ESTIMATING OUTLAYS FOR THE INTEREST ON THE PUBLIC DEBT

Technical Analysis Paper

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Congress of the United States
Congressional Budget Office
Washington, D.C.

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PREFACE

As part of its responsibilities under Sections 308(b) and (c) of the Congressional Budget Act (Public Law 93-344), the Congressional Budget Office (CBO) periodically issues scorekeeping estimates and five-year projections for interest on the public debt. This paper presents the econometric model CBO uses in arriving at these budget estimates.

The total interest-bearing public debt is composed of several different kinds of securities. The model estimates the interest cost for each type of security in order to reach its estimate of total interest cost. It employs certain assumptions about the composition of the debt outstanding and regression equations for the effective interest rate for each type of security. Estimated interest cost for each type of security is calculated as the product of the debt outstanding times the effective interest rate.

The model was formulated and the paper written by Daniel L. Rubenson, formerly of the Budget Analysis Division of CBO, with the assistance of James Capra. Darrel Cohen, formerly of the Fiscal Analysis Division, reviewed the paper and gave many useful suggestions. Johanna Zacharias and Robert L. Faherty edited the manuscript, which was typed for publication by Paula Spitzig.

Alice M. Rivlin
Director

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CHAPTER I. ESTIMATION OF THE SIZE AND COMPOSITION OF THE PUBLIC DEBT

The interest on the public debt represents the interest payments that must be made to holders of federal debt securities. As such, it includes payments to federal agencies (such as trust funds) as well as to the public. In fiscal year 1976, the total cost of this interest amounted to $37.1 billion, roughly 10 percent of total unified budget outlays for the year. Because of the magnitude of this sum, estimating the interest cost and the interest cost implications of larger or smaller deficits is an important element of budget analysis.

The total interest-bearing public debt is not composed of just one type of security. Rather, it contains different instruments that vary according to maturity and other characteristics. The most general breakdown of the total debt is into marketable and nonmarketable securities. Marketable securities can be transferred between individual investors on the open market. 1/ Nonmarketable securities can only be bought from and sold to the U. S. Department of the Treasury.

Each of these two categories can be subdivided into specific classes of debt. The marketables are made up of bills, notes, and bonds. These three have maturities of less than one year, one to ten years, and more than seven years, respectively. The nonmarketables consist of special issues and all other nonmarketables. Special issues are securities sold to trust funds for the purpose of investing their surpluses. Other nonmarketables are savings bonds, foreign government series, and other miscellaneous securities. Savings bonds make up roughly 70 percent of this group.

Table 1 presents a breakdown of the interest-bearing debt into outstanding amounts at the end of fiscal years 1966 and 1976. As the table shows, the composition of the debt has changed significantly over time: bills and notes have increased dramatically; bonds as a financing tool have declined in popularity.

Table 2 shows a breakdown of the interest cost in fiscal years 1966 and 1976 by type of security. The trends between the two years in this table roughly follow the changes in debt composition illustrated in Table 1. They do not correspond exactly because there are different effective interest rates for different securities.

1/ Throughout this paper, the Federal Reserve is treated as an individual private investor.

TABLE 1. INTEREST-BEARING PUBLIC DEBT BY TYPE OF SECURITY: BY FISCAL YEARS, IN BILLIONS OF DOLLARS

	1966	Percent of Total	1976	Percent of Total
Special Issues	51.1	16.2	130.6	21.1
Other Nonmarketables a/	56.8	18.0	96.1	15.5
Total Nonmarketable	(107.9)	(34.2)	(226.7)	(36.6)
Outstanding Bonds	101.9	32.3	39.6	6.4
Outstanding Notes	50.6	16.1	191.8	31.0
Outstanding Bills	54.9	17.4	161.2	26.0
Total Marketable	(207.5)	(65.8)	(392.6)	(63.4)
TOTAL Interest-bearing Public Debt b/	315.4	100.0	619.2	100.0

a/ Includes certificates of indebtedness in 1966.

b/ Components may not add to totals due to rounding.

TABLE 2. INTEREST ON THE PUBLIC DEBT BY TYPE OF SECURITY: BY FISCAL YEARS, IN BILLIONS OF DOLLARS

Security	1966	Percent of Total	1976	Percent of Total
Special Issues	1.7	13.8	7.8	21.2
Other Nonmarketables a/	2.1	17.4	5.6	15.1
Bonds	3.8	31.5	2.1	5.7
Notes	2.0	16.8	12.3	33.2
Bills	2.5	20.6	9.2	24.8
TOTAL b/	12.0	100.0	37.1	100.0

a/ Includes certificates of indebtedness in 1966.

b/ Components may not add to total due to rounding.

THE CBO MODEL FOR ESTIMATION

Estimating the total interest cost involves several steps. First, the size and composition of the interest-bearing public debt must be projected. With this done, effective interest rates can be computed, based on an exogenous macroeconomic forecast. Finally, the vectors of outstanding securities and interest rates are multiplied to yield the total interest cost.

In order to calculate the total debt, the existence of several identities must be assumed. Trust fund holdings of cash and of agency debt are assumed to remain constant, as are federal fund and off-budget holdings of Treasury debt. Given this, the total debt grows according to the following relationships (expressed in end-of-year terms).

(1) Total Treasury Debt = Publicly Held Treasury Debt
+ Federally Held Treasury Debt

(2) Publicly Held Treasury Debt = Publicly Held Treasury Debt_{-1}
+ Borrowing from the Public - Change in Agency Debt
Held by the Public

(3) Borrowing from the Public = Unified Budget Deficit
+ Off-Budget Deficit + Other Means

(4) Federally Held Treasury Debt = Federally Held Treasury Debt_{-1}
+ Combined Trust Fund Surplus

Making the assumptions that the change in agency debt and the change in non-interest-bearing debt will equal zero, equations (1) through (4) can be combined to yield:

(5) Interest-Bearing Debt = Interest-Bearing Debt_{-1}
+ Unified Budget Deficit + Off-Budget Deficit
+ Combined Trust Fund Surplus + Other Means.

The unified and off-budget deficits and the trust fund surplus are expressed as absolute magnitudes and are exogenous inputs to the interest-estimating model. "Other Means" in equations (3) and (5) represents the net total of other modes of financing, changes in Treasury cash balances, and other related mechanisms. Other Means is assumed to sum to zero over time but can take on large positive or negative values in any year to incorporate formally into the model subjective assessments of the Treasury Department's financing strategy.

Using equation (5) over the 1960 through Transition Quarter interval, with Other Means equal to zero, the total debt is calculated to be $634.7 billion — $1.1 billion higher than the actual figure. This represents an error of 0.3 percent in the total change in outstanding debt. Therefore, the assumption that Other Means is equal to zero over time is supported by this test.

A similar test was conducted for equation (3) to validate its assumptions, and with equal success. Both of these tests confirm the set of assumptions incorporated into equations (1) through (5).

Exogenous inputs of Treasury financing requirements, such as the deficits and surpluses, are estimated as annual (fiscal year) figures. The model, however, produces quarterly estimates of the interest cost. Therefore, some assumption of Treasury's seasonal financing patterns must be explicitly incorporated into the model. At present, the total debt is assumed to grow at a constant rate during each fiscal year. Variations to this assumption can be incorporated into the estimating procedure in any quarters in which they are deemed appropriate.

Because the total interest-bearing debt is not homogenous, it must be broken down by type of security before estimates of the interest cost can be made. The categories used are the same as those in Tables 1 and 2: special issues, other nonmarketables, bonds, notes, and bills.

SPECIAL ISSUES

Special issues are nonmarketable securities issued to trust funds and government agencies. As noted in Chapter I, the econometric model assumes that trust fund holdings of other securities and cash remain constant; therefore, trust fund surpluses or deficits translate directly into changes in the amount of outstanding special issues. Analysis of historical data indicates that the growth in special issues exhibits a stable seasonal pattern. Testing with the X-11 technique yields an F-statistic of 144.9, which is clearly greater than the critical value of 4.1 necessary to demonstrate stable seasonality at the 1 percent level. The model therefore projects growth in outstanding special issues to have a seasonal component.

Estimation of the interest cost from special issues is done by multiplying the average value of outstanding special issues in any quarter by the average effective interest rate. The average amount outstanding is computed as 0.974 times the arithmetic mean of the start-of-quarter and end-of-quarter figures. The 0.974 adjustment, which is based on analysis of historical data, is required because the timing of certain benefit payments causes outstanding special issues to average slightly higher at the end of each month than during the month.

The average effective interest rate for special issues is determined by the following linear regression equation fit on historical data: 1/

$$\text{(6)} \quad RSI = \underset{(1.663)}{0.07132} + \underset{(33.23)}{0.9347} \times RSI_{-1} - \underset{(-1.939)}{0.01443} \times RU + \underset{(3.138)}{0.01454} \times RCPI + \underset{(2.202)}{0.04767} \times \overline{RAAA}$$

Interval: Quarterly, 1960-1976
R-Bar Squared = 0.9978
Durbin-Watson = 1.9458
Durbin h = 0.2297
Standard Error = 0.06600

RSI = Effective rate on special issues
RU = Unemployment rate
RCPI = Quarter-to-quarter percent change in the consumer price index
$\overline{RAAA}$ = Four-quarter moving average of the Moody's AAA corporate bond rate

1/ Throughout this paper, t-statistics are shown in parentheses.

The corporate bond rate is used explicitly to capture the movements of the middle- to long-term financial markets. The change in consumer prices is a proxy for inflationary expectations, and the unemployment rate is a proxy for demand-side pressures in the economy that can push interest rates up or down. All coefficients are clearly of the correct sign.

A lagged dependent variable is used in this equation to capture the structure of a partial adjustment model. Fluctuations in market interest rates affect new special issues, but these marginal changes are slow to transfer fully into changes in the average rate. This functional form, called the Koyck lag, puts geometrically declining weights on all the independent variables. Their effect therefore fades smoothly over time rather than disappearing abruptly after one or two quarters. 2/

OTHER NONMARKETABLE SECURITIES

Other nonmarketable securities make up a mixed category. As mentioned earlier, roughly 70 percent of this group is savings and retirement bonds, and most of the remainder is miscellaneous foreign accounts series. Rather than trying to predict savings bond sales and redemptions, the model assumes that this category will grow at the same rate as the total interest-bearing public debt. Therefore, other nonmarketables as a percent of the total will be constant in all projected periods. Analysis of the historical data supports this assumption. The percentage has remained quite constant over time, with a mean of 17.4 percent and a standard deviation of 1.3 percent. Furthermore, any changes in the percentage have been slow rather than erratic.

Unfortunately, the published average interest rate for savings bonds is not accurate for calculating the interest cost, so an equation for this rate cannot be estimated and used. Instead, an equation is used to compute directly the accrued interest on this category. Because savings bonds make up the bulk of other nonmarketables, the entire category is estimated in this way.

Savings bonds accrue interest every six months after their issue. It is reasonable therefore to assume that the interest on savings bonds is a function of the interest lagged two quarters and the increase in outstanding savings bonds two quarters ago. 3/ In addition, a linear time trend is used as

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2/ See J. Johnston, Econometric Methods, Second Edition (New York: McGraw-Hill Book Company, 1972).

3/ The methodology used in this section is based on that described in Office of Management and Budget, "The OMB Model to Project Interest on the Public Debt," Technical Paper series BRD/FAB 75-5, 1975.

a proxy to capture the slowly changing mix of nonmarketables from savings bonds towards foreign series. This proxy is used as an explanatory variable to eliminate the need to predict separately what increases may occur in outstanding foreign series securities.

(7) $$\underset{}{INM} = \underset{(-2.811)}{-0.6058} + \underset{(6.583)}{0.7087} \times INM_{-2} + \underset{(1.978)}{0.009250} \times (NM_{-2} - NM_{-3})$$
$$+ \underset{(2.947)}{0.009093} \times \text{Time}$$

Interval: Quarterly, 1967-1977:1
R-Bar Squared = 0.9738
Durbin-Watson = 1.5014
Standard Error = 0.05745

INM = Accrued interest on other nonmarketables
NM = Value of other nonmarketables outstanding
Time = Linear time trend

TREASURY BONDS

Treasury bonds are marketable securities with maturities greater than seven years. The Second Liberty Bond Act of 1917 put a 4.25 percent interest rate ceiling on publicly held bonds, and as market rates have risen above this level Treasury bonds have declined in usefulness as a financing tool. Subsequent amendments to the act, the most recent being the Public Debt Limit Bill (Public Law 94-334), have granted exceptions to the ceiling for fixed quantities of bonds. CBO's current policy assumption is that Treasury bonds will continue to be constrained by the act, and that the average effective interest rate on all bonds is maintained at its most recent level, but it can be adjusted to include the effects of recent bond issues.

The interest cost on bonds is calculated as the product of the average value of bonds outstanding during a quarter and the average effective interest rate. 4/

Since the total interest-bearing public debt is equal to the sum of bills, notes, bonds, special issues, and other marketables, the bill and note

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4/ This method, used for bonds and notes, excludes any effect on the interest cost of discounts and premiums. These errors are very small in magnitude and largely cancel each other out.

portion can be computed at this point as a residual. Splitting up this residual into its two components is largely a matter of judgment. Because bills have shorter maturities than notes and tend to have lower interest rates, the bill-to-note ratio is an important element of Treasury financing strategy. A least-cost strategy would find the Treasury Department leaning more heavily toward bills. In contrast, a strategy designed to minimize credit market intervention by intentionally lengthening the average maturity of the public debt, would call for more financing with notes. In its baseline form, the model projects the bill-to-note ratio as a moving average of its values for the previous four quarters. A four-quarter moving average minimizes the effect on this ratio of seasonal variations in past issuances of bills and notes. This baseline projection is then adjusted to conform to announced Treasury Department financing strategies and expectations of changes in those strategies. Once this bill-to-note ratio is determined, the levels of outstanding bills and notes are easily calculated.

TREASURY NOTES

Treasury notes are marketable securities of one to ten years' maturity. The interest cost is calculated by the same formula used for bonds: average amount outstanding times the average effective interest rate. The effective interest rate is estimated with a regression equation fit on historical data.

Like the equation for the interest rate on special issues, the equation for the rate on notes uses several variables to capture the movements of relevant portions of the financial market. A moving average of the rate on notes of three to five years' maturity explicitly picks up market changes. The change in M2, the money supply, is a proxy for the Federal Reserve's monetary policy. Finally, the unemployment rate is used as a gauge of the tightness of demand in the economy.

The structural form of this equation incorporates two distinct features to model accurately the partial adjustment process by which marginal changes in the financial markets are incorporated into the average effective rate. Like the equation used for special issues, this equation uses a Koyck lag specification to impose a geometrically declining weight structure on all the explanatory variables. While the lagged dependent variable insures that the regression will simulate a partial adjustment process, the response time of changes in the average rate to marginal changes is still unspecified. This response time is explicitly included into the regression equation through an additional feature of its structural form. The response time is clearly a positive function of the level of outstanding notes. When there are few notes, the response time is very short; as the number of notes increases, so does the response time. This can be specified in the regression by dividing each explanatory variable by the level of outstanding notes. As the level of notes increases, the total magnitude of the explanatory variable will decrease, thereby simulating a lengthened response time. Finally, the equation is estimated using the Cochrane-Orcutt technique to correct for serial correlation in the residuals.

The resulting equation is:

$$\text{(8)} \quad RNOTE = \underset{(1.800)}{0.3134} + \underset{(44.44)}{0.9645} \times RNOTE_{-1} + \underset{(2.066)}{2.241} \times \frac{\overline{R35}_{-2}}{NOTE}$$

$$- \underset{(1.077)}{0.3767} \times \frac{RM2}{NOTE} - \underset{(-2.813)}{2.786} \times \frac{RU}{NOTE}$$

Interval: Quarterly, 1960-1976
R-Bar Squared = 0.9985
Durbin-Watson = 1.5715
Durbin h = 1.796
Standard Error = 0.05092
RHO = 0.6806

RNOTE = Average effective interest rate on notes
NOTE = Value of outstanding notes
$\overline{R35}$ = Four-quarter moving average of rate on Treasury notes of three to five years' maturity (market rate)
RM2 = Four-quarter change in M2
RU = Unemployment rate

TREASURY BILLS

Treasury bills are marketable securities with maturities of up to one year. Because bills are generally sold at a discount, the formula used to calculate the interest cost from special issues, bonds, and notes -- multiplying the amount outstanding by the effective rate -- will not give the correct results. The model for estimating the bill cost is therefore slightly different.

If a "true" effective rate for bills were assumed to exist, it would be defined by the identity R = I/A, where R is the true rate, I is the interest cost, and A is the average value of bills outstanding. Using this relationship, an historical series for the true effective bill rate was calculated. A regression equation was estimated to project this rate, and the interest cost was computed by multiplying that rate by the value of outstanding bills.

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Because the longest maturity a bill can have is one year, the equation for the interest rate on bills does not need the same partial adjustment form used for special issues and notes. Most Treasury bills have maturities of either three months, six months, or nine to twelve months. The true effective rate for all bills can therefore be estimated as a linear combination of the market yields on the component bills. The regression equation is:

$$\text{(9)} \quad RBILLT = \underset{(-0.8663)}{-0.07947} + \underset{(7.935)}{0.2976} \times R3_{-1} + \underset{(9.083)}{0.3226} \times \overline{R6} + \underset{(11.63)}{0.4128} \times \overline{R9}$$

Interval: Quarterly, 1966:3-1976
R-Bar Squared = 0.9948
Durbin-Watson = 1.7510
Standard Error = 0.08386

RBILLT = True effective interest rate for bills
R3 = Average market yield on three-month bills
$\overline{R6}$ = Two-quarter moving average of yields on six-month bills
$\overline{R9}$ = Four-quarter moving average of yields on nine- to twelve-month bills

All coefficients are of the correct sign, and their sum is not significantly different from unity.

Although a simulation model such as that just described may be sound in theory, its true test is in practice. To verify its accuracy, three tests were performed. The first was an historical simulation of the model over the 1970-1976 interval. This long-term projection demonstrated that the estimates of interest costs do not diverge significantly from actual data over time. The second test was a short-term simulation outside the estimation interval of the model. This indicated that the relationships assumed are accurate representations and that they are apt to remain accurate into the future. Finally, confidence intervals for the estimates were empirically derived using a statistical technique called "jackknifing." This process is described more fully in the Appendix.

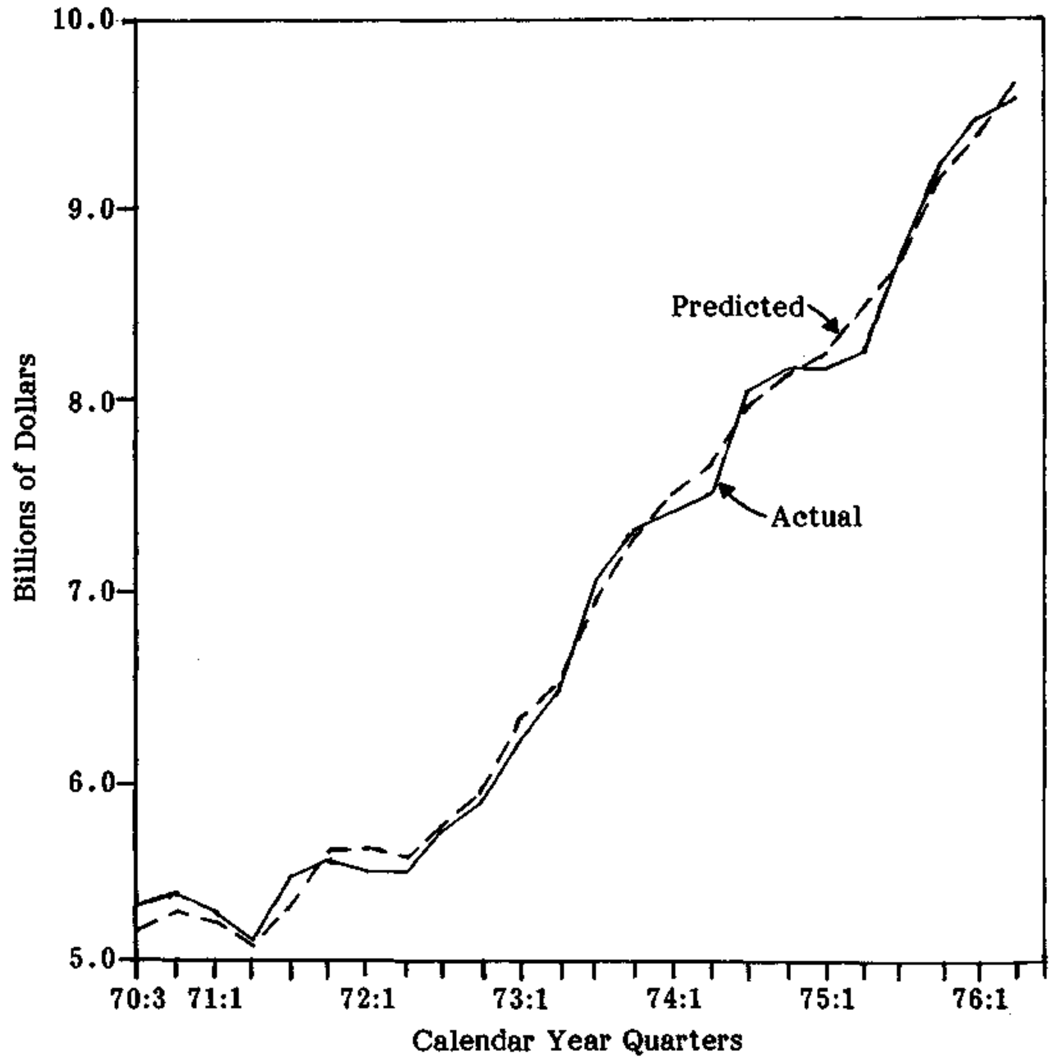
For the long-term simulation, outstanding amounts of the different securities were exogenous, and the actual historical data were used. The simulation was therefore testing the long-run response of the effective interest rate equations for bills, notes, and special issues, and the equation for accrued interest of other nonmarketables. For bonds, the actual effective interest rate was used in this simulation because it is a subjective input into the projections. In addition, this test was validating the assumption that amount times rate equals interest cost, which was used for bills, notes, bonds, and special issues. Table 3 summarizes the actual and predicted total interest cost for fiscal years 1971-1976.

TABLE 3. ACTUAL AND PREDICTED TOTAL INTEREST COST: BY FISCAL YEARS, IN MILLIONS OF DOLLARS

	1971	1972	1973	1974	1975	1976
Actual	20,959	21,848	24,167	29,319	32,665	37,063
Predicted	20,651	21,874	24,457	29,381	32,654	36,923
DIFFERENCE	308	-26	-290	-62	11	138

The errors do not follow any trend, so the actual and predicted values do not diverge consistently over time. The total error over the six-year period is $79 million, or 0.048 percent of the total. Figure 1 shows a plot of actual and predicted quarterly values.

FIGURE 1. TOTAL INTEREST COST: BY CALENDAR YEAR QUARTER, IN BILLIONS OF DOLLARS



Because the total interest cost is estimated by type of security, it is important to show how well the model performed for each component. Table 4 summarizes this information for fiscal years 1971-1976.

TABLE 4. SUMMARY OF ERRORS BY TYPE OF SECURITY IN THE LONG-TERM INTEREST MODEL SIMULATION: IN MILLIONS OF DOLLARS

	Mean	Mean Error	Median Error	Mean Absolute Error	RMS Error a/	RMS Percent Error b/
Bills	1,680	-6	-7	22	28	1.96
Notes	2,114	-6	-33	69	79	3.61
Bonds	487	1	1	2	3	0.64
Special Issues	1,502	14	35	53	61	4.35
Other Non-marketables	1,134	-1	-10	50	58	5.68
TOTAL	6,917	3	1	77	88	1.40

a/ RMS = root mean square.

b/ RMS percent error $= \sqrt{\frac{1}{T} \sum_{t=1}^{T} \left(\frac{Y_t^s - Y_t^a}{Y_t^a} \right)^2}$

where Y_t^s = predicted value of Y_t

Y_t^a = actual value of Y_t

T = number of periods in simulation

As the table shows, the errors are all small and not significantly biased. The model should therefore exhibit no tendency to underestimate or overestimate consistently the total interest cost or any of its components. It should also be noted that the errors do not exhibit any trend over time; they are no worse in the sixth year of simulation than they were in the first year. The model is therefore well suited to both short- and long-term simulations.

The short-term simulation test was constructed in the same way as the previous example. Because the test covered only two quarters, the results can be presented in more detailed form than they were in the long-term simulation. The actual and predicted values are shown in Table 5. These results confirm those reported from the long-term simulation test: the model contains no large errors or systematic biases.

TABLE 5. SHORT-TERM INTEREST MODEL SIMULATION: BY FISCAL YEAR QUARTER, IN MILLIONS OF DOLLARS

	1977:1	1977:2	Total	Weighted Total Percent Error a/
Bills				
Actual	2,248	2,059	4,307	
Predicted	2,192	2,078	4,270	
Difference	56	-19	37	0.18
Notes				
Actual	3,864	4,034	7,898	
Predicted	3,897	4,120	8,017	
Difference	-33	-86	-119	-0.57
Bonds				
Actual	588	584	1,172	
Predicted	584	596	1,180	
Difference	4	-12	-8	-0.04
Special Issues				
Actual	2,133	2,121	4,254	
Predicted	2,147	2,153	4,300	
Difference	-14	-32	-46	-0.22
Other Nonmarketables				
Actual	1,600	1,549	3,149	
Predicted	1,502	1,531	3,033	
Difference	98	18	116	0.56
TOTAL				
Actual	10,433	10,347	20,780	
Predicted	10,322	10,478	20,800	
Difference	111	-131	-20	-0.10

a/ The weighted total percent error is the total error as a percentage of the total interest in a category, weighted by that category's percentage of the total interest cost.

The jackknife simulation allowed the calculation of confidence intervals for the estimates of the total interest cost. The 95 percent confidence limits for a one-year estimate were computed to be \$144 million. These limits are assumed to increase somewhat with the time horizon of the estimate.

CHAPTER IV. SENSITIVITY OF THE ESTIMATES TO ALTERNATIVE ASSUMPTIONS

The basic purpose of the interest model is to provide estimates of the total cost of interest under various economic and budgetary assumptions. Anyone using the model or its results should therefore understand the sensitivity of the estimates to changes in those assumptions.

The estimated size of the deficit in both current and projected years clearly has a positive effect on the volume of interest-bearing debt, and therefore a significant impact on the interest cost. The precise extent of this impact depends upon the nature of the deficit (federal funds or trust funds), the timing of the deficit, and a series of subjective judgments concerning the financing strategy of the Treasury Department.

The unified budget deficit is the sum of the federal funds and trust funds deficits. Since fiscal year 1960, the federal funds have been consistently in deficit, and the trust funds have run a surplus in all but one year. The unified budget deficit is therefore generally smaller in magnitude than the federal funds deficit because of the opposite sign of the trust funds surplus. According to relationships (4) and (5) presented in Chapter I, both the federal funds and the trust funds surpluses enter somewhat differently into calculations of the volume and composition of the interest-bearing debt.

(4) Federally Held Treasury Debt = Federally Held Treasury Debt_{-1}
+ Combined Trust Fund Surplus

(5) Interest-Bearing Debt = Interest-Bearing Debt_{-1}
+ Unified Budget Deficit + Off-Budget Deficit
+ Combined Trust Fund Surplus + Other Means

Increases in the magnitude of the unified budget deficit lead to increases in the total debt. Increases in the trust funds surplus, however, translate directly into increases in the volume of outstanding special issues. To the extent that the average effective interest rate on special issues is different from that on the total debt, the composition (federal versus trust) of the unified budget deficit will have different implications for the total interest cost. For example, an estimated federal funds deficit of $60 billion and trust funds surplus of $10 billion (unified budget deficit equal to $50 billion) would increase the total debt by $60 billion, with $10 billion of the increase going to special issues. Leaving the unified budget deficit at $50 billion, but changing its composition to increase the federal funds portion and decrease the trust funds portion, would probably decrease the total interest cost. This is because the average rate on special issues is generally higher than the average rate on all other debt.

The timing of the assumed deficits also has effect on the interest cost. Because the model computes cost based on average amounts outstanding during some period, a change in the estimated deficit for the current year will have less impact than the same change in estimates for a future year, all other things held equal. This is because in the former case the end-of-year figures for outstanding debt are changed, but not the start-of-year figures. In the case of a future year, however, both start-of-year and end-of-year amounts are changed. Since the year average is the arithmetic mean of starting and ending values, the difference in impacts is clear. For example, suppose that the model is assuming total financing requirements of $60 billion and $50 billion in fiscal years 1977 and 1978, respectively, with 1977 being the current year. Since the total interest-bearing debt at the end of fiscal year 1976 was $619 billion, the model will estimate it to be $679 billion and $729 billion at the ends of 1977 and 1978. The average outstanding debt during these years would then be $649 billion and $704 billion. Reducing the 1977 financing requirements by $10 billion would decrease the average outstanding debt by $5 billion in 1977 but by $10 billion in 1978.

The impacts discussed so far, from the composition and timing of the deficits, are straightforward in concept and computation. The impact on the interest cost of subjective judgments of Treasury financing strategy is more variable and difficult to evaluate. An estimater using this model must determine the extent to which the Treasury will finance deficits by net changes in cash balances instead of by borrowing from the public. This determination cannot be reliably based on any behavioral or explanatory equation estimated from historical data; rather, it must be made from an appraisal of recent trends and current developments in Treasury actions. For this reason, the linkage in actual use of the model between deficits and debt level stated in relationship (5) is not obvious. Rather, the Other Means variable is used to capture the net total of these subjective judgments.

Besides responding to changes in the deficit assumptions, the interest cost is sensitive to varying economic assumptions. Here the linkages are more direct than in the previous case, but a priori expectations can be used to modify the model's estimates for any variable. Although the causal route is straightforward, it is far from simple. Different government securities respond to changes in different parts of the financial markets. In addition, there is often a long lag before changes in market rates are fully transferred to the effective rates on government securities. Finally, some components of the public debt do not respond at all to changes in the market interest rate.

Table 6 presents estimates of the interest cost under three alternative financial market scenarios. These alternative scenarios come from different simulations of a large econometric model, so they will be divergent but consistent pictures of the economy. In all three versions, the economic variables are different during fiscal year 1977, but are the same during 1978. This allows the lagged effects of the market on some components of the interest cost to be evaluated more distinctly. The first section of the table shows some relevant financial market and other variables that influence the interest cost estimates. Interest rates shown are fiscal year averages. The second section presents a breakdown of the interest cost by category of security.

TABLE 6. ALTERNATIVE ESTIMATES OF THE INTEREST COST: IN BILLIONS OF DOLLARS, RATES IN PERCENT

	Fiscal Year 1977			Fiscal Year 1978		
	Base	Pessimistic	Optimistic	Base	Pessimistic	Optimistic
Economic Variables						
91-day Bill Rate	4.85	4.77	4.83	5.75	5.75	5.75
6-month Bill Rate	5.05	4.98	5.03	5.95	5.95	5.95
9- to 12-month Bill Rate	5.26	5.20	5.23	6.15	6.15	6.15
3- to 5-year Note Rate	6.60	6.59	6.55	7.21	7.21	7.21
Moody's AAA Corporate Bond Rate	8.09	8.10	8.05	8.34	8.34	8.34
Percent Increase in the CPI	6.6	7.7	6.4	5.6	4.5	5.9
M2	758.1	758.5	758.0	828.6	828.6	828.6
Interest Cost						
Bills	8.481	8.449	8.464	10.073	9.980	10.047
Notes	16.281	16.279	16.282	17.845	17.839	17.844
Bonds	2.483	2.483	2.483	2.759	2.759	2.759
Special Issues	8.514	8.543	8.508	9.162	9.155	9.157
Other Non-marketables	6.465	6.465	6.465	6.979	6.979	6.979
TOTAL a/	42.225	42.219	42.202	46.819	46.712	46.786

a/ Components may not add to totals due to rounding.

As the table shows, and as the construction of the model would indicate, response to financial market changes is not consistent for the different securities. Treasury bonds are not sensitive to financial market fluctuations because of the constraints of the Second Liberty Bond Act and its subsequent amendments. Other nonmarketables (which are mostly savings and retirement bonds) are also not sensitive. These two categories constitute roughly 21 percent of the total interest cost. The three remaining debt categories (bills, notes, and special issues) are all responsive to financial market fluctuations, but in different ways. Consistent with the model's theoretical basis, bills are sensitive to the shorter-term credit markets and respond rapidly to changes in them. Notes and special issues move with the longer-term market and with a more extended lag because of their partial adjustment nature: market-induced changes in their marginal rate are slow to transfer fully into changes in their average rate.

These simulations show that the model responds in a reasonable fashion to changes in its exogenous inputs. Because of the complexity of the model, the precise magnitude of any response cannot always be anticipated. Instead, it should be simulated again under alternative assumptions and the magnitude of partial responses should be inferred from the results.

APPENDIX. EMPIRICAL ESTIMATION OF PREDICTION CONFIDENCE INTERVALS

In order to make the best use of the model's estimates of the total interest cost, one should have some idea of their accuracy. Determining rough confidence limits for a single-equation regression model is relatively straightforward, based on the equation's error characteristics; this approach, however, cannot be readily extended to the multi-equation case. For example, suppose:

$$A = b_1 x_1 + e_1$$

$$B = b_2 x_2 + e_2$$

$$C = f(A,B)$$

The variances for A and B can be readily computed based on e_1 and e_2. If $C = A + B$, the variance for C is equal to var (A) + var (B) + 2 x covar (A,B). Unless A and B are both normally distributed, however, confidence intervals for C cannot be easily calculated. This problem can be circumvented by means of the jackknife technique, which enables one to estimate directly prediction intervals for the end product of the model (C in the above example).

This appendix will not present a rigorous derivation of jackknifing; that can be found in the literature. 1/ Instead, a brief description of the technique will be given, and then the application of the technique to this specific problem will be discussed.

The jackknife is constructed by dividing the data into groups, making estimates based on combinations of these groups, and computing statistics for those estimates. Assume first a linear model of k independent equations with $Y = X\beta + e$, where Y is a k x 1 vector, X is a k x m matrix, β is an m x 1 vector, and e is a k x 1 vector. Let $\hat{\beta}$ be equal to the least-squares estimate of β. Also, let $\theta = f(\beta)$ and $\hat{\theta} = f(\hat{\beta})$. If the data is divided into n groups so that each group is one observation, the jackknife can be based on n estimates of β, each one computed by dropping one group (observation) from

1/ See R. G. Miller, "A Trustworthy Jackknife," Annals of Mathematical Statistics, December 1964; and R. G. Miller, "An Unbalanced Jackknife," The Annals of Statistics, September 1974.

the data. Define $\hat{\beta}_{-i}$ as the least squares estimate of β with the ith row deleted from the X matrix and the Y vector. Therefore, n new estimates (called pseudovalues) can be defined as

$$\tilde{\theta}_i = nf(\hat{\beta}) - (n - 1)f(\hat{\beta}_{-i})$$

and computed. It can be proved (under relatively weak conditions) that the probability distribution of the pseudovalues ($\tilde{\theta}_i$) is asymptotically normal, even though the distribution of the errors (e) from the original linear model might not be so. 2/ Therefore, the standard error of the pseudovalues is a consistent estimate of the asymptotic standard deviation of $\tilde{\theta}$. Given this, confidence intervals can be computed as t-intervals using the standard error of the pseudovalues, defined as

$$S = \sqrt{\frac{\Sigma(\tilde{\theta}_i - \tilde{\theta})^2}{n(n - 1)}}$$

where $\tilde{\theta}$ is the arithmetic mean of the pseudovalues.

The application of this technique to the interest-estimating model was generally straightforward. The quarterly estimate of the total interest cost was θ. Since $\theta = f(\beta)$, the β were the coefficients of the estimated equations of the interest model. Each of the estimated equations was fit n times, leaving out an observation each time. The equations were then solved for each of the n sets of coefficient values. The pseudovalues and their standard error were computed in the usual manner. Data availability made necessary one variation concerning the estimation interval of the equations. The four regressions fit for the interest model were fit over different intervals, reflecting different amounts of available historical data. Since the jackknife requires the same number of observations for all of the equations, it had to be performed over a shorter interval than in the model, so that n would be consistent across equations.

Since the pseudovalues are a function of the fit estimates $f(\hat{\beta})$ and $f(\hat{\beta}_{-i})$, they can be computed (and therefore confidence limits can be derived) at any point along the data set. In this test they were computed for the most recent data and also for ten forecast points. This interval was chosen because it would yield confidence limits in the same region in which working estimates are made with the actual model.

2/ See Miller, "An Unbalanced Jackknife."

This led to a second variation in computing the variance of the pseudovalues. The interest model produces quarterly estimates; the purpose of this exercise, however, was to determine confidence intervals for annual estimates of the interest cost. Simply adding up the variances of the pseudovalues for four quarters to get the variance of the annual estimate was not a satisfactory solution because this would ignore any possible covariances between quarters. Rather, the quarterly $f(\hat{\beta})$ and $f(\hat{\beta}_{-i})$ were summed to get annual estimates of those two variables, and then pseudovalues were computed for those annual figures. As can be seen in Table A-1, the variance for the annual estimate pseudovalues is quite close to the sum of the variances for the same four quarters. This indicates that if there is any covariance between quarters it is very small.

TABLE A-1. VARIANCE AND STANDARD ERRORS FOR PSEUDO-VALUES: BY FISCAL YEAR, STANDARD ERROR IN MILLIONS OF DOLLARS

Year:Quarter	S^2	S
77:3	8	91
77:4	10	102
78:1	36	188
78:2	24	156
78:3	60	244
78:4	54	233
79:1	83	288
79:2	119	345
79:3	152	389
79:4	195	442
1977	5	73
1978	146	381
1979	528	726

Since the pseudovalues are asymptotically normal, confidence limits were constructed as t-intervals using the standard errors in Table A-1. These limits are shown in Table A-2. The confidence limits are also shown as a percentage of current estimates of the interest cost, so that a rough notion of expected absolute and percentage prediction errors can be formed. It should be noted that these estimated prediction intervals only account for one of two potential sources of error: that stemming from the statistical properties of the model's equations. Any error resulting from erroneous economic or budgetary assumptions which are exogenous to the model is additional and is not included in the jackknife estimates.

TABLE A-2. CONFIDENCE LIMITS FOR ESTIMATES OF THE TOTAL INTEREST COST: BY FISCAL YEAR, IN MILLIONS OF DOLLARS

	1977	1978	1979
Confidence Limits			
68 percent	78	381	726
95 percent	144	747	1,423
Total Interest Cost	42,000	47,100	51,600
Confidence Interval 95 percent	41,850 to 42,150	46,350 to 47,850	50,175 to 53,025
Confidence Limits as Percent of Total Interest Cost			
68 percent	0.19	0.81	1.41
95 percent	0.34	1.59	2.76

It should be noted that the confidence intervals account for the uncertainty implicit in the regression equations for the rates on bills, notes, bonds, special issues, and other nonmarketables. Clearly they do not account for the uncertainty implicit in some of the inputs to those equations, such as the projected CPI or Moody's AAA bond rate.

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