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Quantitative Implications of Indexed Bonds in Small Open Economies*

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Abstract

Recent studies have proposed setting up a benchmark market for indexed bonds to prevent "Sudden Stops," emerging-market crises initiated by sudden reversals of capital inflows. This paper analyzes the macroeconomic implications of such bonds, which would be indexed to the terms of trade or GDP, using a general equilibrium model of a small open economy with financial frictions. Although indexed bonds provide a hedge to income fluctuations and can thereby mitigate the effects of financial frictions, they introduce interest rate fluctuations. Because of this tradeoff, there exists a nonmonotonic relation between the "degree of indexation" (i.e., the percentage of the shock reflected in the return) and the effects of these bonds on macroeconomic fluctuations. Therefore, indexation can improve macroeconomic conditions only if the degree of indexation is less than a critical value. When the degree of indexation is higher than this threshold, it strengthens the precautionary savings motive and increases consumption volatility and the impact effect of Sudden Stops. The threshold degree of indexation depends on the volatility and persistence of income shocks as well as on the relative openness of the economy.

JEL Classification: *F41, F32, E44*

Keywords: *indexed bonds, degree of indexation, financial frictions, sudden stops*

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1 Introduction

Liability dollarization[1] and frictions in world capital markets have played a key role in the emerging-market crises or Sudden Stops. Typically, these crises are triggered by sudden reversals of capital inflows that result in sharp real exchange rate (RER) depreciations and collapses in consumption. Figures 1 and 2 and Table 4 document the Sudden Stops observed in Argentina, Chile, Mexico, and Turkey in the last decade. For example in 1994, Turkey experienced a Sudden Stop characterized by: 10 percent current account-GDP reversal, 10 percent consumption and GDP drops relative to their trends, and 31 percent RER depreciation.[2]

In an effort to remedy Sudden Stops, Caballero (2002) and Borensztein and Mauro (2004) propose the issuance of state-contingent debt instruments by emerging-market economies. Caballero (2002) argues that crises in some emerging economies are driven by external shocks (e.g., terms of trade shocks) and that, contrary to their developed counterparts, these economies have difficulty absorbing the shocks as a result of imperfections in world capital markets. He argues that most emerging countries could reduce aggregate volatility in their economies and cut precautionary savings if they possessed debt instruments for which returns are contingent on the external shocks that trigger crises.[3] He suggests creating an indexed bonds market in which bonds' returns are contingent on terms of trade shocks or commodity prices. Borensztein and Mauro (2004) argue that GDP-indexed bonds could reduce the aggregate volatility and the likelihood of unsustainable debt-to-GDP levels in emerging economies. Hence, they argue that such bonds can help these countries avoid procyclical fiscal policies.

This paper introduces indexed bonds into quantitative models of small open economies to analyze the implications of these bonds for macroeconomic fluctuations and Sudden Stops. Our analysis consists of three steps. First, we start with a simple two-period small open economy model and illustrate the tradeoffs that indexed bonds introduce. Second, to conduct a formal quantitative exercise and to explore further aspects of indexation, we move to a quantitative one-sector economy in which infinitely lived agents receive persistent endowment shocks, credit markets are perfect but insurance markets are incomplete (henceforth, the *frictionless one-sector*

[1]Liability dollarization refers to the denomination of debt in units of tradables (i.e., hard currencies). Liability dollarization is common in emerging markets, where debt is denominated in units of tradables but partially leveraged on large nontradables sectors.

[2]See Figures 1 and 2 and Table 4 for further documentation of these empirical regularities (see Calvo et al., 2003, among others for a more detailed empirical analysis).

[3]Precautionary savings refers to extra savings caused by financial markets being incomplete. Caballero (2002) points out that precautionary savings in emerging countries arise as excessive accumulation of foreign reserves.

model), and analyze the implications of indexed bonds on precautionary savings motive, consumption volatility, co-movement of consumption with income. Third, we move to a two-sector model, which incorporates financial frictions proposed in the Sudden Stops literature (Calvo, 1998; Mendoza, 2002; Mendoza and Smith, 2005; Caballero and Krishnamurthy, 2001; among others). This model (henceforth, the *two-sector model with financial frictions*) can produce Sudden Stops endogenously through a debt-deflation mechanism similar to Mendoza (2002). Using this framework, we explore the implications of indexed bonds on Sudden Stops and RER fluctuations.

Our analyses establish that there exists a nonmonotonic relation between the "degree of indexation" of the bonds (i.e., the percentage of the shock that is passed on to the bonds' return) and the total effects of the bonds on macroeconomic variables. Therefore, indexed bonds can improve welfare and reduce precautionary savings, volatility of consumption, and correlation of consumption with income or smooth Sudden Stops only if the degree of indexation is lower than a critical value. If it is higher than that threshold (as with full-indexation), indexed bonds worsen these macroeconomic variables.

The two-period model is fairly standard (see Vegh, 2006, chapter 2). Agents receive a non-stochastic endowment in the first period and a stochastic endowment in the second period. Assuming, without loss of generality, that the mean endowment in the second period is higher than that in the first period, agents transfer a portion of the second-period endowment to the first period by borrowing from the rest of the world. When agents are endowed with nonindexed bonds, insurance markets are incomplete. Hence, they cannot attain the first best consumption allocations that they would achieve if they were endowed with the full set of state-contingent assets, such as Arrow securities. If the returns of the bonds are indexed, they pay lower return in the event that the second-period endowment is low (higher return if the second-period endowment is high). In a sense, indexed bonds provide a partial insurance to the fluctuations in the second period endowment. This hedge is imperfect, however, if, for example, the high-endowment state is more likely in the second period, indexation would have two effects working in opposite directions: the *consumption-smoothing effect* and the *wealth effect*. The consumption-smoothing effect arises, because by lowering (or increasing) debt repayments in low (or high) states, consumption levels become closer to each other across states. This effect is stronger when the degree of indexation is higher. Conversely, indexation also reduces the lifetime wealth of households because, given that the high-endowment state in the second period is more likely than the low-endowment state, agents transfer a portion of their wealth to the rest of the world. Our analysis suggests that the wealth effect could dominate the consumption-smoothing effect for degrees of indexation greater than a certain threshold. Hence, agents may end up worse off with indexation. The two-period model illustrates a potential tradeoff that indexation can introduce if the high-endowment states are more likely. We will see that in a dynamic framework, even if the high and low-endowment states are equally likely in the long run, indexation introduces tradeoffs but this time different than consumption-smoothing effect versus wealth effect tradeoff, as we explain below.

Our formal quantitative analysis starts with exploring the frictionless one-sector model. In this model, when the only available instruments are nonindexed bonds with constant exogenous returns, agents try to insure away income fluctuations with trade balance adjustments. Because insurance markets are incomplete, agents are not able to attain full consumption smoothing, consumption is volatile, and correlation of consumption with income is positive. Moreover, agents try to self-insure by engaging in precautionary savings. If the returns of the bonds are indexed to the exogenous income shock only, the insurance markets are only partially complete. As mentioned above, to have complete markets, either the full set of state-contingent assets, such as Arrow securities, must be available (i.e., there are as many assets as the states of nature) or the returns of the bonds must be state contingent (i.e., contingent on both the exogenous shock and the debt levels; see Section 2.2 for further discussion). Although indexed bonds partially complete the market, the hedge they provide is imperfect because they introduce interest rate fluctuations. Our quantitative analysis establishes that the interaction of these two effects implies a nonmonotonic relation between the degree of indexation of the bonds and the overall effects of bonds on macroeconomic variables. Therefore, as mentioned above, indexed bonds can reduce precautionary savings, the volatility of consumption, and the correlation of consumption with income only if the degree of indexation is lower than a critical value.

The changes in precautionary savings are driven by changes in the "catastrophic level of income." Risk-averse agents have strong incentives to avoid attaining levels of debt that the economy cannot support when the income is at catastrophic level.[4] Otherwise, agents would have non-positive consumption in the worst state of the economy which in turn would lead to infinitely negative utility. The degree of indexation has a significant effect on the state of nature

[4]The largest debt that the economy can support to guarantee non-negative consumption in the event that income is almost surely at its catastrophic level is referred to as *natural debt limit*.

that defines catastrophic levels of income and whether these income levels are higher or lower than what they would be without indexation. With higher degrees of indexation, these income levels can be determined at a positive shock; for example, if agents receive positive income shocks forever, they will receive higher endowment income but will also pay higher interest rates. Our analysis shows that for higher values of the degree of indexation, the latter effect is stronger, leading to lower catastrophic income levels. This effect in turn creates stronger incentives for agents to build up buffer stock savings.

The effect of indexation on consumption volatility can be analyzed by decomposing the variance of consumption. (Consider the budget constraint of such an economy: $c_t = (1+\varepsilon_t)y - b_{t+1} + (1+r+\varepsilon_t)b_t$.[5] Using this budget constraint, $var(c_t) = var(y_t) + var(tb_t) - 2cov(tb_t, y_t)$). On one hand, for a given income volatility, indexation increases the covariance of trade balance with income (since in good (bad) times indexation commands higher (lower) repayments to the rest of the world), which lowers the volatility of consumption. On the other hand, indexation increases the volatility of the trade balance (because of introduction of interest rate fluctuations), which increases the volatility of consumption. Our analysis suggests that at high levels of indexation, increase in the variance of the trade balance dominates the increase in the covariance of the trade balance with income, which in turn increases consumption volatility.

To understand the implications of indexed bonds on Sudden Stops, we finally introduce them into a two-sector economy, which incorporates financial frictions that can account for the key features of Sudden Stops. In particular, the economy suffers from liability dollarization, and international debt markets impose a borrowing constraint on the small open economy. This constraint limits debt to a fraction of the economy's total income valued at tradable goods prices. As established in Mendoza (2002), when the only available instrument is nonindexed bonds, an exogenous shock to productivity or to the terms of trade that renders the borrowing constraint binding triggers a Fisherian debt deflation mechanism.[6] A binding borrowing constraint leads to a decline in tradables consumption relative to nontradables consumption, inducing a fall in the relative price of nontradables as well as a depreciation of the RER. The decline in RER makes the constraint even more binding, because it creates a feedback mechanism that induces collapses in consumption and the RER as well as a reversal in capital inflows.

The tradeoffs mentioned in the frictionless one-sector model are preserved in the two-sector

[5] Here, b is bond holdings, r is risk-free net interest rate, y is endowment income, ε_t is the income shock, and c is consumption.

[6] See Mendoza and Smith (2005), and Mendoza (2005) for further analysis on Fisherian debt deflation.

model with financial frictions. Moreover, in the two-sector model, the interaction of the indexed bonds with the financial frictions leads to additional benefits *and* costs. Specifically, when indexed bonds are in place, negative shocks can result in a relatively small decline in tradable consumption; as a result, the initial capital outflow is milder and the RER depreciation is weaker than in a case with nonindexed bonds. The cushioning in the RER can help contain the Fisherian debt deflation process. Although the indexed bonds help relax the borrowing constraint in case of negative shocks, this time, an increase in debt repayment following a *positive* shock can lead to a larger need for borrowing, which can make the borrowing constraint suddenly binding, triggering a debt deflation. Quantitative analysis of this model suggests, once again, that the degree of indexation needs to be lower than a critical value to smooth Sudden Stops. When indexation is higher than this critical value, the latter effect dominates the former, hence leading to more detrimental effects of Sudden Stops. The degree of indexation that minimizes macroeconomic fluctuations and the impact effect of Sudden Stops depends on the persistence and volatility of the exogenous shock triggering Sudden Stops as well as the size of the nontradables sector relative to its tradables sector; this finding suggests that the indexation level that maximizes benefit of indexed bonds needs to be country-specific. An indexation level that is appropriate for one country in terms of its effectiveness at preventing Sudden Stops may not be effective for another and may even expose that country to higher risk of facing Sudden Stops.

Debt instruments indexed to real variables (i.e., GDP, commodity prices, etc.) have not been widely employed in international capital markets.[7] As Table 3 shows, only a few countries have issued this type of instrument in the past. Moreover, most of those countries stopped issuing them: for example, Bulgaria swapped its GDP-indexed bonds for nonindexed bonds. Although the literature has emphasized the problems on the demand side as the primary reason for the limited issuance of indexed bonds, the supply of such bonds has always been thin, because countries have exhibited little interest in issuing them. Our results may help to illuminate why that has been the case: countries may have been reluctant because of the imperfect hedge that indexed bonds provide.

Several studies have explored the costs and benefits of indexed debt instruments in the context of public finance and optimal debt management.[8] As mentioned above, Borensztein and Mauro (2004) and Caballero (2002) drew attention to such instruments as possible vehicles to provide

[7] CPI-indexed bonds may not provide a hedge against income risks, because inflation is procyclical.

[8] See, for instance, Barro, 1995; Calvo, 1988; Fischer, 1975; Magill and Quinzil, 1995; among others

insurance benefits to emerging countries. Moreover, Caballero and Panageas (2003) quantified the potential welfare effects of credit lines offered to emerging countries. They used a one-sector model with collateral constraints in which Sudden Stops are exogenous to explore the benefits of such credit lines in smoothing Sudden Stops, interpreting them as akin to indexed bonds. This paper contributes to this literature by modeling indexed bonds explicitly in a dynamic stochastic general equilibrium model in which Sudden Stops are endogenous. Endogenizing Sudden Stops reveals that, depending on the structure of indexation, indexed bonds may amplify the effects of Sudden Stops.[9]

This paper is related to studies in several strands of macroeconomics and international finance literature. The model has several features common to the literature on precautionary saving and macroeconomic fluctuations (e.g., Aiyagari 1994, Hugget 1993). The paper is also related to studies exploring business cycle fluctuations in small open economies (e.g., Mendoza, 1991; Neumeyer and Perri, 2005; Kose, 2002; Oviedo, 2005; Uribe and Yue, 2005) from the perspective of analyzing how interest rate fluctuations affect macroeconomic variables. In addition to the papers in the Sudden Stops literature, this paper is also related to follow-up studies to this literature, including Calvo et. al. (2003), Durdu and Mendoza (2006), and Caballero and Panageas (2003), which investigate the role of relevant policies in preventing Sudden Stops. Durdu and Mendoza (2006) explore the quantitative implications of price guarantees offered by international financial organizations on emerging-market assets. They find that these guarantees may induce moral hazard among global investors and conclude that the effectiveness of price guarantees depends on the elasticity of investors' demand as well as on whether the guarantees are contingent on debt levels. Similarly, in this paper, we explore the potential imperfections that indexation can introduce and derive the conditions under which such a policy could be effective in preventing Sudden Stops.

Earlier seminal studies in the financial innovation literature, such as Shiller (1993) and Allen and Gale (1994), analyze how creation of a new class of "macro markets" can help manage economic risks such as real estate bubbles, inflation, and recessions and discuss what sorts of frictions can prevent the creation of such markets. This paper emphasizes possible imperfections in global markets and points out under which conditions issuance of indexed bonds may not improve macroeconomic conditions for a given emerging-market.

[9] Krugman (1988) and Froot et al. (1989) emphasize moral hazard problems that GDP indexation can introduce. Here, we point out other adverse effects that indexation can cause, even in the absence of moral hazard.

The next section starts with description of the models used for analyses and presents quantitative results. Section 3 provides conclusions and offers extensions for further research.

2 Quantitative models of Small Open Economies

This section describes the models we use for our analysis. We start with the basic two-period model to illustrate the tradeoffs that indexed bonds could introduce. In light of those results, we move to the frictionless one-sector model to conduct a disciplined quantitative exercise and to explore further aspects of indexation, particularly, to explore how indexation alters precautionary savings motive, consumption volatility, co-movement of consumption with income, and welfare of the agents in the economy. Finally, we consider the ultimate two-sector model to explore whether indexed bonds could be a remedy for Sudden Stops.

2.1 The two-period model

We introduce indexed bonds to a typical two-period small open economy model as presented in Vegh (2006, chapter 2). Representative households live for two periods and receive an endowment of tradable goods every period. The first period endowment y_1 is nonstochastic. The second-period endowment, however, is stochastic and is equal to $(1+\varepsilon)y_2$ with probability p, and $(1-\varepsilon)y_2$ with probability $1-p$, where ε denotes the endowment shock. The households can borrow from or lend to international capital markets to smooth their consumption using a single type of bonds (nonindexed or indexed) depending on their expectations of the second-period endowment. They maximize their utility,

$$U = u(c_1) + \beta E\left[u(c_2)\right], \tag{1}$$

subject to the following budget constraints

$$c_1 = y_1 - b_1, \tag{2}$$

$$c_2^H = (1+\varepsilon)y_2 + (1+r+\phi\varepsilon)b_1, \tag{3}$$

$$c_2^L = (1-\varepsilon)y_2 + (1+r-\phi\varepsilon)b_1, \tag{4}$$

where utility function is in constant relative risk-aversion (CRRA) form, and $u(c) = (c^{1-\sigma} - 1)/(1-\sigma)$, ϕ is the degree of indexation; b_1 denotes bond holdings (regardless of whether bonds are indexed or nonindexed); and c_1, c_2^H, and c_2^L denote consumption in the first period, the second-period high state of nature, and the second-period low state of nature, respectively. When households' current bond holdings are negative (i.e., households are debtors) in the second period, they pay less (or more) in the event that the second-period endowment is low (or high). By introducing the degree of indexation ($\phi \in [0, 1]$), we introduce the flexibility to analyze and compare the cases that have no indexation ($\phi = 0$), full-indexation ($\phi = 1$), and the cases in between.[10] When the degree of indexation is zero, the return of the bonds does not respond to endowment shocks. However, if the degree of indexation is positive, and for instance is equal to 0.15, then 15 percent of the endowment fluctuation is passed on to bonds' return.

Without loss of generality, we assume that $y_1 < y_2$ (i.e., the economy has good prospects). Hence, households in equilibrium would like to transfer a portion of the second-period income to the first period, that is, they will be debtors ($b_1 < 0$).

The optimality conditions of the problem facing households are as follows:

$$u'(c_1) = \beta p(1 + r + \phi\varepsilon)u'(c_2^H) + \beta(1-p)(1 + r - \phi\varepsilon)u'(c_2^L),$$

$$y_1 + \frac{(1+\varepsilon)y_2}{1 + r + \phi\varepsilon} = c_1 + \frac{c_2^H}{1 + r + \phi\varepsilon},$$

$$y_1 + \frac{(1-\varepsilon)y_2}{1 + r - \phi\varepsilon} = c_1 + \frac{c_2^H}{1 + r - \phi\varepsilon}.$$

The above system of equations does not have a closed-form solution. Hence, we conduct our analysis by solving them numerically. For that purpose, we assign numerical values to the parameters in the model. We do not perform a formal calibration exercise (which we pursue later) but use reasonable parameter values to illustrate the tradeoffs indexation could introduce.

We set the CRRA parameter (σ) to 2, the net interest rate (r) to 5 percent, the discount factor to the inverse of the gross interest rate ($\beta = 1/(1+r)$), the first period endowment (y_1) to 0, the mean second-period endowment (y_2) to 1, the endowment shock (ε) to 3 percent, and the probability of second-period endowment being high (p) to 0.6.

The graph on the left in Figure 3 shows the percentage difference in utilitarian welfare in indexed economies relative to the nonindexed economy for various degrees of indexation. The

[10]No theoretical restriction exists for the degree of indexation taking values higher than 1, but to make the analysis as close as possible to real life applications, we set the upper bound to 1.

graph on the right plots the second-period consumption in the high-endowment and the low-endowment states. Without indexation, insurance markets are incomplete, because a single type of nonindexed bonds cannot fully hedge away the endowment shocks (notice that the second-period consumption in the high and low states is not equalized). Given the above parameter values, if the households were endowed with a full set of state-contingent assets such as Arrow securities (i.e., the insurance markets were complete), the consumption allocations would be equalized across state and time (consumption in the first and the second period would be equal to 0.5). These findings are in line with those of the studies in the literature, such as Vegh (2006, chapter 2).

When the households are endowed with indexed bonds, the insurance markets are still incomplete (notice from Figure 3 that regardless of what degree of indexation value is used, the second-period consumption in the high and in the low states is still not equalized) because the number of available instruments is still not equal to the number of states of nature. (We elaborate more on this issue in the next section). Moreover, as the degree of indexation increases, households first attain higher welfare with $0 \leq \phi < 0.4$ (the highest welfare is achieved with $\phi = 0.39$), thereafters, welfare decreases as the degree of indexation increases.[11] Indeed, households attain lower welfare with degrees of indexation higher than 0.8 relative to the nonindexed economy. This result suggests that the degree of indexation is a crucial variable that can affect the total implications of indexation and that households can actually be worse off with indexation than without it.

These results are driven by two effects of indexation working in opposite directions: the consumption-smoothing effect and the wealth effect. The consumption-smoothing effect is in place, because, agents can smooth the second-period consumption across states by indexation, this effect is stronger if the degree of indexation is higher. (Notice that consumption in the high and in the low states converge as the degree of indexation increases.) Conversely, indexation also reduces the lifetime wealth of the households because, given that the high-endowment state in the second period is more likely than the low-endowment state, agents transfer a portion of their wealth to the rest of the world with indexation and the higher the degree of indexation is, the stronger is the wealth effect. For degrees of indexation higher than 0.8, the wealth effect dominates the consumption smoothing effect; hence, the agents are worse off with indexation.

[11]Not surprisingly, given the incompleteness of the asset market, even the highest welfare achieved in indexed economies is lower than the welfare achieved with complete markets.

The simple framework described in this section illustrates that the degree of indexation could play a crucial role in determining the implications of indexation, and more important that agents could actually be worse off with indexation than without it. In light of these results, we further analyze implications of indexed bonds in more formal settings and with more structured calibration exercises. The next section introduces our frictionless one-sector model in which we study the implications of indexation on precautionary savings, consumption smoothing, and welfare. Finally, we move to our final two-sector model with financial frictions to study the effects of indexation on Sudden Stops. As we explain below, in these dynamic frameworks, the tradeoffs introduced by indexation is different than the ones introduced in the two-period model. Because, in a dynamic framework, indexation introduces persistent fluctuations in the interest rate, which does not exist in the two-period model. Moreover, the adverse effects of indexation are in place even if the long-run probability of low-endowment and high-endowment states are equal.

2.2 The frictionless one-sector model

The frictionless one-sector model is a quantitative one-sector small open economy model, of which the benchmark model with nonindexed bonds is an endowment economy version of the model described in Mendoza (1991), and a small open economy version of the model used in Hugget (1993) and Aiyagari (1994).

Representative households receive a stochastic endowment of tradables, which is denoted as $(1+\varepsilon_t)y^T$. ε_t is a shock to the world value of the mean tradables endowment that could represent either a productivity shock or a terms-of-trade shock. $\varepsilon \in \mathcal{E} = [\varepsilon_1 < ... < \varepsilon_m]$ (where $\varepsilon_1 = -\varepsilon_m$) evolves according to an m-state symmetric Markov chain with transition matrix $\mathcal{P}$. Households derive utility from aggregate consumption (c, which equals to tradable consumption, c^T, in this frictionless one-sector model), and they maximize Epstein's (1983) stationary cardinal utility function:

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \exp\left[-\sum_{\tau=0}^{t-1} \gamma \log(1+c_t) \right] u(c_t) \right\}. \tag{5}$$

where

$$u(c_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma}. \tag{6}$$

The instantaneous utility function (6) is in CRRA form and has an intertemporal elasticity of substitution $1/\sigma$. $\exp\left[-\sum_{\tau=0}^{t-1}\gamma\log(1+c_t)\right]$ is an endogenous discount factor that is introduced to induce stationarity in consumption and asset dynamics. γ is the elasticity of the subjective discount factor with respect to consumption. Mendoza (1991) introduced preferences with endogenous discounting to quantitative small open economy models, and such preferences have since been widely used.[12]

The households' budget constraint is

$$c_t^T = (1+\varepsilon_t)y^T - b_{t+1} + (1+r+\phi\varepsilon_t)b_t, \tag{7}$$

where b_t is current bond holdings, and $(1+r+\phi\varepsilon_t)$ is the gross return on bonds. The indexation mechanism works follows: the returns of the indexed bonds are low in the low state of nature and high in the high one, but the mean of the returns remains unchanged and equal to $R = 1+r$. When households' current bond holdings are negative (i.e., when households are debtors) they pay less (more) in the event of a negative (positive) endowment shock. As in the previous two-period model, we introduce the degree of indexation, $\phi \in [0,1]$, to have flexibility to analyze the cases with no indexation, full-indexation and the cases in between. Notice that ϕ affects the variance of the bonds' returns (since $var(1+r+\phi\varepsilon_t) = \phi^2 var(\varepsilon_t)$). As ϕ increases, the bonds provide a better hedge against negative income shocks, but at the same time they introduce additional volatility by increasing the returns' variance.

The optimality conditions of the problem facing households can be reduced to the following standard Euler Equation:

$$U_c(t) = \exp\left[-\gamma\log(1+c_t)\right] E_t\left\{(1+r+\phi\varepsilon_t)U_c(t+1)\right\} \tag{8}$$

along with the budget constraint (7), and the standard Kuhn-Tucker conditions. U_c is the derivative of lifetime utility with respect to consumption.

As discussed in the two-period model, indexed bonds with returns indexed to the exogenous shock are not able to complete the market; they just partially complete it by providing the agents with the means to hedge against fluctuations in endowment income. If we call $(1+r+\phi\varepsilon)b_t$ financial income, the underlying goal to complete the market would be to keep the sum of endowment and financial incomes constant and equal to the mean endowment income (i.e.,

[12]See Schmitt-Grohé and Uribe (2003) for other specifications used for this purpose. See Kim and Kose (2003) for a comparison of quantitative implications of endogenous discounting with that of constant discounting.

$(1+\varepsilon_t)y^T + (1+r+\phi\varepsilon)b_t = y^T)$. Clearly, one can keep this sum constant only if the bonds' returns are state-contingent (i.e., contingent on both the exogenous shock and the debt stock, which requires $R_t(b,\varepsilon) = -\frac{\varepsilon_t y^T}{b_t}$) or if agents can trade Arrow securities (i.e., there are as many assets as the number of state of nature). Moreover, indexed bonds introduce a tradeoff: on one the hand, they hedge income fluctuations but on the other hand, they introduce interest rate fluctuations.

Given the income uncertainty, and the incompleteness of the insurance market, households' engage in precautionary savings to hedge away the risk of attaining "catastrophic" levels of income. They accomplish this task by imposing on themselves a debt limit (i.e., the natural debt limit), given by the annuity value of the worst income realizations. Indexation of the return reduces the incentives for precautionary savings against low realizations of income shocks but it might introduce incentives to save against high realizations of income shocks if the degree of indexation is such that the repayments to the rest of the world outweigh the additional income received in those states. (We provide a formal analysis of this point below).

Exploring the overall implications of indexation in different dimensions requires a detailed analysis of the model economy presented above. For this purpose, we perform a series of numerical exercises presented below.

2.2.1 Dynamic programming representation

The dynamic programming representation of the household's problem is as follows:

$$
\begin{aligned}
V(b,\varepsilon) = \max_{b'} \left\{u(c) + (1+c)^{-\gamma} E\left[V(b',\varepsilon')\right]\right\} \quad s.t. \\
c^T = (1+\varepsilon)y^T - b' + (1+r+\phi\varepsilon)b.
\end{aligned}
\tag{9}
$$

Here, the endogenous state-space is given by $\mathcal{B} = \{b_1 < ... < b_{NB}\}$, which is constructed using $NB = 1,000$ equidistant grid points. The exogenous Markov process is assumed to have two states for simplicity: $\mathcal{E} = \{\varepsilon_L < \varepsilon_H\}$. Optimal decision rules, $b'(b,\varepsilon) : \mathcal{E} \times \mathcal{B} \rightarrow \mathcal{R}$, are obtained by solving the above problem via a value-function iteration algorithm.

2.2.2 Calibration

The parameter values used to calibrate the model are summarized in Table 1. The CRRA parameter σ is set to 2, the mean endowment y^T is normalized to one, and the gross interest rate is set to the quarterly equivalent of 6.5 percent, following values used in the small open economy RBC literature (see, for example, Mendoza, 1991). The steady state debt-to-GDP ratio is set to 35 percent, which is in line with the estimate for the net asset positions of Turkey (see Lane and Milesi-Ferretti, 1999). The elasticity of the subjective discount factor follows from the Euler Equation for consumption evaluated in steady state:

$$(1+\bar{c})^{-\gamma}(1+r) = 1 \Rightarrow \gamma = \log(1+r)/\log(1+\bar{c}). \tag{10}$$

The standard deviation of the endowment shock is set to 3.51 percent and the autocorrelation is set to 0.524; those values are the standard deviation and the autocorrelation of tradable output for Turkey given in Table 4.

Table 1: Parameter Values

σ	2	Relative risk-aversion	RBC parametrization
y^T	1	Tradable endowment	Normalization
σ_ε	0.0351	Tradable output volatility	Turkish data
ρ_ε	0.524	Tradable output autocorrelation	Turkish data
R	1.0159	Gross interest rate	RBC parametrization
γ	0.0228	Elasticity of discount factor	Steady state condition

Using the "simple persistence" rule, we construct a Markovian representation of the time series process of output. The transition probability matrix $\mathcal{P}$ of the shocks follows:

$$\mathcal{P}(i,j) = (1-\rho_\varepsilon)\Pi_i + \rho_\varepsilon \mathcal{I}_{i,j} \tag{11}$$

where $i, j = 1, 2$; Π_i is the long-run probability of state i; and $\mathcal{I}_{i,j}$ is an indicator function, which equals 1 if $i = j$ and 0 otherwise, ρ_ε is the first-order serial autocorrelation of the shocks.

2.2.3 Simulation results

To show the effect of indexation on consumption smoothing, we report long-run values of the key macroeconomic variables, such as mean bond holdings (a measure of precautionary savings), volatility of consumption, correlation of consumption with income (which measures the extent to which income fluctuations affect consumption fluctuations) and serial autocorrelation of consumption (which measures the persistence of consumption, see Table 5). Without indexation ($\phi = 0$), mean bond holdings are higher than the case with perfect foresight (-0.35) (a value that implies precautionary savings); volatility of consumption is positive; and consumption is correlated with income.

When the degree of indexation is in the [0.015, 0.25) range, households engage in less precautionary savings (as measured by the long-run average of b) and the standard deviation of consumption declines relative to the case without indexation. Moreover, in this range, correlation of consumption with GDP falls slightly and its serial autocorrelation increases slightly. The results suggest that when the degree of indexation is in this range, indexation improves these macroeconomic variables from the consumption-smoothing perspective. When the degree of indexation is greater than 0.25, however, the improvements reverse. In the full-indexation ($\phi = 1$) case, for example, the standard deviation of consumption is 4.8 percent, four times the standard deviation in the no-indexation case. The persistence of consumption also declines at higher degrees of indexation. The autocorrelation of consumption in the full-indexation case is 0.886, compared with 0.978 in the no-indexation case and the high of 0.984 when $\phi = 0.10$. Not surprisingly, the ranking of welfare (calculated as compensating variations in consumption) is in line with the ranking of consumption volatility, as the last row of Table 5 reveals. However, the absolute values of the differences in welfare are quite small.[13]

The above results are driven by the changes in the ability to hedge income fluctuations with indexed bonds. This hedging ability is affected by the degree of indexation because indexation alters the incentives for precautionary savings. In particular, it has a significant effect on determining the state of nature that defines the catastrophic level of income at which household reach their natural debt limit. The natural debt limit (ψ) is the largest debt that the economy can support to guarantee non-negative consumption in the event that income remains at its catastrophic level almost surely; that is,

$$\psi = -\frac{(1-\varepsilon)y^T}{r}. \tag{12}$$

With nonindexed bonds, the catastrophic level of income is realized with a negative endowment shock. When the bond holdings approach the natural debt limit, consumption approaches zero, which leads to infinitely negative utility. Hence, agents have strong incentives to avoid holding levels of bonds lower than the natural debt limit. To guarantee positive consumption almost surely in the event that income remains at its catastrophic level, agents engage in strong

[13]As pointed out by Lucas (1987), the welfare implications of altering consumption fluctuations in this type of model are quite low.

precautionary savings. An increase (or decrease) in this debt limit strengthens (or weakens) the incentive to save, because the level of bond holdings that agents would try to avoid would be higher (or lower). With indexation, the natural debt limit can be determined at either negative or positive realization of the endowment shock, depending on which yields the lower income (i.e., determines the catastrophic level of income). To see this effect, notice that using the budget constraint, when the shock is negative,

$$c_t \geq 0 \Rightarrow (1-\varepsilon)y - b_{t+1} + b_t(1+r-\phi\varepsilon) \geq 0 \Rightarrow \psi_L \geq -\frac{(1-\varepsilon)y}{r-\phi\varepsilon},\ if\ r-\phi\varepsilon > 0. \tag{13}$$

For the ranges of values of ϕ where $r - \phi\varepsilon < 0$, Equation 13 yields an upper bound for the bond holdings; i.e., $\psi_L \leq -(1-\varepsilon)y/(r-\phi\varepsilon)$. Hence, in this range, negative shock will not play any role in determining the natural debt limit. Again using the budget constraint, positive endowment shock implies the following natural debt limit:

$$c_t \geq 0 \Rightarrow (1+\varepsilon)y - b_{t+1} + b_t(1+r+\phi\varepsilon) \geq 0 \Rightarrow \psi_H \geq -\frac{(1+\varepsilon)y}{r+\phi\varepsilon}. \tag{14}$$

Combining the two equations yields the following formula:

$$\psi = \begin{cases} \max\{-\frac{(1-\varepsilon)y}{r-\phi\varepsilon}, -\frac{(1+\varepsilon)y}{r+\phi\varepsilon}\},\ if\ \phi < r/\varepsilon \\ -\frac{(1+\varepsilon)y}{r+\phi\varepsilon},\ if\ \phi > r/\varepsilon. \end{cases} \tag{15}$$

Further algebra suggests that when $\frac{1-\varepsilon}{1+\varepsilon} < \frac{r-\phi\varepsilon}{r+\phi\varepsilon}$ or $\phi < r$, the natural debt limit is sound in the state of nature with a negative endowment shock. In this case, $\partial\psi/\partial\phi < 0$; that is, increasing the degree of indexation decreases the natural debt limit or weakens the precautionary savings incentive. However, if $\frac{1-\varepsilon}{1+\varepsilon} > \frac{r-\phi\varepsilon}{r+\phi\varepsilon}$ or $\phi > r$, then $\partial\psi/\partial\phi > 0$, that is, increasing the degree of indexation increases the natural debt limit or strengthens the precautionary savings incentive.

Table 6 shows calculations for these natural debt limits as functions of the degrees of indexation, along with the corresponding returns in both states ($R_t^i = 1 + r + \phi\varepsilon_t$), and confirms the analytical results derived above. When the degree of indexation is less than 0.0159, the natural debt limit is determined by the negative shock; and it decreases (i.e., becomes looser) as ϕ increases. When ϕ is greater than 0.0159, the debt limit is determined by the positive shock, and it increases (i.e., becomes tighter) as ϕ increases (the corresponding limits are shown in bold in Table 6). In the full-indexation case, for example, this debt limit is -20.09, whereas the corresponding value is -61.49 in the nonindexed case. In other words, in the full-indexation case, positive endowment shocks decrease the catastrophic level of income to one third of the value in the nonindexed case. This decrease, in turn, sharply strengthens the precautionary savings motive.

To understand the role of indexation on volatility of consumption, we perform a variance decomposition analysis. Higher indexation provides a better hedge to income fluctuations by increasing the covariance of the trade balance ($tb = b' - R_t^i$b) with income (because in good (or bad) times agents pay more (or less) to the rest of the world). Higher indexation, however, also increases the volatility of the trade balance because it introduces interest rate fluctuations. To pin down the effect of indexation on these variables, we perform a variance decomposition using the following identity:

$$var(c^T) = var(y^T) + var(tb) - 2cov(tb, y^T).$$

Table 7 presents the corresponding values for the last two terms in the above equation for each of the indexation levels.[14] Clearly, both the variance of the trade balance and the covariance of the trade balance with income monotonically increase with the level of indexation. However, the term $var(tb) - 2cov(tb, y^T)$ fluctuates in the same direction as the volatility of consumption, suggesting that at high levels of indexation, the rise in the variance of the trade balance offsets the improvement in the co-movement of the trade balance with income (i.e., the effect of increased fluctuation in interest rate dominates the effect of hedging provided by indexation). Hence, consumption becomes more volatile for higher degrees of indexation.

In summary, when the degree of indexation is higher than a critical value (as with full-indexation), the precautionary savings motive is stronger and the volatility of consumption is higher than in the nonindexed case. These results arise because the natural debt limit is higher at higher levels of indexation and because the increased volatility in the trade balance far outweighs the improvement in the co-movement of the trade balance with income.

The results suggest that to improve macroeconomic variables, the indexation level should be low. When ϕ is lower than 0.25, agents can better hedge against fluctuations in endowment income than when ϕ is at higher levels. In this case, the precautionary savings motive is weaker, the volatility of consumption is smaller, and consumption is more persistent. When ϕ is in

[14] Because the endowment is not affected by changes in the indexation level, its variance is constant.

the $[0.10, 0.25]$ range, the correlation of consumption with income approaches zero and the autocorrelation of consumption nears unity. These values resemble the results that could be attained in the full-insurance scenario, and they suggest that partial indexation is optimal.

The results using a frictionless one-sector model shed light on the implications of indexed bonds. The findings in this section suggest that the hedge provided by indexed bonds is imperfect and that the implications of indexed bonds depend on the degree of indexation of the bonds. As illustrated in the two-period model and here in the frictionless one sector model, the implications of indexation could be a nonmonotonic function of the degree of indexation. For values of this variable that are higher than a certain threshold, households may end up being worse off with indexation than without it.

2.3 The two-sector model with financial frictions

We build on the previous frictionless one-sector model by introducing a non-tradable sector and a borrowing constraint. Foreign debt is denominated in units of tradables, and imperfect credit markets impose a borrowing constraint that limits external debt to a share of the value of total income in units of tradables (this constraint therefore reflects changes in the relative price of nontradables that is the model's RER). With these new features, the model with nonindexed bonds is the same as described in Mendoza (2005) (an endowment economy version of Mendoza, 2002).

Representative households receive a stochastic endowment of tradables and a nonstochastic endowment of nontradables, which are denoted $(1+\varepsilon_t)y^T$ and y^N, respectively. As in the previous model, ε_t is a shock to the world value of the mean tradables endowment, which could represent a productivity shock or a terms-of-trade shock, $\varepsilon \in \mathcal{E} = [\varepsilon_1 < ... < \varepsilon_m]$ (where $\varepsilon_1 = -\varepsilon_m$) evolves according to an m-state symmetric Markov chain with transition matrix $\mathcal{P}$. Households derive utility from aggregate consumption (c), and they maximize Epstein's (1983) stationary cardinal utility function (see Equation (5), where the utility function (6) is in CRRA form). The consumption aggregator is represented in constant elasticity of substitution (CES) form as follows:

$$c_t(c_t^T, c_t^N) = \left[\omega(c_t^T)^{-\mu} + (1-\omega)(c_t^N)^{-\mu}\right]^{-\frac{1}{\mu}}. \tag{16}$$

where $1/(1+\mu)$ is the elasticity of substitution between consumption of tradables and nontradables and where ω is the CES weighting factor.

The households' budget constraint is

$$c_t^T + p_t^N c_t^N = (1+\varepsilon_t)y^T + p_t^N y^N - b_{t+1} + (1+r+\phi\varepsilon_t)b_t \tag{17}$$

where p_t^N is relative price of nontradables. (The rest of the variables are defined as in the frictionless one-sector model). Here, the returns of the bonds are indexed to the terms of trade shock.[15]

In addition to the budget constraint, foreign creditors impose the following borrowing constraint, which limits debt issuance as a share of total income at period t not to exceed κ:

$$b_{t+1} \geq -\kappa \left[(1+\varepsilon_t)y^T + p_t^N y^N\right]. \tag{18}$$

The borrowing constraint takes a similar form to those used in the Sudden Stops literature to mimic the tightening of the available credit to emerging countries (see, for example, Caballero and Krishnamurthy, 2001; Mendoza, 2002; Mendoza and Smith, 2005; Caballero and Panageas, 2003). As Mendoza and Smith (2005) explain, even though these types of borrowing constraints are not based on a contracting problem between lenders and borrowers, they are realistic in the sense that they resemble the risk management tools used in international capital markets, such as the Value-at-Risk models that investment banks use.

The optimality conditions are:

$$U_c(t)\left(1-\frac{\nu_t}{\lambda_t}\right) = \exp\left[-\gamma \log(1+c_t)\right] E_t\left\{\frac{(1+r+\phi\varepsilon_t)p_t^c}{p_{t+1}^c}U_c(t+1)\right\}, \tag{19}$$

$$\frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{1+\mu} = p_t^N, \tag{20}$$

the budget constraint (17), the borrowing constraint (Equation 18), and the standard Kuhn-Tucker conditions. ν and λ are the Lagrange multipliers of the borrowing constraint and the budget constraint, respectively. U_c is the derivative of lifetime utility with respect to aggregate consumption. p_t^c is the CES price index of aggregate consumption in units of tradable consumption, which equals $\left[\omega^{\frac{1}{\mu+1}} + (1-\omega)^{\frac{1}{\mu+1}}(p^N)^{\frac{\mu}{\mu+1}}\right]^{\frac{1+\mu}{\mu}}$. Equation 19 is the standard Euler equation

[15] Although returns are indexed to terms of trade shock, our modeling approach potentially sheds light on the implications of RER indexation as well. In this model, the aggregate price index (i.e., the RER) is an increasing function of the relative price of nontradables (p^N), which is determined at equilibrium in response to endowment shocks.

equating marginal utility at date t to that of date $t+1$. Equation 20 equates the marginal rate of substitution between tradables consumption and nontradables consumption to the relative price of nontradables.

We conduct a series of numerical exercises to explore the implications of indexed bonds on Sudden Stops. Those results are presented in the next section.

2.3.1 Dynamic programming representation

With introduction of liability dollarization and the borrowing constraint, the dynamic programming of the households' problem is updated as follows:

$$\begin{aligned} V(b,\varepsilon) &= \max_{b'} \left\{ u(c) + (1+c)^{-\gamma} E\left[V(b',\varepsilon')\right] \right\} \quad s.t. \\ c^T &= (1+\varepsilon)y^T - b' + (1+\phi\varepsilon)Rb \\ c^N &= y^N \\ b' &\geq -\kappa\left[(1+\varepsilon)y^T + p^N y^N\right]. \end{aligned} \tag{21}$$

As in the previous one-sector model, the endogenous state-space is given by $\mathcal{B} = \{b_1 < ... < b_{NB}\}$, and the exogenous Markov process is assumed to have two states: $\mathcal{E} = \{\varepsilon_L < \varepsilon_H\}$. Optimal decision rules, $b'(b,\varepsilon) : \mathcal{E} \times \mathcal{B} \rightarrow \mathcal{R}$, are obtained by solving the above dynamic programming problem (DPP).

2.3.2 Solving the model

We solve the stochastic simulations using value-function iteration over a discrete state-space in the [-2.5, 5.5] interval with 1,000 evenly spaced grid points. We derive this interval by solving the model repeatedly until the solution captures the ergodic distribution of bond holdings. The endowment shock has the same Markov properties described in the previous section. The solution procedure is similar to that described in Mendoza (2002). We start with an initial conjecture for the value-function and solve the model without imposing the borrowing constraint for each coordinate (b, ε) in the state-space, we then check whether the implied b' satisfies the borrowing constraint. If so, the solution is found and we calculate the implied value-function, which is then used as a conjecture for the next iteration. If not, we impose the borrowing constraint with equality and solve a system of non-linear equations defined by the three constraints given in the DPP (Equation 21) as well as the optimality condition given in Equation (20). Then, we calculate the implied value-function using the optimal b' and iterate to convergence.

2.3.3 Calibration

We calibrate the model such that aggregates in the non-binding case match certain aggregates of Turkish data. In addition to the parameters used in the frictionless one-sector model, we introduce the following parameters, the values of which we summarize in Table 2.: y^N is set to 1.3418, which implies a share of nontradables output in line with the average ratio of the non-tradable output to tradable output between 1987 and 2004 for Turkey; μ is set to 0.316, which is the value Ostry and Reinhart (1992) estimate for emerging countries; the steady-state relative price of nontradables is normalized to unity, which implies a value of 0.4027 for the CES share of tradable consumption (ω), calculated using the condition that equates the marginal rate of substitution between tradables and nontradables consumption to the relative price of nontradables (Equation 20). The elasticity of the subjective discount factor (γ) is recalculated to include the new variables in the solution of the non-linear system of equations implied by the steady-state equilibrium conditions of the model given in Equation 10. κ is set to 0.3 (i.e., households can borrow up to 30 percent of their current income), which is found by solving the model repeatedly until the model matches the empirical regularities of a typical Sudden Stop episode at a state where the borrowing constraint binds with a positive probability in the long-run.

Table 2: Parameter Values

μ	0.316	Elasticity of substitution	Ostry and Reinhart (1992)
y^N/y^T	1.3418	Share of NT output	Turkish data
p^N	1	Relative price of NT	Normalization
κ	0.3	Constraint coefficient	Set to match SS dynamics
ω	0.4027	CES weight	Calibration
γ	0.0201	Elasticity of discount factor	Calibration

2.3.4 Simulation results

The stochastic simulation results are divided into three sets. In the first set, which we refer to as the *frictionless economy*, the borrowing constraint never binds. In the second set of results, which we refer to as the *constrained economy*, the borrowing constraint occasionally binds and households can issue only nonindexed bonds. In the last set, which we refer to as the *indexed economy*, borrowing constraint occasionally binds but households can issue indexed bonds.

Our results, which compare the frictionless and constrained economies are analogous of those presented by Mendoza (2002). Hence, we emphasize the results that are specific and crucial to the analysis of indexed bonds and refer the interested reader to Mendoza (2002) for further details. Because at equilibrium, the relative price of nontradables is a convex function of the ratio of tradables consumption to nontradables consumption, a decline in tradables consumption relative to nontradables consumption as the result of a binding borrowing constraint leads to a decline in the relative price of nontradables, which makes the constraint more binding and leads to a further decline in tradables consumption.

Figure 4 shows the ergodic distributions of bond holdings. The distribution in the frictionless economy is close to normal and symmetric around its mean. The mean bond holding is -0.299, higher than the steady state bond holding of -0.35; this level reflects the precautionary savings motive that arises as a result of uncertainty and the incompleteness of financial markets. The distribution of bond holdings in the constrained economy is shifted right relative to that of the frictionless economy. Mean bond holdings in the constrained economy are 0.244, which reflects a sharp strengthening in the precautionary savings motive due to the borrowing constraint.

Table 8 presents the long-run business cycle statistics for the simulations. Relative to the frictionless economy, the correlation of consumption with the tradables endowment is higher in the constrained economy. In line with this strong co-movement, the persistence (autocorrelation) of consumption is lower in the constrained economy.

Behavior of the model can be divided into three ranges. In the first range, debt is sufficiently low that the constraint is not binding. In this case, the response of the constrained economy to a negative endowment shock is similar to that of the frictionless economy, and a negative endowment shock is smoothed by a widening in the current account deficit as a share of GDP. In addition debt levels are too high in a range of bond holdings. In this range, the constraint always binds regardless of the endowment shock. At more realistic debt levels, however, where the constraint only binds when the economy suffers a negative shock, the model with nonindexed bonds roughly matches the empirical regularities of Sudden Stops. This range, which we call the "Sudden Stop region" following Mendoza and Smith (2005), corresponds to grid points 218 to 230.

In Figure 5, we plot the conditional forecasting functions of the frictionless and constrained economies for tradables consumption, aggregate consumption, the relative prices of nontradables, and the current account-GDP ratios, in response to an endowment shock of one-standard deviation. These forecasting functions are conditional on the 229th bond grid, which is one of the Sudden Stop states and has a long-run probability of 0.47 percent, and they are calculated as percentage deviations from the long-run means of their frictionless counterparts.[16]

As the graphs suggest, the response of the constrained economy is dramatic. The endowment shock results in a 4.1 percent decline in tradable consumption, compared with a decline of only 0.9 percent in the frictionless economy. In line with the larger collapse in the tradables consumption, the responses of aggregate consumption and the relative price of nontradables are more dramatic in the constrained economy than in the frictionless economy. Whereas households in the frictionless economy are able to absorb the shock via adjustments in the current account (the current account deficit slips to 1.4 percent of GDP), households in the constrained economy cannot because of the binding borrowing constraint (the current account shows a surplus of 0.02 percent of GDP). These figures also suggest that the effects of Sudden Stops are persistent. It takes more than 40 quarters for these variables to converge back to their long-run means.

Figures 6, 7, and 8 compare the detrended conditional forecasting functions of the constrained economy with that of the indexed economy to illustrate how indexed bonds can help smooth Sudden Stop dynamics (the degrees of indexation are provided on the graphs).[17] As Figure 6 suggests, when the degree of indexation is 0.05, indexed bonds provide little improvement over the constrained case; indeed, the difference in the forecasting functions is not visible. When indexation reaches 0.10, however, the improvements are minor yet noticeable. At this degree of indexation, aggregate consumption rises 0.11 percent, tradables consumption rises 0.24 percent, and the relative price of nontradables increases 0.30 percent.

With increases in the degree of indexation to 0.25 and 0.45, the initial effects are relatively small. Figure 7 suggests that the improvements in tradables consumption are close to 1 percent and 1.8 percent when the degrees of indexation are 0.25 and 0.45, respectively. Figure 8 suggests that when the degree of indexation becomes higher, 0.7 and 1.0, for example, tradables consumption and aggregate consumption fall below the constrained case after the fourth quarter and stay below for more than 30 quarters, despite the initially small effects of a negative endowment shock. In other words, degrees of indexation higher than 0.45 in an indexed economy

[16] Bond holdings on this grid point are equal to -0.674, which implies a debt-to-GDP ratio of 30 percent.

[17] These forecasting functions are detrended by taking the differences relative to the frictionless case.

imply more pronounced detrimental Sudden Stop effects than in a constrained economy.

Table 9 summarizes the initial effects of both a negative and a positive shock conditional on the same grid points used in the forecasting functions. When indexed bonds are in place, our results suggest that if the degree of indexation is within [0.05, 0.25], indexed bonds help to smooth the effects of Sudden Stops. As Table 9 suggests, when the degree of indexation is 0.05, indexed bonds provide little improvement. As the degree of indexation increases, the initial impact of a negative endowment shock on key variables decreases. In this case, debt relief accompanies a negative endowment shock, and that relief helps reduce the initial impact of a binding borrowing constraint. Hence, the depreciation in the relative price of nontradables is milder, an effect that, in turn, prevents Fisherian debt deflation.

Table 9 also suggests that although the smallest initial impact of a negative endowment shock occurs when the degree of indexation is unity (full-indexation), this level of indexation has significant adverse effects if a positive shock occurs. In this case, households must pay a significantly higher interest rate over and above the risk-free rate. Although the constrained economy is not vulnerable to a Sudden Stop when a positive endowment shock occurs, agents in such an economy face a Sudden Stop from a sudden jump in debt-servicing costs.

Hence, our analysis suggests that households face a tradeoff when they engage in debt contracts with high degrees of indexation. If the households are hit by a negative endowment shock, highly indexed bonds can allow them to absorb the shock without suffering severely in terms of consumption. Such a shock might trigger a Sudden Stop if households were to borrow instead using nonindexed bonds (the initial effects are closest to the frictionless case when the degree of indexation is 1). If households receive a positive endowment shock, however, the initial effects are larger in the indexed economy (where the degree of indexation equals 1) than in the constrained economy (e.g., the impact on tradable consumption jumps from -1.1 percent to -6.7 percent). Analyzing the results in columns 3-9 of Table 9 shows that degrees of indexation in the [0.45, 1.0] interval lead to stronger Sudden Stop effects. If one takes the average of initial responses across the high and the low states in this range of values, one finds that the minimum of those averages is attained when the degree of indexation is 0.25, a result suggesting that households with concave utility functions would attain a higher utility with this consumption profile than ones achieved with indexation levels higher than 0.25.

In Figure 9, we plot the time-series simulations of the frictionless, constrained, and indexed economies. The simulations are derived first by generating a random, exogenous endowment-shock process using the transition matrix, $\mathcal{P}$, and then by feeding these series into each of the respective economies. As the graphs reveal, although patterns of consumption in each economy mostly move together, in some cases (around periods 2000, 3600, 6500, and 8800), sharp declines in constrained economy are seen. Those declines correspond to Sudden Stop episodes. In those cases, a consecutive series of negative endowment shocks makes the constraint binding, which in turn triggers a debt deflation that leads to a collapse in consumption.

When the return is indexed and the degree of indexation is 0.05 (top right graph), the volatility of consumption is noticeably lower than in the constrained case, and collapses in consumption during Sudden Stop episodes are milder. When the degree of indexation increases to 0.45, however, the volatility of consumption significantly increases, and more frequent collapses occur than the constrained case. When the degree of indexation is 1.0, a spike in volatility and much more frequent and sizeable collapses in consumption occur than the economies with lower degrees of indexation. The simulations illustrate that when indexation is full, the effect on consumption can be significantly negative, and moreover that indexation can yield benefits for consumption volatility only if the degree of indexation is quite low.

Table 8 suggests that in addition to the tradeoff of gains in the low state for losses in the high state, a short-run versus long-run tradeoff exists with respect to issuing indexed bonds with high degrees of indexation. With higher indexation levels, indexed bonds can generate substantial short-run benefits, but higher indexation levels also introduce more severe adverse effects in the long-run (i.e., consumption volatility and its co-movement with income increase with greater degrees of indexation). Consistent with our findings in the frictionless one-sector model, the value of indexation that minimizes the co-movement of consumption with GDP and yields more persistent consumption is low (in the range of [0.05, 0.1] for this calibration). These results also suggest that, depending on the objectives, the optimal degree of indexation level may vary. As illustrated earlier, the level of indexation that would minimize the effect of Sudden Stops is in the [0.25, 0.45] interval, whereas the level that minimizes long-run fluctuations is in the [0.05, 0.1] range. Regardless of whether one would like to smooth Sudden Stops or long-run fluctuations, full-indexation is undesirable.

2.4 Sensitivity Analysis

This section presents the results of analysis aimed at evaluating the robustness of our results to several variations in model parameterization. Due to space limitations, for the first three sensitivity analysis we present result of the the frictionless one-sector model. These results are summarized in Table 10.

We first analyze the robustness of the results to changes in the number of exogenous state variables. For this analysis, we use a seven-state Markov chain that maintains the same autocorrelation and standard deviation of the shock as in the previous framework. Note that the simple persistence rule can be applied only if the number of exogenous-state variables is two. To create the transition matrix with seven exogenous states, we employ the method described in Tauchen and Hussey (1991). The first block in Table 10 presents key long-run statistics, which are nearly identical to the ones presented in Table 5; in fact, for a given indexation level, the statistics are the same out to two decimal points. Hence, the results are robust to the number of state variables used in the Markov process.

Second, we increase the standard deviation of the exogenous endowment shock to 4.5 percent. As Table 10 suggests, when bonds are not indexed, the precautionary savings motive is stronger, and consumption is more volatile; consumption displays greater correlation with income when variation in the magnitude of the exogenous endowment shock increases. Comparing Table 10 with Table 5 for the indexed case, we conclude that the optimal indexation level that minimizes long-run macroeconomic fluctuations is in the [0.05, 0.1] interval in the high-volatility case, whereas it is in the [0.1, 0.25] interval in the low-volatility case. In other words, the optimal degree of indexation decreases with increases in the volatility of the exogenous endowment shock.

Next, we evaluate the changes in results that arise when one lowers the autocorrelation of the endowment shock. Compared with the baseline results given in Table 5, with an endowment shock autocorrelation of 0.4, agents engage in less precautionary savings. Moreover, consumption volatility and its co-movement with income are lower. When indexed bonds are in place, the lower the persistence of the shock, the higher the degree of indexation that would minimize the co-movement of consumption with income. For instance, when the indexation is 0.1, the correlation of consumption with income is 0.07 when the autocorrelation of the shock is 0.4. By comparison, at the same indexation level, the correlation of consumption with income is 0.017 when the autocorrelation is 0.524.

As a final robustness check, we examine the effect of increasing the size of nontradables sector. The results are summarized in Table 11. We set the y^N/y^T ratio to 1.6, implying that the degree of openness of the country is lower than in the baseline case. Not surprisingly, the model in this case captures the empirical regularities of an economy with less financial integration than the baseline case. In particular, consumption is more volatile than in the baseline case (for instance, the volatility of the tradables consumption in the frictionless economy increases to 1.6 percent, compared to the baseline value of 1.5 percent), and the co-movement of consumption with income is stronger (the correlation of tradables consumption with income in the frictionless economy increases to 0.75 from the baseline value of 0.69). Comparing the initial responses of each of these economies to a 1-standard-deviation endowment shock, one finds that the response of the constrained economy with a higher share of nontradable output is stronger than that of the one with baseline parameters, an outcome that suggests that the debt-deflation process is more severe in the economy with higher share of nontradable output. This result is consistent with the empirical evidence on the relationship between the degree of openness and the severity of Sudden Stops (see Calvo et al., 2003). To compare the optimal indexation levels across different parameterizations, we compare the average responses of the economies in the high and the low states to a 1-standard-deviation endowment shock. The results suggest that the minimum average response is attained when the degree of indexation is 0.25, which is the same degree of indexation in the baseline results. This result, however, depends on the coarseness of the indexation intervals with which one is solving the problem. Economic intuition suggests that lower financial integration would require higher indexation levels to smooth exogenous shocks better.

The sensitivity analysis presented in this section suggests that the optimal indexation level depends on the properties of the exogenous shock, including its persistence and its volatility. Hence, the optimal degree of indexation must be country specific, because it is highly likely that each emerging country receives shocks with different statistical properties. The findings of this paper suggest that although indexed bonds might aid many countries in averting or at least mitigating the effects of Sudden Stops in emerging markets, an indexation level appropriate for one country might not be optimal for another.

3 Conclusion

Recent policy proposals argue that indexing the debt of emerging markets could help prevent the sudden reversals of capital inflows accompanied by RER devaluations that were typical of the emerging-market crises of the past decade. This paper explores the quantitative implications of this policy in various quantitative models of small open economies.

We conducted quantitative experiments to evaluate the effects of indexed bonds in three steps. First, we illustrated the tradeoffs that indexation could introduce in a simple two-period model. We showed that in this framework, if the low-endowment state that agents try to hedge away is less likely than a high endowment state, indexation would introduce a wealth effect working in a direction opposite to the consumption smoothing effect. Hence, improvement in welfare is a nonmonotonic function of the degree of indexation. Moreover, after a threshold value, the agents are actually worse off with indexation than without it.

In the second step, we studied the effects of bonds indexed to output in a canonical one-sector infinite-horizon small open economy model with varying degrees of indexation. The introduction of indexed bonds partially completes the insurance market in such an economy, and whether those bonds help reduce precautionary savings, the volatility of consumption, and the correlation of consumption with income depends on the degree of indexation. When this degree is higher than a critical threshold (as with full-indexation, for example), indexation can, in fact, make agents worse off. Increase in the variance of trade balance (resulting from higher interest rate fluctuations) outweighed the improvement in the covariance of trade balance with income, which then led to higher volatility of consumption; catastrophic income levels decreased, which in turn led to an increase in precautionary savings.

In the third step, we analyzed the role of indexed bonds in smoothing Sudden Stops and RER fluctuations. Indexed bonds can reduce the initial capital outflow in the event of an exogenous shock that otherwise triggers a Sudden Stop in an economy with only nonindexed bonds. Indexed bonds can, in turn, reduce the depreciation in the RER and break the Fisherian debt deflation mechanism. Once again, however, the benefit of those bonds depends critically on the degree of indexation. When the level of indexation is lower than a critical value, indexed bonds weaken Sudden Stops. If indexation is higher than this critical value, indexed bonds can provide some temporary relief in the event of a negative shock, but the initial improvement is short lived. Moreover, in the event of a positive shock, the economy is vulnerable to a Sudden Stop even though such a shock would never trigger a Sudden Stop in an economy in which household facing borrowing constraints could issue only nonindexed bonds. In this case, positive shock commands higher repayment, which increases the need for larger amount of borrowing, this shift can make the borrowing constraint suddenly binding, and trigger a debt-deflation.

To conclude, indexed bonds may not provide benefits to emerging countries if the degree of indexation is higher than a critical value (as with full-indexation). Indexed bonds with an optimal degree of indexation, however, can help those countries smooth Sudden Stops. This optimal value depends on the persistence and the volatility of the exogenous shocks a given country experiences as well as on the size of the country's nontradables sector relative to its tradables sector (i.e., the openness of the country). Hence, in terms of policy implications, our analysis reveals that the degree of indexation is a key variable that should optimally be chosen to smooth Sudden Stops; moreover this value should be country specific.

In our analysis, we assumed that investors are risk neutral and that indexing debt repayments would not require them to obtain country specific information. Indexed returns, however, may affect investors' incentives to collect country specific information. The implications of introducing risk-averse investors or informational costs in a dynamic framework are left for future research. The model also can be used to explore the implications of indexation to relative price of nontradables, or to CPI, but that is left for further research. Analyzing whether trading in option or futures markets can help emerging countries for mitigating Sudden Stops is an avenue of research. Such analysis would require a richer model, and that is left for further research as well. Another avenue for future research could be analyzing the implications of indexed bonds on default probabilities. To carry out such an analysis, indexed bonds could be introduced into "willingness to pay" models, such as those of Eaton and Gersovitz (1981), Arellano (2004), Aguiar and Gopinath (2005).

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Table 3: Previous Attempts with Indexed Bonds

Country	Date Issued	Indexation Clause	Note
Argentina	1972 – 1989	CPI	
Australia	1985 – 1988	CPI	
Bosnia and Herzegovina	1990s	GDP	Issued as part of Brady Plan, VRRs.
Brazil	1964–	CPI	
Bulgaria	1990s	GDP	Issued as part of Brady Plan, VRRs.
Colombia	1967–	CPI	
Costa Rica	1990s	GDP	Issued as part of Brady Plan, VRRs.
Chile	1956–	CPI	
Israel	1955–	CPI	
France	1973	Gold	Debt servicing cost increased significantly from depreciation of French franc against gold.
	1970s	Oil	Issued as petro-bonos.
Mexico	1990s	Oil	Issued as part of Brady Plan, VRRs.
	1989–	CPI	
Turkey	1994–	CPI	
United Kingdom	1975–	CPI	
Venezuela	1990s	Oil	Issued as part of Brady Plan, VRRs.

Sources: Borensztein and Mauro (2004), Campell and Shiller (1996), Kopcke and Kimball (1999).

Table 4: Business Cycle Facts for Emerging Countries

Variable:x	$\sigma(x)$	$\sigma(x)/\sigma(Y)$	$\rho(x)$	$\rho(x,Y)$	Sudden Stop	Sudden Stop relative to std.
Argentina					$2002:1-2$	
GDP (Y)	4.022	1.000	0.865	1.000	−12.952	3.220
tradables GDP	4.560	1.134	0.667	0.923	−15.100	3.311
nontradables GDP	3.977	0.989	0.894	0.990	−12.169	3.060
consumption	4.475	1.113	0.830	0.975	−17.063	3.813
real exchange rate	15.189	3.777	0.754	0.454	−48.177	3.172
CA/Y	0.916	0.228	0.837	−0.802	1.353	1.476
Chile					$1998:4-1999:1$	
GDP (Y)	2.093	1.000	0.731	1.000	−4.492	2.147
tradables GDP	1.833	0.876	0.473	0.762	−5.068	2.764
nontradables GDP	2.520	1.204	0.796	0.961	−4.840	1.921
consumption	4.184	1.999	0.748	0.898	−8.410	2.010
real exchange rate	0.007	0.003	0.649	0.372	−0.019	2.578
CA/Y	3.302	1.578	0.352	−0.512	10.932	3.311
Mexico					$1994:4-1995:1$	
GDP (Y)	2.261	1.000	0.799	1.000	−7.440	3.290
tradables GDP	2.682	1.186	0.712	0.921	−8.976	3.347
nontradables GDP	2.189	0.968	0.832	0.978	−6.178	2.822
consumption	4.222	1.867	0.841	0.973	−11.200	2.653
real exchange rate	8.627	3.816	0.726	0.599	−32.844	3.807
CA/Y	0.698	0.309	0.831	−0.475	2.220	3.180
Turkey					$1994:1-2$	
GDP (Y)	3.695	1.000	0.667	1.000	−10.383	2.001
tradables GDP	3.511	0.950	0.524	0.962	−10.925	3.112
nontradables GDP	4.021	1.088	0.680	0.982	−10.007	2.489
consumption	4.134	1.119	0.746	0.919	−10.098	2.443
real exchange rate	9.110	2.465	0.675	0.602	−31.630	3.472
CA/Y	2.744	0.743	0.633	−0.591	9.704	3.375

Source: Argentinean Ministry of Finance (MECON), Bank of Chile, Bank of Mexico, Central Bank of Turkey, International Financial Statistics. The data cover periods 1993:Q1-2004:Q4 for Argentina, 1986:Q1-2001:Q3 for Chile, 1987:Q1-2004:Q4 for Mexico, 1987:Q1-2004:Q4 for Turkey. Data are quarterly seasonally adjusted real series. GDP and consumption data are logged and filtered using an HP filter with a smoothing parameter 1600. Real exchange rates are calculated using the IMF definition ($RER_i = NER_i \times CPI_i/CPI_{US}$ for country i).

Table 5: Long-Run Business Cycle Statistics of the One-Sector Model

	Degree of Indexation (ϕ)								
	0.00	0.015	0.02	0.05	0.10	0.25	0.45	0.70	1.0
$E(b)$	−0.328	−0.349	−0.355	−0.385	−0.428	−0.042	0.522	1.458	2.026
$\sigma(cons)$	1.243	1.242	1.240	1.236	1.209	1.474	2.119	3.291	4.731
$\sigma(tb/y)$	3.486	3.516	3.527	3.590	3.674	4.211	4.820	5.724	6.755
$\rho(cons,y)$	0.186	0.160	0.151	0.097	0.017	−0.311	−0.409	−0.381	−0.304
$\rho(tb/y,y)$	0.936	0.937	0.937	0.939	0.945	0.943	0.916	0.849	0.752
$\rho(cons)$	0.978	0.980	0.980	0.981	0.984	0.909	0.870	0.876	0.886
$\rho(tb/y)$	0.549	0.549	0.548	0.546	0.541	0.542	0.562	0.601	0.646
$welfare$	n.a.	0.0025	0.0034	0.0090	0.0146	−0.0032	−0.0092	−0.0120	−0.0136

Notes: Standard deviations are percentages of the mean. Welfare gains are percentages relative to the nonindexed model. n.a. refers to not applicable.

Table 6: Returns and Natural Debt Limits

	Degree of Indexation (ϕ)								
	0.00	0.01	0.015	0.05	0.10	0.25	0.45	0.70	1.0
$R^i(L)$	1.016	1.016	1.015	1.014	1.012	1.007	1.000	0.991	0.981
$R^i(H)$	1.016	1.016	1.016	1.018	1.019	1.025	1.032	1.040	1.051
$NDL(L)$	**-61.487**	**-62.182**	**-62.894**	-68.503	-78.431	-138.754	5440.508	106.131	48.760
$NDL(H)$	-64.517	-63.819	-63.136	**-58.642**	**-53.262**	**-41.767**	**-32.434**	**-25.353**	**-20.089**

Notes: The first two rows are the corresponding gross returns in each states. In the last two rows, the implied natural debt limits are in bold.

Table 7: Variance Decomposition Analysis for Consumption

	Degree of Indexation (ϕ)								
	0.00	0.015	0.02	0.05	0.10	0.25	0.45	0.70	1.0
$\sigma(cons)$	1.243	1.242	1.240	1.236	1.209	1.474	2.119	3.291	4.731
$var(tb)$	12.241	12.463	12.540	13.008	13.638	17.707	22.903	31.959	44.788
$cov(tb, y)$	11.508	11.620	11.660	11.897	12.248	13.929	15.365	16.724	17.364
$var(tb)$ $-2cov(tb, y)$	-10.775	-10.777	-10.781	-10.792	-10.857	-10.147	-7.827	-1.488	10.061

Table 8: Long-Run Business Cycle Statistics of the Two-Sector Model

	Degree of Indexation (ϕ)							
	F	C	0.05	0.10	0.25	0.45	0.70	1.0
$E(b)$	-0.299	0.244	0.122	0.276	0.594	1.599	2.328	2.516
$\sigma(c^T)$	1.530	1.268	1.251	1.389	1.851	2.835	3.914	5.266
$\sigma(c)$	0.775	0.638	0.631	0.697	0.923	1.392	1.889	2.508
$\sigma(p^N)$	2.026	1.682	1.660	1.845	2.467	3.804	5.291	7.162
$\sigma(tb/y)$	1.534	1.467	1.491	1.610	1.799	2.113	2.398	2.755
$\rho(c^T, y)$	0.687	0.663	0.636	0.567	0.609	0.773	0.875	0.930
$\rho(c, y)$	0.687	0.664	0.637	0.567	0.608	0.770	0.870	0.924
$\rho(p^N, y)$	0.687	0.663	0.636	0.567	0.609	0.774	0.877	0.933
$\rho(tb/y, y)$	0.512	0.648	0.646	0.548	0.290	-0.141	-0.404	-0.580
$\rho(c^T)$	0.986	0.971	0.976	0.967	0.953	0.926	0.911	0.907
$\rho(c)$	0.986	0.971	0.976	0.967	0.953	0.925	0.909	0.903
$\rho(p^N)$	0.986	0.971	0.976	0.967	0.953	0.927	0.912	0.909
$\rho(tb/y)$	0.581	0.546	0.540	0.546	0.572	0.609	0.631	0.661

Notes: The first column is the frictionless economy, the second column is the constrained economy, and the rest of the columns are for the economy with borrowing constraints and indexed bonds (with given degrees of indexation). Standard deviations are percentages.

Table 9: Initial Responses to a 1-Standard-Deviation Endowment Shock

	Non-Indexed		Degree of Indexation (ϕ)					
	F	C	0.05	0.10	0.25	0.45	0.70	1.0
Negative Shock								
tradable consumption	-0.907	-4.126	-4.007	-3.888	-3.531	-3.056	-1.657	-1.748
aggregate consumption	-0.384	-1.780	-1.728	-1.676	-1.520	-1.312	-0.706	-0.745
relative price of nontradables	-1.197	-5.398	-5.244	-5.090	-4.626	-4.007	-2.179	-2.299
Positive Shock								
tradable consumption	-0.291	-1.095	-2.019	-2.138	-2.494	-2.970	-4.369	-6.691
aggregate consumption	-0.120	-0.464	-0.862	-0.913	-1.068	-1.275	-1.887	-2.919
relative price of nontradables	-0.387	-1.444	-2.653	-2.808	-3.274	-3.895	-5.714	-8.716

Notes: The first column is the frictionless economy, the second column is the constrained economy, and the rest of the columns are for the economy with borrowing constraints and indexed bonds (with given degrees of indexation). Initial responses are calculated as percentage deviations relative to the long-run mean of the frictionless economy.

Table 10: Sensitivity Analysis of the One-Sector Model

	Degree of Indexation (ϕ)								
	0.00	0.015	0.02	0.05	0.10	0.25	0.45	0.70	1.0
1. seven-state markov chain									
$E(b)$	-0.320	-0.345	-0.351	-0.369	-0.371	-0.083	0.548	1.459	1.968
$\sigma(cons)$	1.246	1.245	1.244	1.243	1.258	1.487	2.147	3.319	4.776
$\rho(cons, y)$	0.182	0.154	0.144	0.079	0.031	-0.301	-0.410	-0.378	-0.293
$\rho(cons)$	0.970	0.971	0.971	0.974	0.982	0.906	0.869	0.870	0.869
2. σ_ε=0.045									
$E(b)$	-0.315	-0.335	-0.343	-0.359	-0.295	-0.017	0.908	1.741	2.064
$\sigma(cons)$	1.567	1.566	1.566	1.560	1.576	1.919	2.899	4.372	6.226
$\rho(cons, y)$	0.208	0.173	0.160	0.085	-0.046	-0.270	-0.357	-0.307	-0.230
$\rho(cons)$	0.983	0.987	0.988	0.991	0.974	0.927	0.892	0.893	0.898
3. ρ_ε=0.4									
$E(b)$	-0.335	-0.357	-0.361	-0.398	-0.477	-0.300	0.180	0.918	1.637
$\sigma(cons)$	1.074	1.069	1.068	1.060	1.034	1.202	1.462	2.229	3.351
$\rho(cons, y)$	0.178	0.157	0.152	0.112	0.070	-0.167	-0.361	-0.367	-0.301
$\rho(cons)$	0.966	0.968	0.969	0.970	0.975	0.944	0.865	0.865	0.885

Note: Resulting transition matrix for seven-state markov chain is approximated using the method described in Tauchen and Hussey (1991). Standard deviations are percentages.

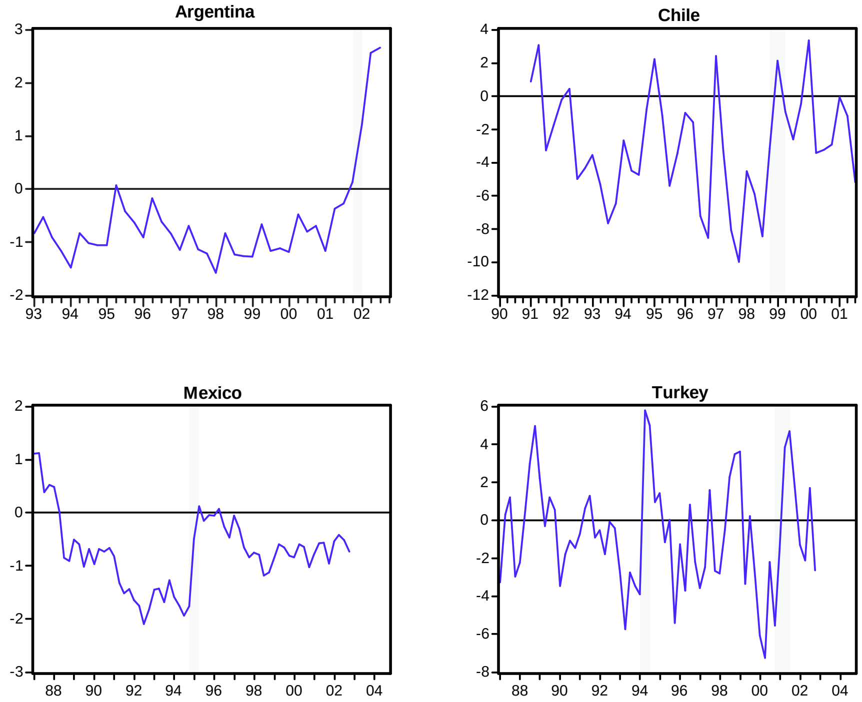
Table 11: Sensitivity Analysis of the Two-Sector Model: Higher Share of Non-tradable Output

	Degree of Indexation (ϕ)							
	F	C	0.05	0.10	0.25	0.45	0.70	1.0
1. Long run statistics								
$E(b)$	-0.290	0.258	0.084	0.682	0.667	1.739	2.399	2.551
$\sigma(c^T)$	1.590	1.306	1.261	1.639	1.957	2.919	3.956	5.300
$\sigma(c)$	0.822	0.671	0.649	0.836	0.994	1.457	1.941	2.565
$\sigma(p^N)$	2.105	1.734	1.672	2.182	2.609	3.920	5.351	7.211
$\rho(c^T, y)$	0.749	0.716	0.691	0.664	0.714	0.844	0.913	0.951
$\rho(c, y)$	0.750	0.718	0.692	0.664	0.713	0.841	0.908	0.945
$\rho(p^N, y)$	0.749	0.716	0.691	0.664	0.714	0.845	0.915	0.953
$\rho(c^T)$	0.987	0.975	0.975	0.973	0.956	0.931	0.914	0.909
$\rho(c)$	0.987	0.976	0.976	0.974	0.956	0.930	0.911	0.905
$\rho(p^N)$	0.987	0.975	0.975	0.973	0.957	0.932	0.915	0.911
2. Initial Responses								
Negative Shock								
tradable consumption	-1.036	-4.254	-4.122	-3.991	-3.596	-3.070	-1.608	-1.623
aggregate consumption	-0.395	-1.655	-1.603	-1.551	-1.395	-1.187	-0.616	-0.622
relative price of non-tradables	-1.366	-5.565	-5.395	-5.224	-4.711	-4.025	-2.115	-2.135
Positive Shock								
tradable consumption	-0.420	-2.029	-2.156	-2.292	-2.686	-3.213	-4.675	-7.074
aggregate consumption	-0.157	-0.780	-0.818	-0.883	-1.037	-1.244	-1.823	-2.788
relative price of non-tradables	-0.557	-2.666	-2.985	-3.010	-3.525	-4.211	-6.111	-9.208

Notes: y^N/y^T ratio is set to 1.6 in this analysis. Standard deviations are percentages of the mean. The first column is the frictionless economy, the second column is the constrained economy, and the rest of the columns are for the economy with borrowing constraints and indexed bonds (with given degrees of indexation).

Figure 1: Sudden Stops in Emerging Markets

a. Current Account-GDP Ratio



b. Real Exchange Rate

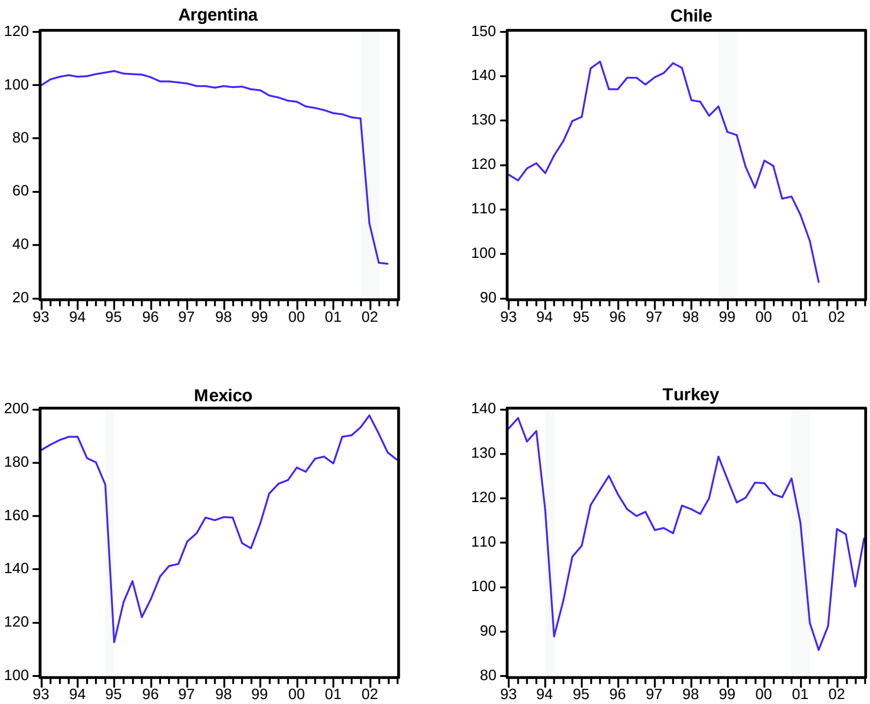
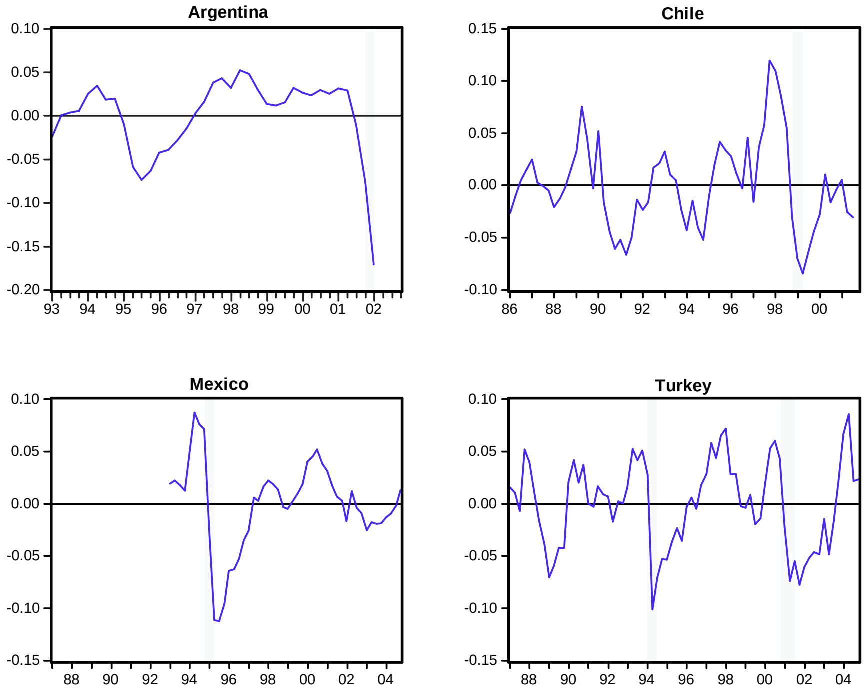


Figure 2: Deviations from Trend in Consumption and Ouput

a. Consumption



b. Output

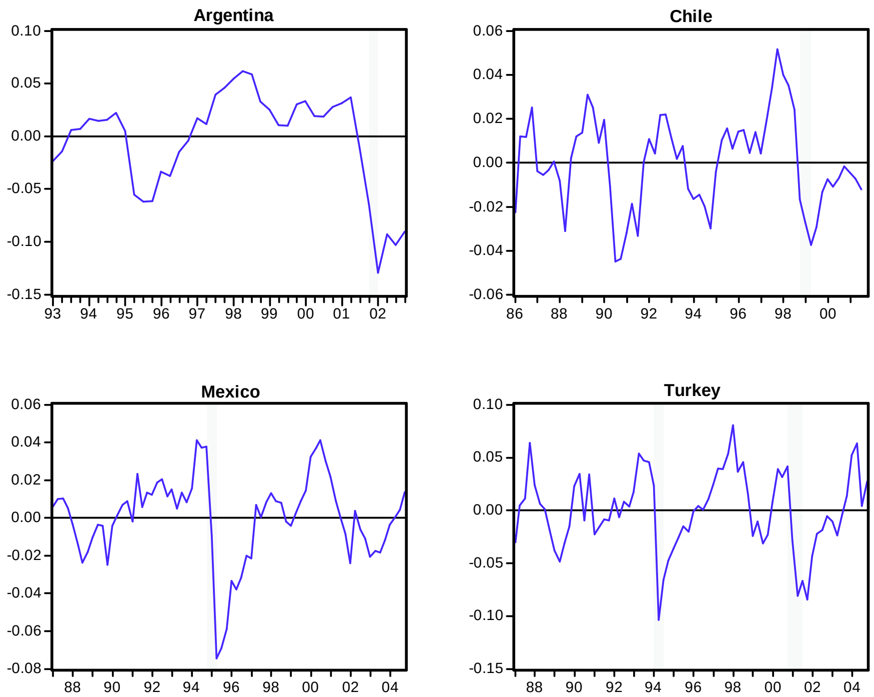
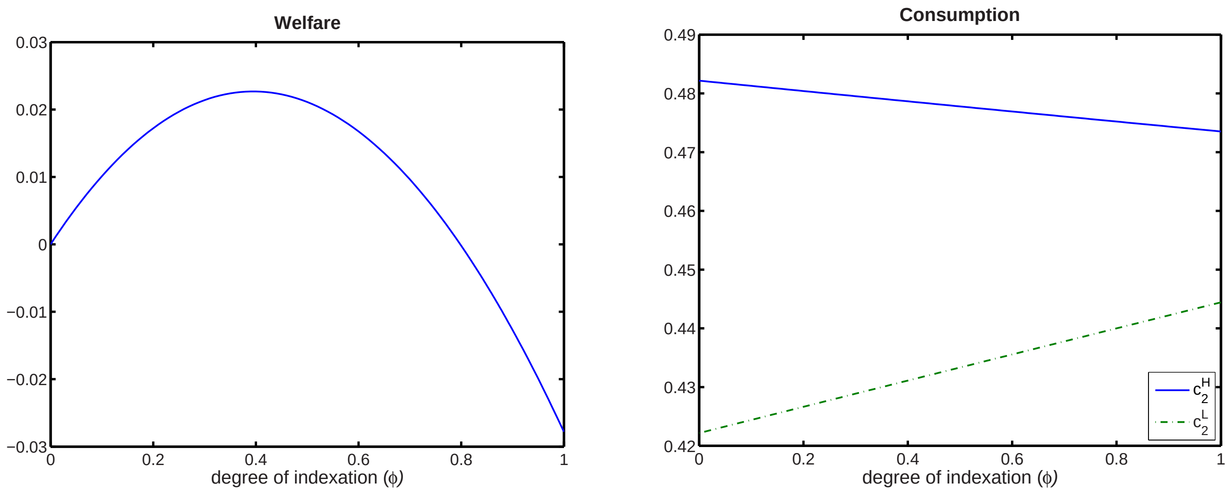


Figure 3: Welfare and Consumption of the Two-Period Model as a Function of the Degree of Indexation



Note: Welfare changes are utilitarian percentage changes relative to the nonindexed economy.

Figure 4: Long-Run Distributions of Bond Holdings in Nonindexed Economies

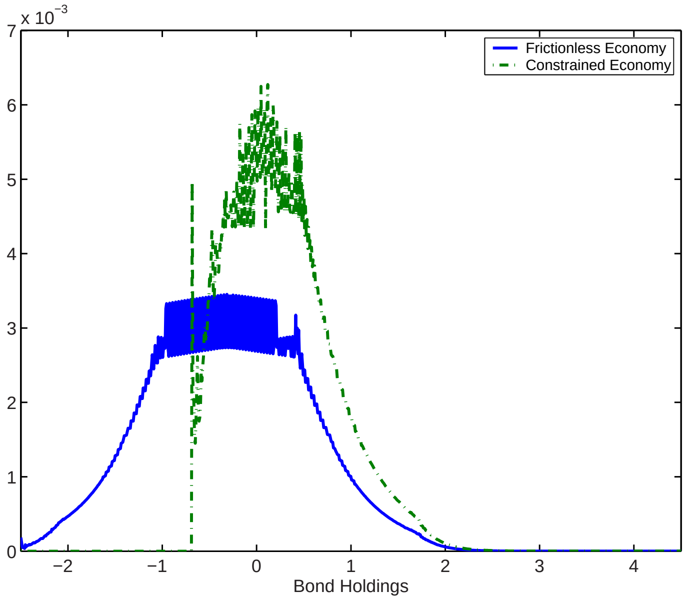
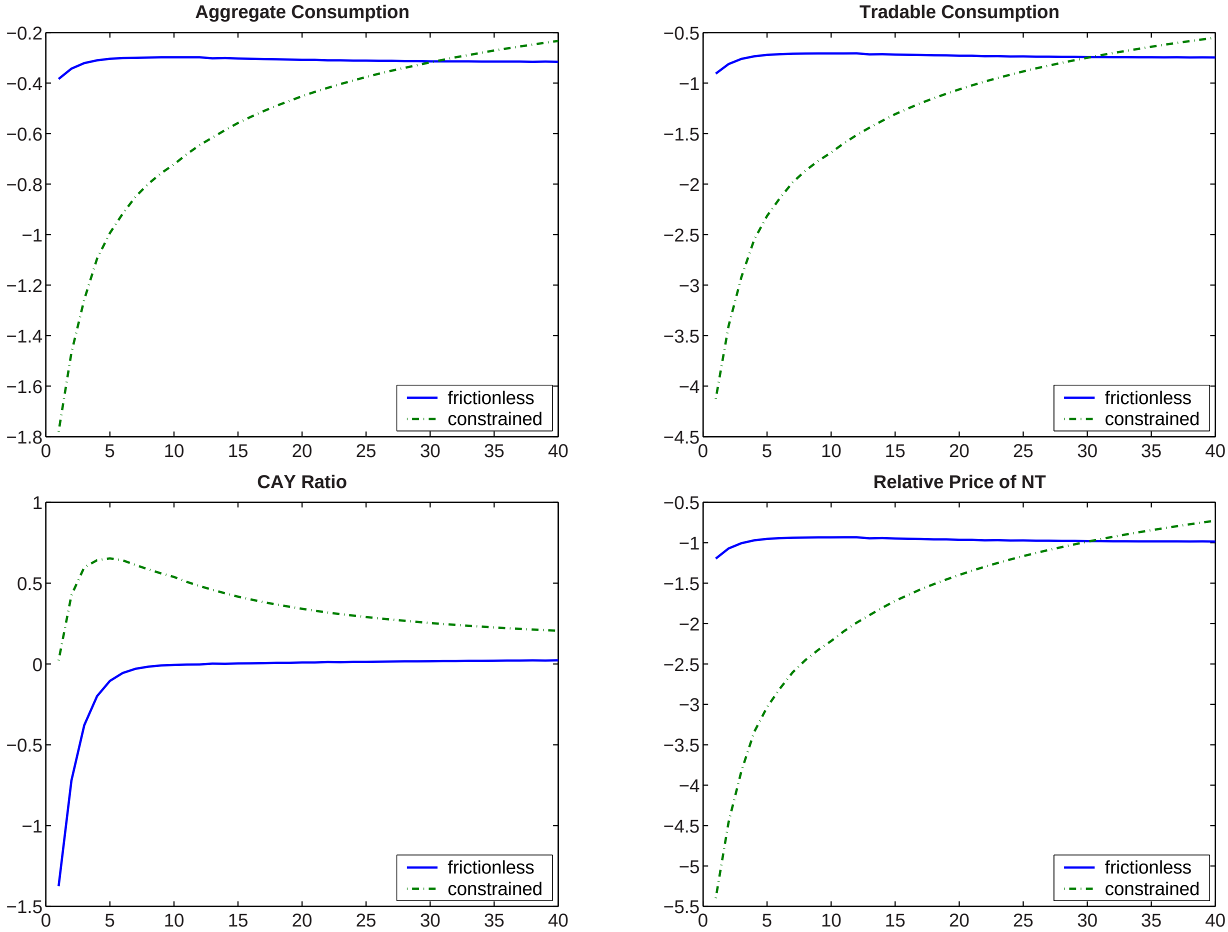
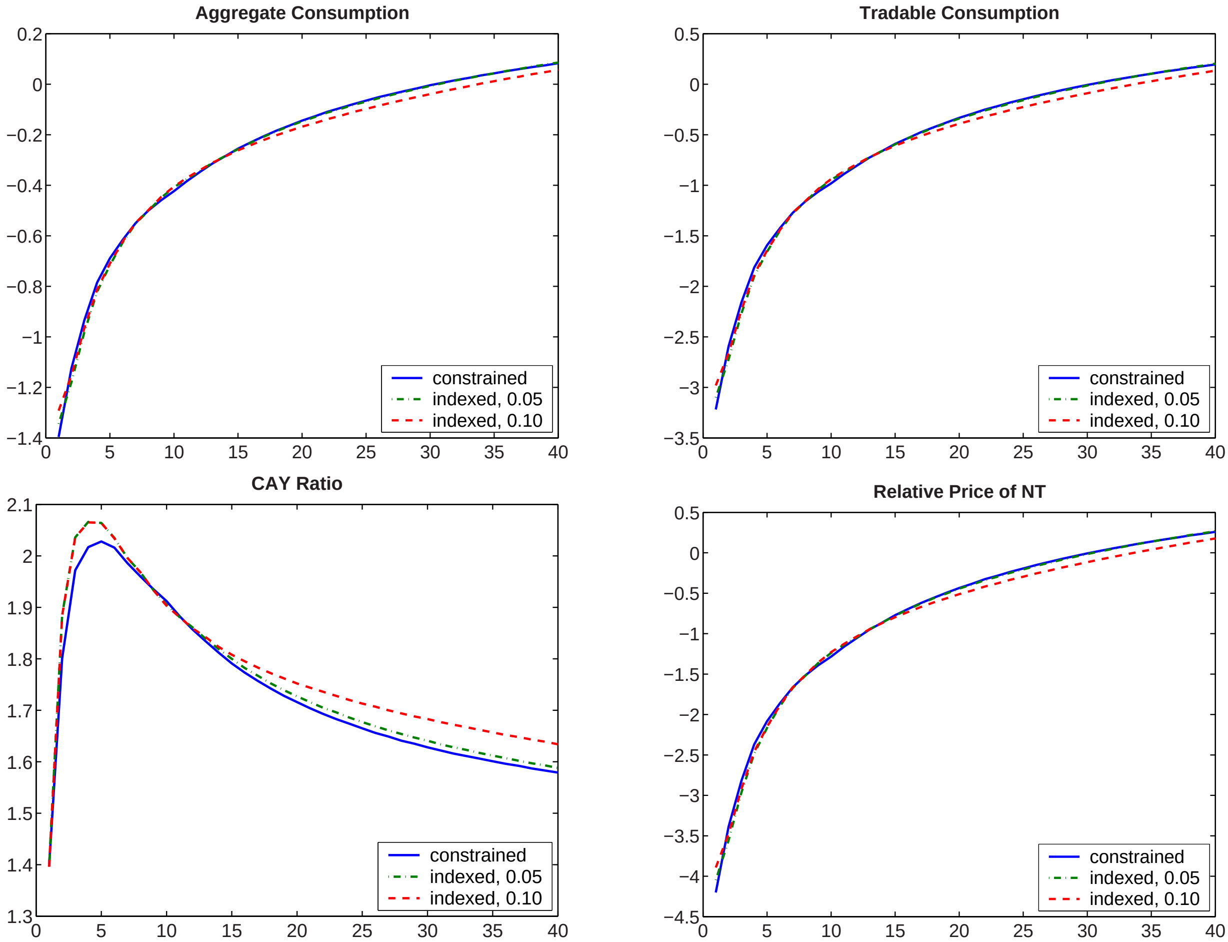


Figure 5: Conditional Forecasting Functions in Response to a 1-Standard-Deviation Negative Endowment Shock



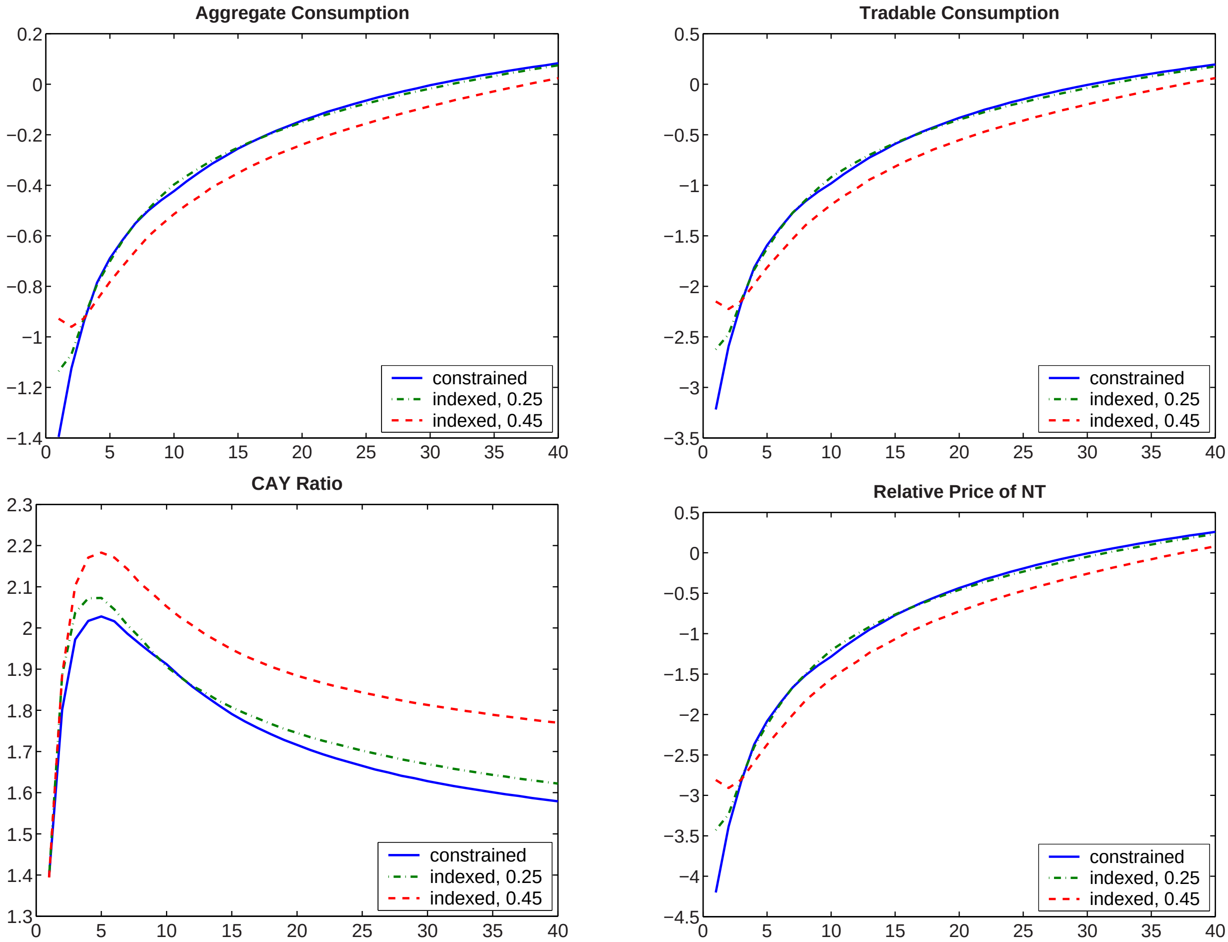
Note: Forecasting functions are conditional on the 229th grid point of the bond holdings, which implies a debt-to-GDP ratio of 30 percent. Solid and dashed lines are forecasting functions of the frictionless and constrained economies, respectively. NT refers to nontradable goods.

Figure 6: Conditional Forecasting Functions in Response to a 1-Standard-Deviation Negative Endowment Shock



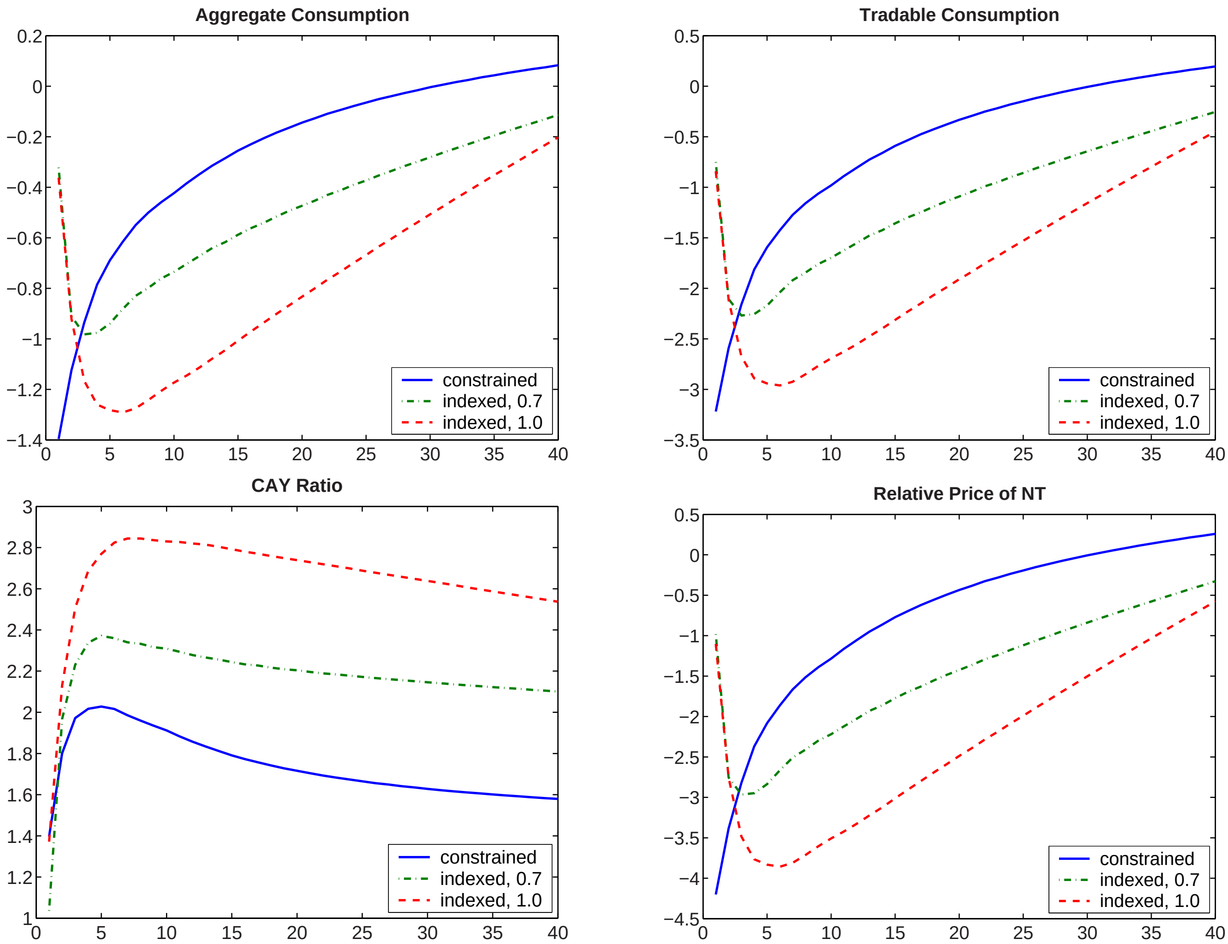
Note: Forecasting functions are conditional on the 229th grid point of the bond holdings, which implies a debt-to-GDP ratio of 30 percent. NT refers to nontradable goods.

Figure 7: Conditional Forecasting Functions in Response to a 1-Standard-Deviation Negative Endowment Shock



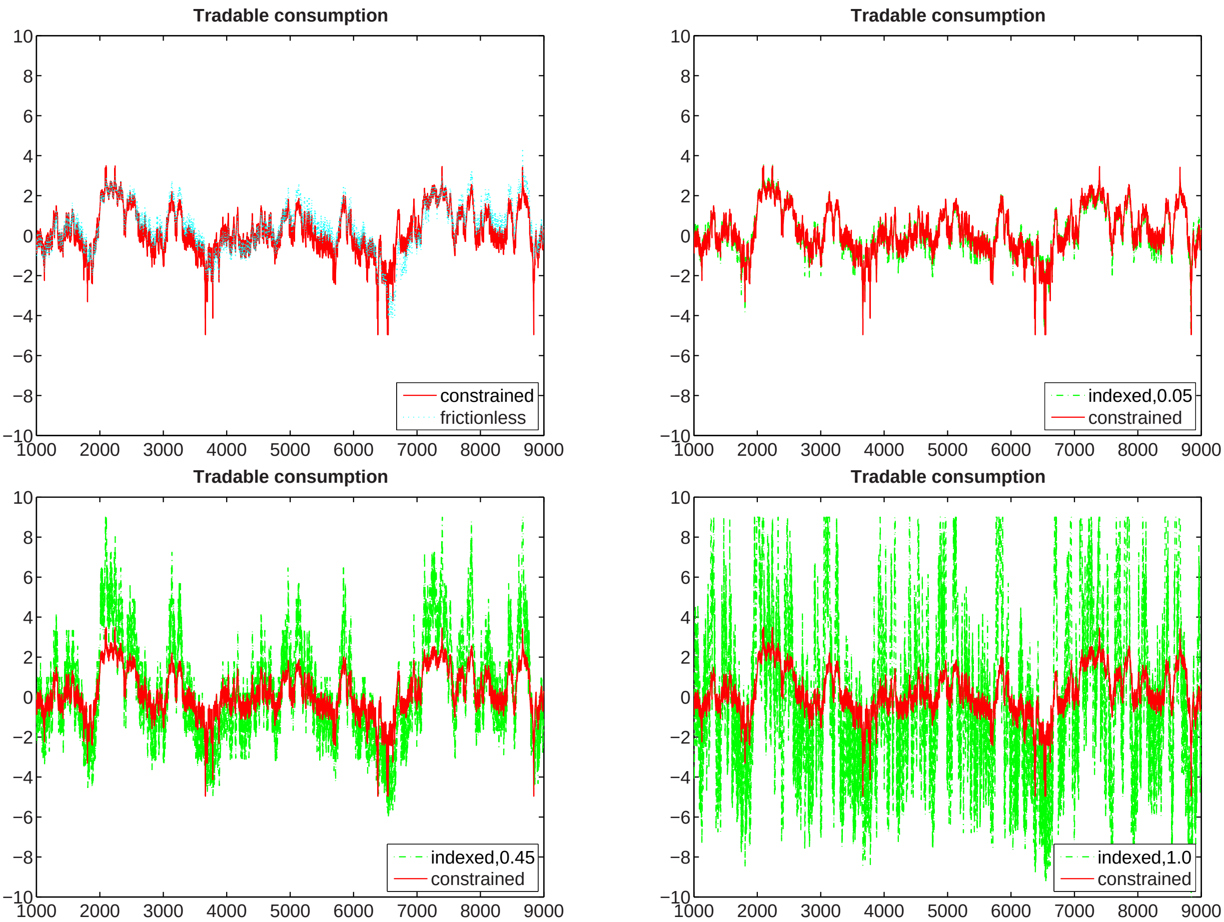
Note: Forecasting functions are conditional on the 229th grid point of the bond holdings, which implies a debt-to-GDP ratio of 30 percent. NT refers to nontradable goods.

Figure 8: Conditional Forecasting Functions in Response to a 1-Standard-Deviation Negative Endowment Shock



Note: Forecasting functions are conditional on the 229th grid point of the bond holdings, which implies a debt-to-GDP ratio of 30 percent. NT refers to nontradable goods.

Figure 9: Time Series Simulation



Note: On the top left graph, the dotted line is the tradable consumption series for the frictionless economy. The solid line is the series for the constrained economy. Consumptions are in percentage deviations from their corresponding means. The first 1,000 periods have been excluded from the graphs to focus on the data which are independent of initial conditions. Space limitations require us to leave out the figures associated with other degrees of indexation.