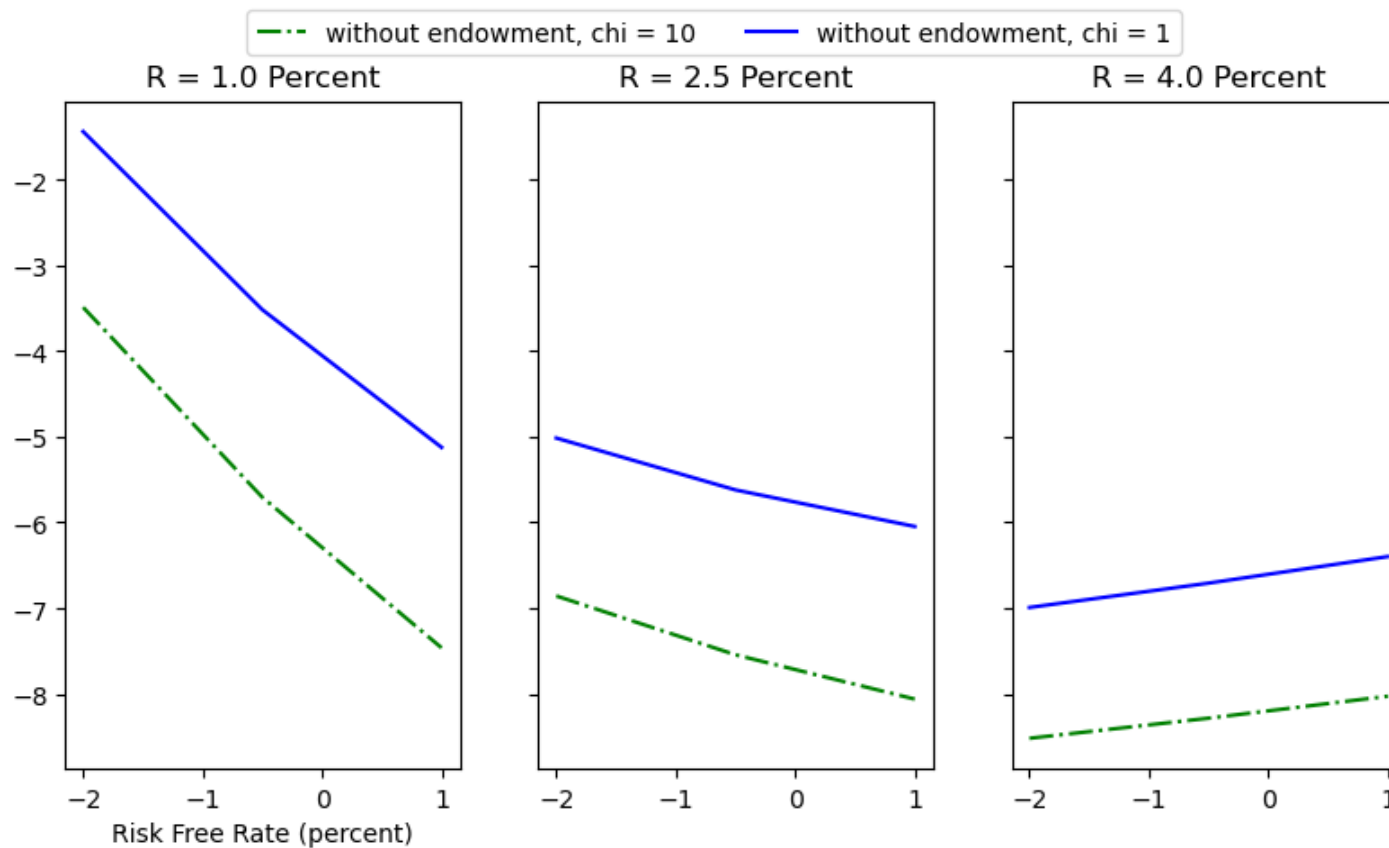


Figure 7. Effects of a 20 Percent Transfer on Lifetime Utility by Level of Persistence of Technology Shocks (Percentage change)

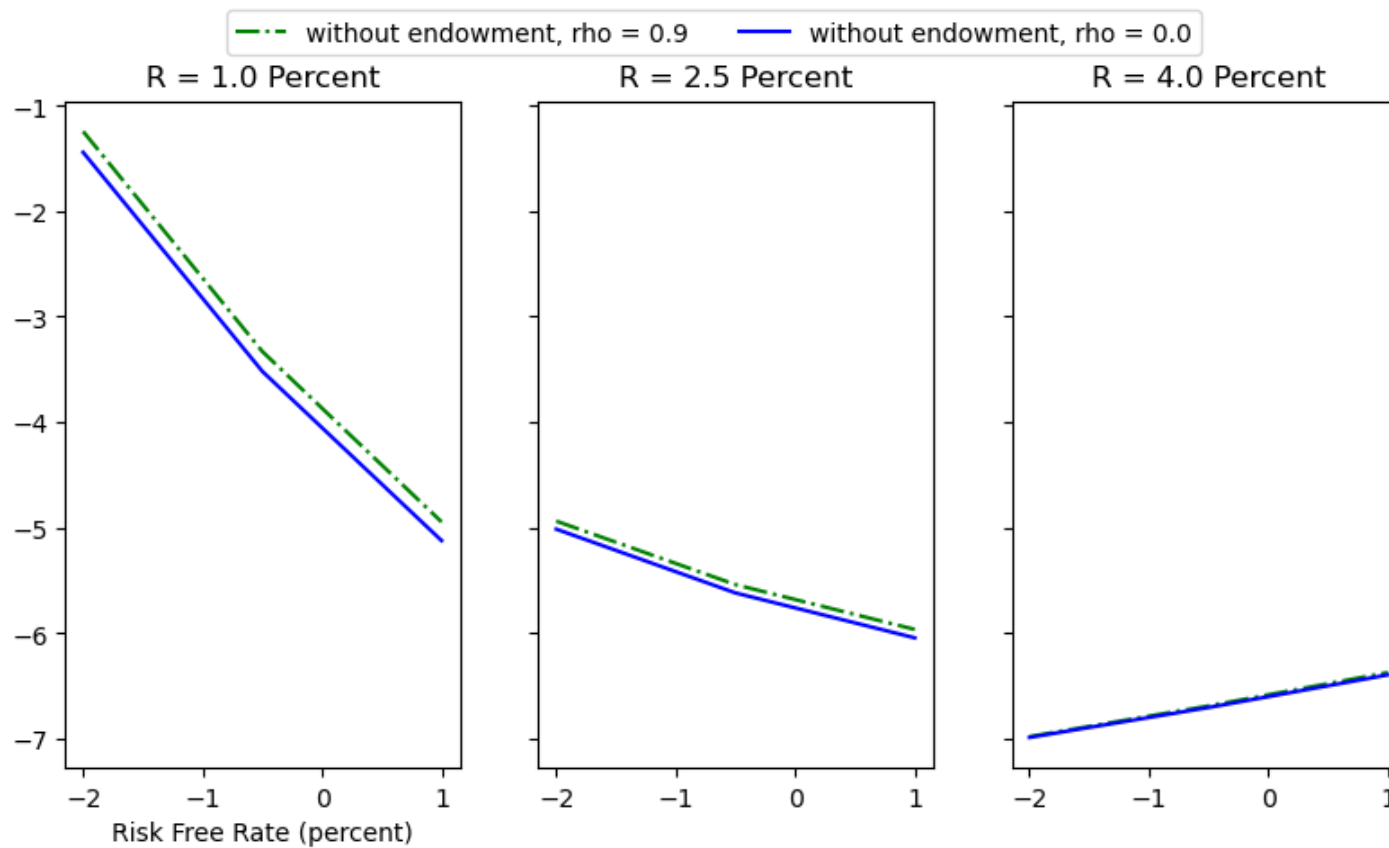


Figure 8. Effects of a 20 Percent Transfer on Lifetime Utility by Level of Persistence of Technology Shocks Under High Aversion to Intergenerational Risk (Percentage change)

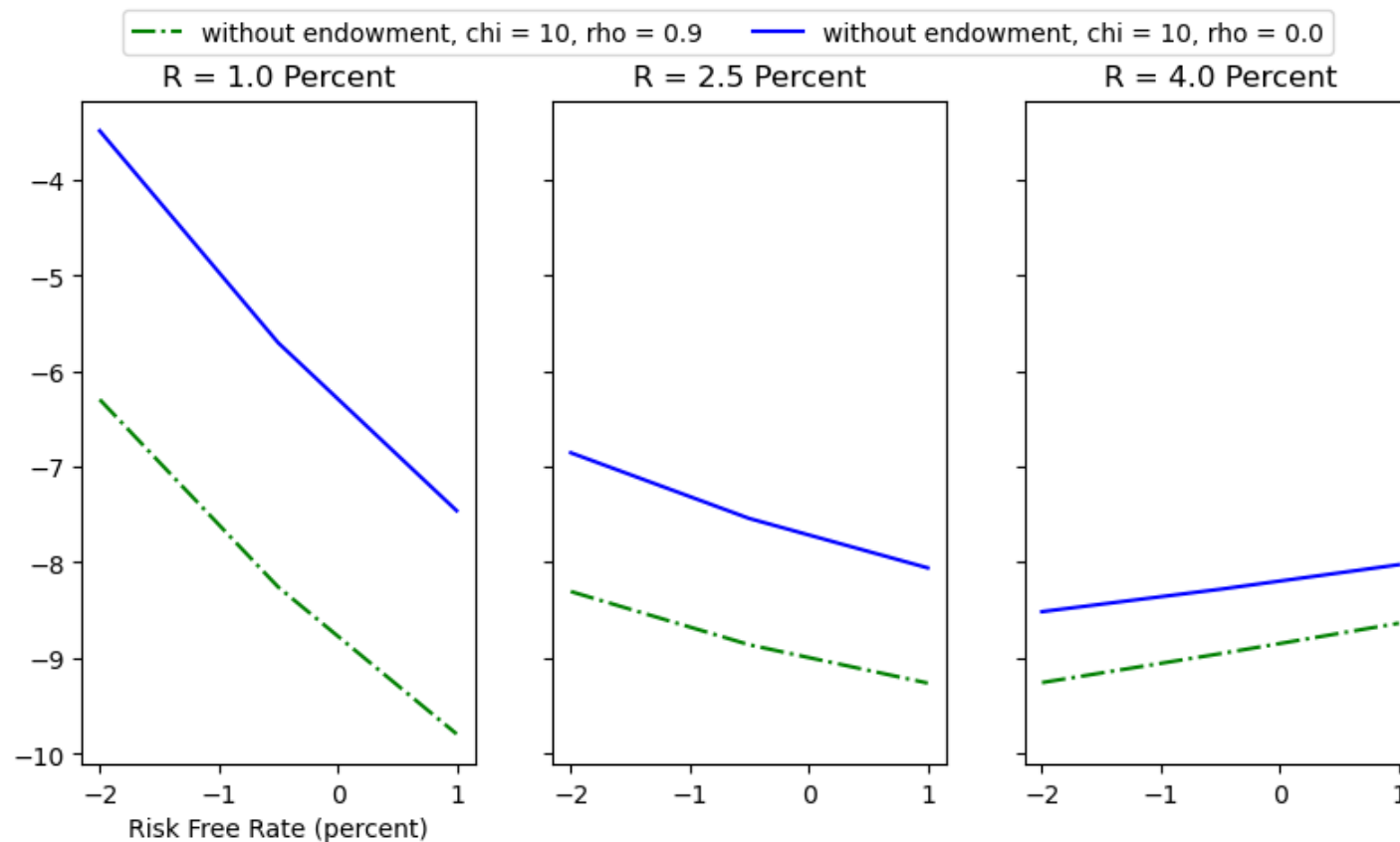


Figure 9. Effects of Transfers on Lifetime Utility Under Different Assumptions

(Percentage change)

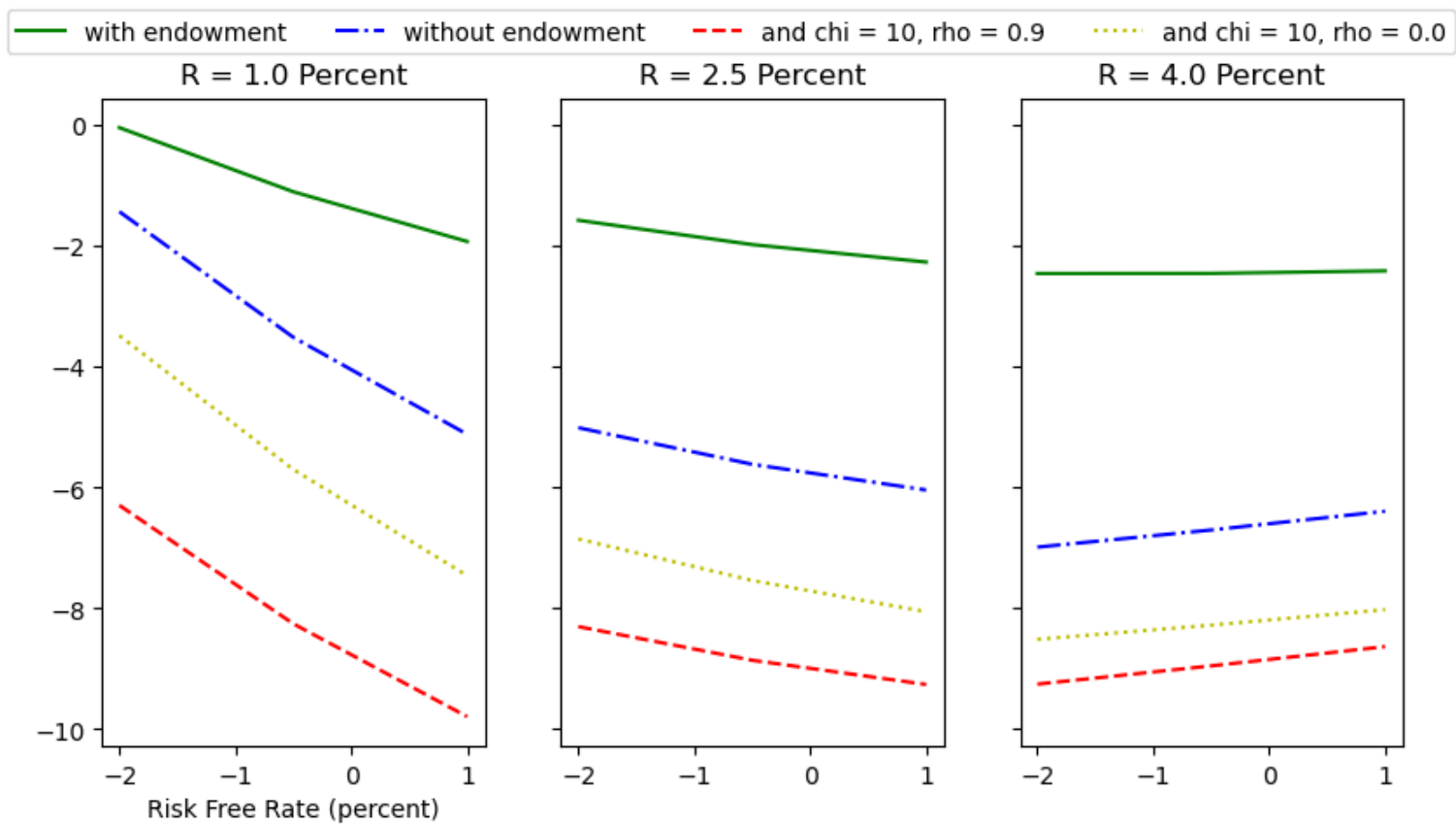


Figure 10. Decomposition of Effects on Lifetime Utility Without Endowment, With Higher Persistence of Technology Shocks and High Aversion to Intergenerational Risk (Percentage change)

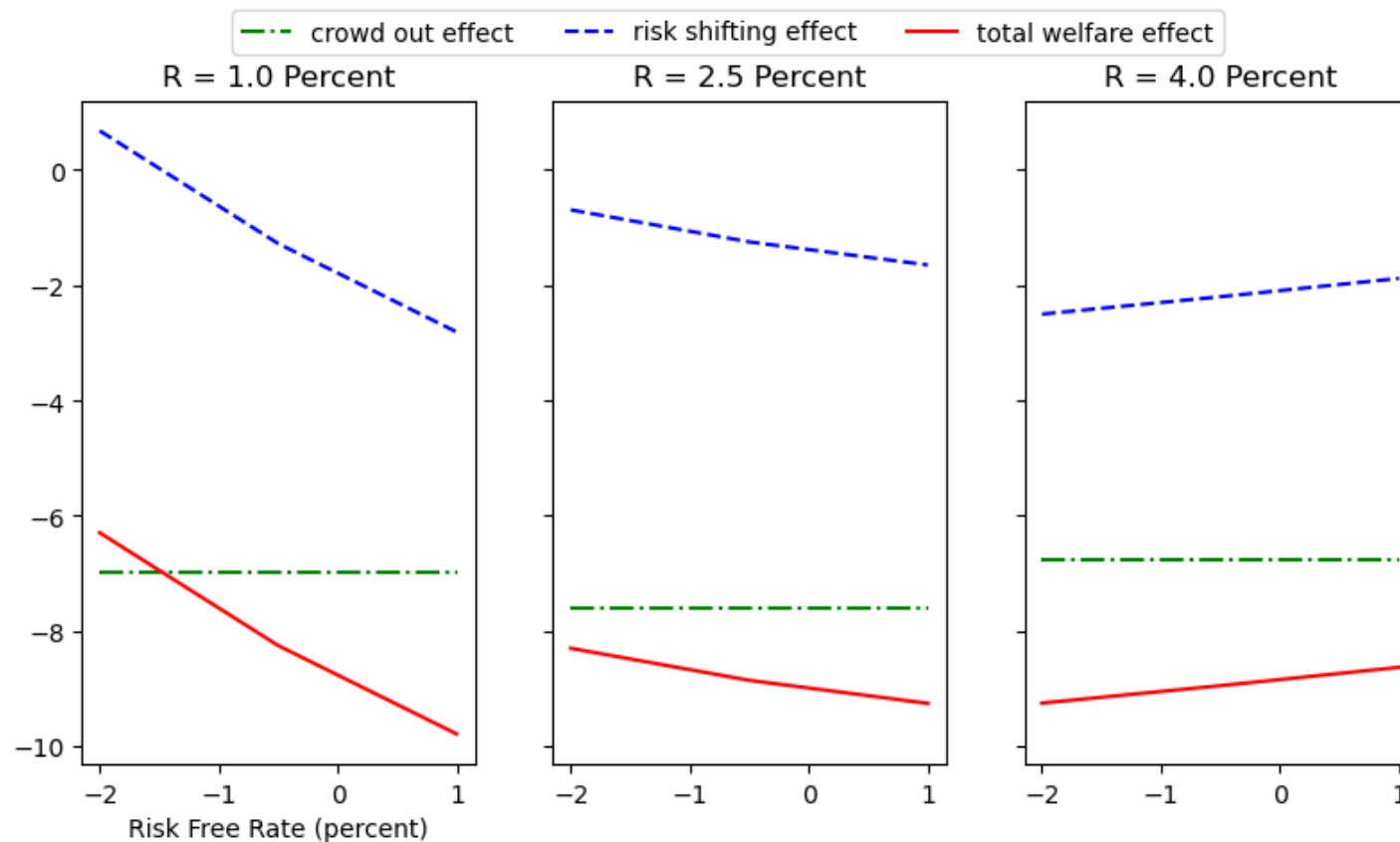
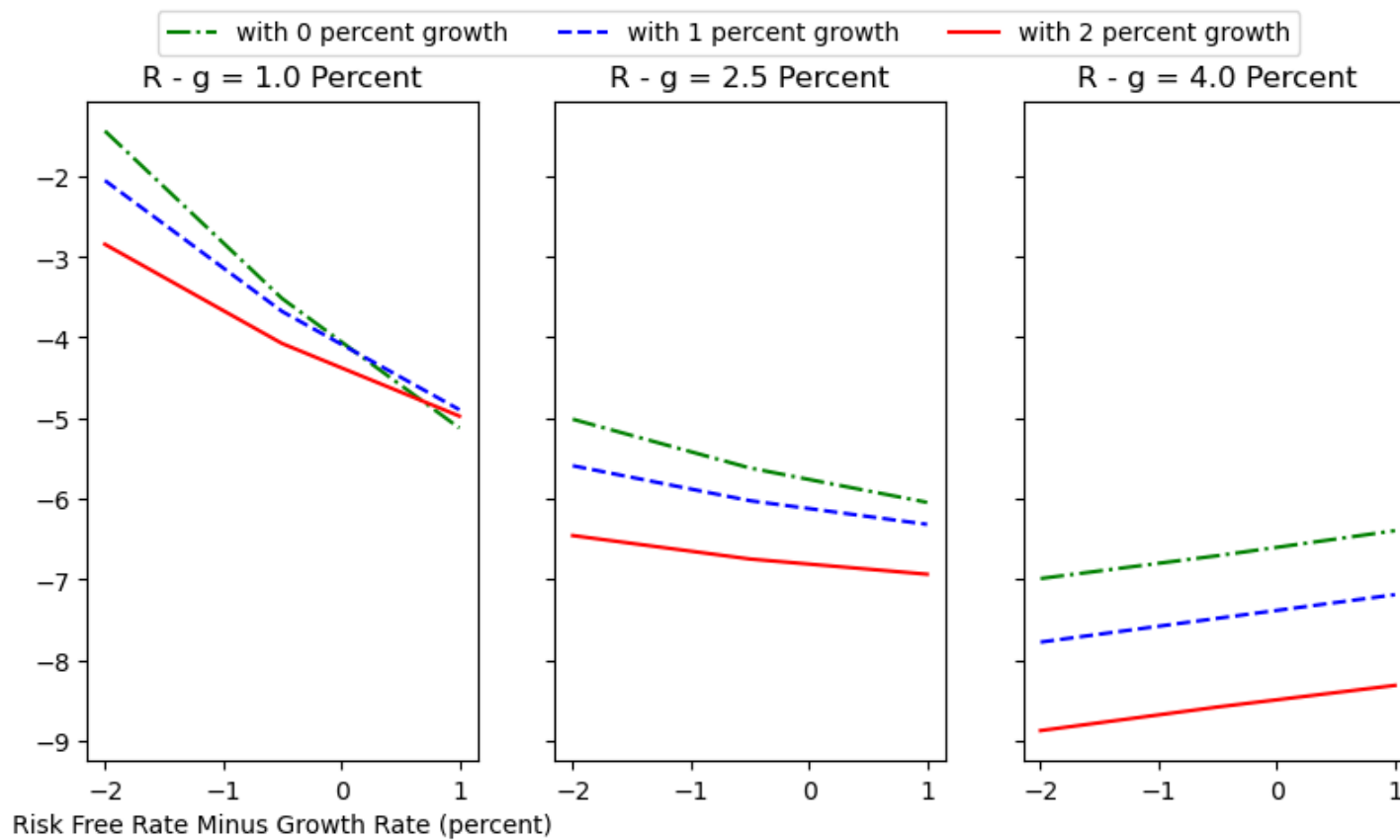


Figure 11. Effects of a 20 Percent Transfer on Lifetime Utility, by Growth Rate (Percentage change)



Table

Table 1. Process for Calibrating Parameters in This Analysis

Parameter	Definition	Process for Setting Value
<i>Calibration Targets</i>		
r	Risk-free interest rate	Calibrated to 0.98, 1.005, and 1.02
R	Marginal product of capital	Calibrated to 1.01, 1.025, and 1.04
<i>Exogenous Parameters</i>		
g	Growth rate in effective labor	Set to 0 except for in sensitivity analysis
μ	Expected log of total factor productivity (A_t)	Set without loss of generality to 3 for Cobb-Douglas and calibrated to R in linear model
σ	Standard deviation of $\log A_t$	Set to 0.2 as discussed in Blanchard 2019
b	Capital's share of income	Set to 1/3 as discussed in Blanchard 2019
χ	Aversion to risk in lifetime consumption	Set to 1 for base simulations (consistent with Blanchard's log specification) and to 10 for sensitivity analysis
<i>Calibrated Parameters</i>		
γ	Level of aversion to consumption risk in old age	Set by formula as a function of σ , R , and r
β	Weight on utility in old age	Calibrated on the basis of simulation of the steady state in Cobb-Douglas and set to 0.325 (following Blanchard) in linear model
α	Transfer as a share of steady-state wages	Calibrated on the basis of simulation of the steady-state economy

Appendix: Calculation of Derivatives With Respect to Transfers and Swaps

Blanchard (2019) found that the derivative of lifetime utility with respect to the size of the transfer was:

$$dU = \{-(1 - \beta)U'(C_{1t}) + \beta E_t[U'(C_{2t+1})]\}dD + \{(1 - \beta)U'(C_{1t})dW_t + \beta K_{t+1}E_t[U'(C_{2t+1})]dR_{t+1}\} \quad (\text{A.1})$$

He defined the first term $\{-(1 - \beta)U'(C_{1t}) + \beta E_t[U'(C_{2t+1})]\}dD$, dU_a and noted that it represented the direct effect of the transfer on utility. He called the second term $\{(1 - \beta)U'(C_{1t})dW_t + \beta K_{t+1}E_t[U'(C_{2t+1})]dR_{t+1}\}$, dU_b and noted that it was the indirect effect of the transfer on utility through its effects on relative prices.

Under the equation for the risk-free rate,

$$(1 - \beta)U'(C_{1t}) = r\beta E_t[U'(C_{2t+1})] \quad (\text{A.2})$$

And, therefore, the direct term equals:

$$(1 - r)\beta E_t[U'(C_{2t+1})]dD \quad (\text{A.3})$$

The derivative of lifetime utility with respect to the size of a risky transfer D_V in the current study is

$$dU = \left\{-(1 - \beta)\frac{A_t}{\bar{A}}U'(C_{1t}) + \beta E_t\left[\frac{A_{t+1}}{\bar{A}}U'(C_{2t+1})\right]\right\}dD_V + \{(1 - \beta)U'(C_{1t})dW_t + \beta K_{t+1}E_t[U'(C_{2t+1})]dR_{t+1}\} \quad (\text{A.4})$$

The indirect effect has the same expression as the indirect effect of a risk-free transfer. The direct effect is different. Given that the marginal rate of return on capital directly varies with the level of technological productivity A_{t+1} , $R_{t+1} = bA_{t+1}K_{t+1}^{b-1}$, I define $\bar{R} \equiv b\bar{A}K_{t+1}^{b-1}$ and note that $\frac{A_t}{\bar{A}} = \frac{R_{t+1}}{\bar{R}}$. The direct effect of an increase in the size of the risky transfer D_V becomes:

$$\left\{-(1 - \beta)\frac{A_t}{\bar{A}}U'(C_{1t}) + \beta E_t\left[\frac{R_{t+1}}{\bar{R}}U'(C_{2t+1})\right]\right\}dD_V \quad (\text{A.5})$$

Under the Euler equation $(1 - \beta)U'(C_{1t}) = \beta E_t[R_{t+1}U'(C_{2t+1})]$, the expression for that effect becomes:

$$\beta E_t[R_{t+1}U'(C_{2t+1})]\left[\frac{1}{\bar{R}} - \frac{A_t}{\bar{A}}\right]dD_V \quad (\text{A.6})$$

Given that a risky transfer and a swap combine to make a risk-free transfer, the derivative of utility with respect to a swap is dU_{St} , $dU_{St} = dU_{at} - dU_{vt}$. The direct effect of a swap on lifetime utility is thus:

$$\beta \left\{ (1-r)E_t[U'(C_{2t+1})] - E_t[R_{t+1}U'(C_{2t+1})] \left[\frac{1}{\bar{R}} - \frac{A_t}{\bar{A}} \right] \right\} dU_{St} \quad (\text{A.7})$$

Given that $E_t[R_{t+1}U'(C_{2t+1})] = rE_t[U'(C_{2t+1})]$, this expression can be simplified in the following steps:

$$\begin{aligned} & \beta \left\{ (1-r)E_t[U'(C_{2t+1})] - rE_t[U'(C_{2t+1})] \left[\frac{1}{\bar{R}} - \frac{A_t}{\bar{A}} \right] \right\} \\ &= \beta E_t[U'(C_{2t+1})] \left\{ (1-r) - r \left[\frac{1}{\bar{R}} - \frac{A_t}{\bar{A}} \right] \right\} \\ &= \beta E_t[U'(C_{2t+1})] \left\{ 1 + \frac{A_t}{\bar{A}} - r \left(\frac{\bar{R} + 1}{\bar{R}} \right) \right\} \end{aligned}$$

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