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Health Shocks and the Demand for Annuities

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Abstract

A new explanation is offered for the thin private market for individual annuities in the United States. Individuals face a risk of health shocks which simultaneously cause large uninsured expenses and shorten the life expectancy. The value of a life annuity then decreases at the same time as the need for cash increases, undermining its effectiveness in providing financial security. When the risk of such health shocks is substantial, it is no longer optimal for risk-averse individuals with uncertain life spans to hold all of their wealth in life annuity form, even if annuity contracts are reversible, and bequest motives, transaction costs and adverse selection are absent.

A dynamic programming model is used to compute the demand for annuities, under conditions involving health shocks, in an overlapping-generations setting calibrated to resemble the United States economy. The model is used to estimate the demand for life annuities and the relative significance of the factors that affect the demand: health shocks, Social Security, bequest motives and premium loads. It is useful for understanding various modes of drawing down retirement savings, measuring the extent of uninsured health-related risks (particularly long-term care expenses) and providing a consistent framework for analysis of various approaches to insure such risks.

1. Introduction

It has become conventional wisdom that life annuities are the “best” retirement savings vehicle for most people. Yaari (1965) proved that a risk-averse individual with uncertain life spans, intertemporally separable utility and no bequest motives would find it optimal to hold all assets in life annuity form if the annuity can be purchased at the actuarially fair premium, namely, without transaction costs (policy loading). Moreover, more recent work has shown that the more restrictive of Yaari’s assumptions are not necessary: Davidoff, Brown and Diamond (2002), for instance, proved that it is sufficient to assume that agents have no desire to leave bequests and that premiums are not too far from being actuarially fair.

In contrast with this theoretical result, the actual market for individual life annuities in the United States is very thin. Premiums for individual immediate annuities totaled $7 billion in 1999. (Brown et al., 2001) Other financial products marketed as individual annuities are far more popular, but they are used primarily as tax-efficient investment vehicles in the accumulation phase, rather than as a lifetime source of income in the payout phase (e.g., Mitchell et al., 1999).[1]

Several explanations have been offered for the discrepancy between the standard theoretical result and the observed state of the market. Premium loading is always present and is an obvious departure from Yaari’s assumptions, but the observed premiums are not so far from actuarially fair that consumers would not want to annuitize at least a significant portion of their wealth at the usually assumed levels of risk aversion (Mitchell et al., 1999). Social Security and defined-benefit pensions provide a mandatory life

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[1] Examples of such investment products are single-premium deferred annuities and variable annuities.

annuity for most Americans, crowding out the market for individual annuities (Townley and Boadway, 1988), but this does not explain why wealthier Americans without defined-benefit pensions do not purchase significantly more life annuities, nor why most elderly individuals did not hold private annuities prior to the introduction of Social Security (Warshawsky 1988). Marriage provides some insurance against the mortality risk (Kotlikoff and Spivak, 1981), but the insurance is incomplete, depends on the assumption of uncorrelated mortality between spouses, and does not apply at all to unmarried individuals. Another common, and powerful, explanation is the bequest motive (Hurd 1989, Jousten 2001), but it again fails to explain non-purchase of annuities by individuals without children.

Another popular explanation has been that the disciplined, gradual income stream of annuities leaves individuals liquidity-constrained. In its simple form, this explanation requires the restriction that life annuities, once purchased, may not be sold, and this situation is not entirely realistic. Even if law or market failure prevents individuals from actually selling annuities on their own life, a negative life annuity can be constructed by simultaneously borrowing and purchasing term life insurance. Since it is legal and feasible for someone to own both an annuity and a life insurance policy, the assumption that annuities are irrevocable may be hard to defend. This paper introduces a closely related situation, in which annuity values fall as liquidity needs rise, but no exogenous restrictions on annuity purchases and sales are assumed.

In Yaari's and all related models, uncertainty is limited to *individual* mortality, but the *probabilities* of survival for any number of future years are known and deterministic. This assumption is not trivial. Anyone who has bought life insurance and gone through the underwriting process likely knows that future life expectancy depends on a number of variables; most of those variables are health-related and many are revealed at random times during a person's life.[2] If, additionally, there is a positive correlation between available assets and life expectancy, Yaari's result does not hold: full annuitization is no longer optimal. This paper focuses on the effect of wealth-affecting stochastic changes in health (henceforth "health shocks").

An idea related to this paper has been analyzed by Brugiavini (1993). The main similarity is that both papers use information updating to explain low annuity demand.[3] The nature of this updating, as well as the model used, is different. Brugiavini considers an adverse selection economy, with ex ante heterogeneous agents who learn their type in the early period of life. The present paper assumes ex ante identical agents exposed to random shocks, and it does not assume asymmetric information (although it is possible to extend the model to include it). Furthermore, Brugiavini limits the analysis to a three-period model and derives a prediction with a complete-market flavor: no trades occur after the first period. We employ a more realistic computational model which can accommodate more flexible assumptions, and the predictions generally turn out to be quite sensitive to the parameters used.

To estimate the magnitude of deviation of optimal asset allocation from full annuitization, individual choices are simulated in an overlapping-generation economy with realistically calibrated relevant parameters. In this model, health shocks can be turned on and off and their magnitude changed; moreover, same can be done to Social

[2] Some of the health information is about genetics (e.g., family history) and doesn't change through life, but other is about personal history and may change (usually for the worse) at any time. Typical questions in life insurance applications might be "Have you been diagnosed with cancer/heart disease/AIDS/etc."

[3] One further connection between the two papers is that neither poses any exogenous restrictions on individuals selling or undoing life annuities.

Security, premium loads and bequest motives, so that the relative sizes of their respective effects on annuity demand can be compared.

2. Basic Framework

2.1. Health Shocks

Consider a currently healthy individual whose entire wealth is annuitized, as Yaari's model would predict, and who faces a risk of illness which would require expensive treatment during the next year. If that individual becomes ill, two adverse effects follow: probability of survival in current and future years decreases, and the need for cash increases due to the medical and related expenses. We will refer to health changes with those two effects as health shocks. The necessary medical expense — required for survival or simply for maintaining a minimally acceptable quality of life — can consume a large portion of the individual's wealth and can easily be larger than the annual income provided by the annuity. Whether these expenses are insurable or not, the aggregate effect of uninsured expenses is significant: one in four petitioners for personal bankruptcy identify an illness or injury as a reason for filing for bankruptcy; among petitioners age 65 or higher, the proportion is roughly one in two (Warren, Sullivan and Jacoby, 2000). Following a health shock, the affected individual might like to sell (or undo) the annuity in order to pay for the health-related expenses. Such a transaction, however, would be based on the updated mortality table, which incorporates the knowledge of illness and consequently higher probabilities of death.[4] The value of the annuity would now be lower than that for a healthy individual of the same age: health

[4] This paper assumes symmetric information throughout.

shocks reduce the value of annuities precisely at the times annuitants would prefer to liquidate. If this effect is significant, it is better not to annuitize all wealth, and it may be best not to annuitize at all.

Various health conditions can cause catastrophic health care expenses, but costs of most hospital stays and medical treatments are typically well insured for a majority of Americans, including virtually all those over age 65, who are covered by Medicare. Many people are still exposed to large out-of-pocket expenses, as the cited bankruptcy data demonstrate, but the actual exposure to expense due to acute illness is hard to estimate. Nursing home stays, however, cost tens of thousands of dollars per year, are usually not eligible for Medicare reimbursement, and are rarely insured in the private market. The practically relevant, and straightforward to calibrate, interpretation of health shocks is thus as onsets of long-term-care (LTC) episodes. In addition, when working-age individuals are considered, the financial part of a health shock may take the form of a loss of earnings in case of disability.

2.2. Assets: Bonds and Annuities

Individual decisions in this model involve the consumption and portfolio choice with two available securities: a bequeathable asset and life annuity. No result will depend critically on the kind of bequeathable asset used; we will use risk-free bonds as the simplest asset to model. There is no loss of generality from this as long as the insurance company that provides annuities can hold the same portfolio of assets backing that annuity as the individuals would hold themselves. No matter what that portfolio is, the expected return on annuity will be higher by the transfer of wealth from those who die to those who survive.[5] Annuities yield a higher expected rate of return, since their payout includes a survivorship premium (transfer from decedents to survivors), but they involve risk. In the traditional model, with deterministic survival probabilities, annuities are effectively riskless because their return, *conditional on survival*, is certain, and those are the only relevant states for individuals not concerned with bequests. With health shocks, annuity returns are risky even when restricted to the states in which the individual survives (and hence cares about them).

3. Analytical Aspects

3.1. A Three-Period Model

An individual lives for up to three periods, $j = 1, 2, 3$. The probability of surviving to period j+1, conditional on being alive in period j, is p_j, where $p_0 = 1$ and $p_3 = 0$. In each period the individual earns income w_j and consumes c_j, while the unconsumed portion of wealth can be saved either as a noncontingent bond yielding interest rate r or as a life annuity with a rate of return $\rho > r$ conditional on being alive. The individual's period utility $u(\cdot)$ is a concave function of consumption only, and the rate of time preference is β. The individual has no desire to bequeath any wealth. At time t the individual has total assets $A_t = a_t + b_t$, where a_t is the actuarial value of the annuity and b_t is the value of bonds. In each period the individual learns his current wealth and income and the available returns r and ρ, and decides how to allocate his resources among a, b, and c.

The individual's problem is:

[5] Furthermore, in an economy with no aggregate uncertainty, a well-diversified market portfolio is also risk-free.

$$\begin{aligned} &\max_{a,b,c} EU = u(c_1) + \beta p_1 u(c_2) + \beta^2 p_1 p_2 u(c_3) \\ &\text{s.t. } A_{j+1} = A_j + w_j - c_j + a_t \rho_j + b_j r_j, \ \ j = 1,2 \\ &\qquad c_j \le A_j + w_j, \ \forall j \\ &c_j, b_j, A_j \ge 0, \ \forall j. \end{aligned} \tag{1}$$

If $u(\cdot)$ satisfies the Inada conditions, the nonnegativity of consumption constraint can be omitted. Obviously for any increasing $u(\cdot)$ the individual consumes all wealth in the last period. In period j the uncertain quantities are $\{w_s, s > j\}$, $\{p_s, s > j\}$, and $\{\rho_s, s \ge j\}$. The last two will generally be closely related since the annuity return normally consists of a bond return and the annuity premium. The general form is:

$$1 + \rho_j = \frac{1 + E_j \pi_{j+1}}{\pi_j}, \tag{2}$$

where E_j denotes conditional expectation with respect to the period j information set, and π_j is the single premium in period j for an annuity paying 1 per period for the rest of the life, the first payment to occur at the beginning of period j+1. If the annuity premiums are actuarially fair, the return can be expressed as:

$$\begin{aligned} 1 + \rho_1 &= \left(\frac{1 + r + p_2}{1 + r + E_1 p_2} \right) \left(\frac{1 + r}{p_1} \right) \\ 1 + \rho_2 &= \frac{1 + r}{p_2} \end{aligned} \tag{3}$$

An important result follows in the simplest case, where the annuity returns ρ_s are deterministic (and, naturally, greater than the risk-free interest rate): the constraint preventing borrowing at the risk-free rate is binding and the individual will hold all savings in the annuity form whenever he holds any savings at all. This is a special case of Yaari's (1965) result and is intuitively obvious because annuities dominate bonds as an asset in all states the individual cares about. Another result under certainty is that the ratio of marginal utilities in consecutive period equals the individual's subjective discount factor: $u'(c_j) / u'(c_{j+1}) = (1+\rho_j)\,\beta\, p_j$, whenever the problem for total savings has an interior solution.[6] This is the well-known optimality condition for saving under certainty. These obvious results illustrate the view of annuities which will be taken throughout the paper: a life annuity is a (generally risky) financial asset, and annuitization decisions are essentially portfolio decisions. The only special feature of this asset is that it is a derivative of the *individual owner's* expected mortality.

Even when there is uncertainty, the full annuitization result will hold in the second-to-last period. This is because $p_3 = 0$ is certain, and hence ρ_2 is also certain, leading to the first-order condition involving the multiplier of the borrowing constraint (λ) that is essentially the same as under certainty, since both returns can be taken outside the expectation:

$$\begin{aligned}\lambda &= \beta p_2 \left\{ E\left[(1+\rho_2)\,u'(c_3)\right] - E\left[(1+r_2)\,u'(c_3)\right]\right\} \\ &= \beta p_2 (\rho_2 - r_2) E u'(c_3).\end{aligned} \tag{4}$$

The right-hand side is strictly positive, so the borrowing constraint is always binding in period 2. This also explains why the simplest model of annuity demand with updating of mortality information must have at least three periods. No demand for noncontingent assets can arise in the penultimate period even if income is uncertain. In the present model, anything interesting can happen only in the first period.

Consider the first-order condition, analogous to (4), for the first period. It can be reduced to:

$$E_1\left[(1+\rho_1)\,u'(c_2)\right] = E_1\left[(1+r)\,u'(c_2)\right] + \lambda/\beta p_1\,, \tag{5}$$

[6] If $(1+r_j)\,\beta > 1$ — and hence $(1+\rho_j)\,\beta\, p_j > 1$ — a sufficient condition for an interior solution, for any increasing utility function, is $A_1 + w_1 \geq w_2$ and $w_1 \geq w_2$.

where λ is the Lagrange multiplier of the period-1 borrowing constraint. Now the return ρ_1 is uncertain and the next-period wealth, and hence also consumption c_2, will depend on it, and will obviously increase as ρ_1 increases. Since $u(c)$ is concave, $u'(c_2)$ will decrease as ρ_1 increases, so it will be (perfectly) negatively correlated with ρ_1. Therefore:

$$E\left[(1+\rho_1)u'(c_2)\right] < E\left[1+\rho_1\right]E\left[u'(c_2)\right], \tag{6}$$

and it may be possible to satisfy (5) with $\lambda = 0$ (borrowing constraint not binding), although $E[1+\rho_1] > E[1+r]$. This simply shows that a life annuity behaves like a risky asset. It turns out that, for realistic parameters and an actuarially fair ρ_1, the magnitude of the correlation is not sufficient to produce an interior solution. If, however, income w_2 is also positively correlated with the annuity return, the correlation can be much greater and the effect will be amplified. The annuity acts as a negative hedge for the risk of changes in future income, and this makes it a less desirable asset.

As an illustration, consider the extreme case where period-2 survival probability may take values 0 or 1. Let the probability in period 1 of those events be $(1-\alpha)$ and α, respectively. According to (3), the annuity return will be either $\rho_{1S} = (1+r)^2 \big/ (1+r+\alpha)p_1 - 1$ or $\rho_{1H} = (1+r)(2+r)\big/(1+r+\alpha)p_1 - 1$. Also assume that utility is CRRA, $u(c) = c^{1-\sigma}\big/1-\sigma$, that $\beta = 1$, and income is zero in periods 1 and 3, while in period 2 it takes on values $w_{2H} = h$ if "healthy" (low mortality) or $w_{2S} = -s$ if "sick" (high mortality), where $h, s \geq 0$. Even with such a simple utility function the first-order conditions are rather cumbersome, but are easily solved numerically for various parameter values.[7] For simplicity, in this discussion we will assume $h = 0$, i.e., no

[7] It may be interesting to note that logarithmic utility, which is easier to solve, cannot give rise to less than full annuitization in this (three-period) model.

unforeseen income, and vary only s, the unforeseen expense in the sick state. Some results are shown in Tables 1 and 2.

With relative risk aversion $\sigma = 2$, initial wealth A_1 normalized to 1, interest rate $r = 0.25$, probability of survival from first to second period $p_1 = 0.9$, and probability of remaining healthy in the second period $\alpha = 0.6$, the critical value of s for which the borrowing constraint ceases to bind is 0.196 — the potential expense in the unfavorable state needs to exceed approximately one-fifth of the initial wealth in order to make it worthwhile to use bonds in the portfolio.[8] The critical value is higher for higher interest rate and lower period-one survival probability, both of which mean more discounting of future shocks. It is lower at higher levels of risk aversion as such individuals assign relatively less importance to the higher expected return of annuities; it reaches a minimum at the intermediate value of the frequency parameter α, because those levels maximize the uncertainty of outcome.

Table 2 shows that the portfolio composition will generally be quite sensitive to the magnitude of the shocks. In most cases where shocks are comparable to one's entire wealth, no long position in annuities is taken; the individual prefers to short a life annuity if allowed.

3.2. Infinitely Many Periods

For another illustration of the implications of updating mortality and health information on the demand for annuities, consider a version of the model of perpetual youth. Individuals live for an uncertain, but unbounded, number of periods. In the standard perpetual youth model (e.g., Blanchard and Fischer 1989, p.115) individuals

[8] Note that one period corresponds to one-third of a lifetime, so an interest rate of 25% is not high.

have a constant (age-invariant) probability of death, so that their distribution of age at death is geometric (or exponential in continuous time). For the present problem, assume this probability of death is itself stochastic, following a two-state Markov process. We can label the state of lower mortality "healthy" and the state of higher mortality "sick", with respective survival probabilities p^H and p^S (where $1 > p^H > p^S > 0$). To simplify the analysis, let the sick state be absorbing, so that the only nontrivial transition probability is $q = Pr\ (\xi_{j+1} = S \mid \xi_j = H)$, which can be interpreted as morbidity. When the transition from healthy to sick state occurs, the individual's wealth is reduced. This outcome can represent health care expense or reduction in income in case of disability.

The problem can be formulated recursively, using the value function $V(A)$:

$$\begin{aligned} V(A,w) &= \max_{a,b,c}\left\{u(c) + \beta pEV'(A',w')\right\} \\ \text{s.t.}\ \ A' &= a(1+\rho) + b(1+r) + w - c \\ c &\le A + w \\ b,c &\ge 0. \end{aligned} \tag{7}$$

where primes denote next-period quantities.

The simplest case involves an individual who has no income, just an initial endowment of wealth to be consumed over the remaining life. For tractability and time invariance, let the reduction in wealth at transition to the sick state be multiplicative: whatever assets A_τ the individual has at the time, the wealth available for future consumption becomes λA_τ, where $0 < \lambda < 1$.[9] This is essentially the simple portfolio problem with two assets (e.g., Sargent 1987), with the simple modifications in the rate of time preference ($p^\xi\beta$, $\xi \in \{H, S\}$, instead of β) and the next-period wealth being multiplied

[9] This is obviously not a very realistic model of health expenses if comparison across wealth levels is a goal. It also distorts the incentive to save; however, its effect on the portfolio choice for a given level of savings is small. We use it here for tractability and to avoid the possibility of the expense exceeding the available wealth. In numerical computations we use a more realistic model of expenses.

by the random variable $\tilde{\lambda}$ which takes the value λ if the individual becomes sick (probability q) and 0 otherwise. The distribution of asset returns is discrete with two states. The bond return, $(1+r)$ is equal in both states, and the annuity return can be shown to be:

$$1+\rho^{0}=1+\frac{1+r-p_{S}}{p_{S}}, \tag{8}$$

if sick in current period,

$$1+\rho^{H}=1+\frac{\left[1+r-(1-q)p^{H}\right]\left(1+r-p^{S}\right)}{p^{H}\left[1+r-(1-q)p^{S}\right]}, \tag{9}$$

if healthy now and next period (probability 1–q), and

$$1+\rho^{S}=1+\frac{(1+r)^{2}+p^{H}\left[(1-q)p^{S}-(2-q)(1+r)\right]}{p^{H}\left[(1+r)-(1-q)p^{S}\right]}, \tag{10}$$

if healthy now and sick next period (probability q).

First note that an individual who is already sick (and this is known to the insurer) would behave just like in the conventional model and hold only annuity and no bonds, since the only uncertainty that is left is that of the time of death. Therefore, in order to study the interesting effects of information updating, it suffices to consider the problem of a healthy individual. The Bellman equation for the individual's problem is:

$$V(A_{t},\xi_{t}=H)=\max_{a_{t},b_{t}}\left\{u(A_{t}-a_{t}-b_{t})+\beta p^{H}E_{t}V\left(\tilde{\lambda}\left[(1+\tilde{\rho})a_{t}+(1+r)b_{t}\right],\xi_{t+1}\right)\right\}. \tag{11}$$

If we further assume logarithmic utility, $u(c) = ln(c)$, then it is straightforward to obtain the expressions for a and b:[10]

[10] For a general CRRA utility, a_t and b_t can also be expressed in closed form. The expressions are more complicated and they depend on λ, but the qualitative behavior is similar.

$$a_t = \frac{\beta p^H (1+r)\left[r - (1-q)\rho^H - q\rho^S \right]}{(r-\rho^H)(r-\rho^S)} A_t \ ,$$

$$b_t = \frac{\beta p^H \left[(1-q-qr+\rho^S)\rho^H + (q-r+qr)\rho^S - r \right]}{(r-\rho^H)(r-\rho^S)} A_t \ .$$

Using expressions(8)–(10) for ρ^H and ρ^S, we can express a_t and b_t in terms of fundamental quantities q, p^H, p^S, r and β. The expressions are rather cumbersome and are not shown. The optimal annuity share in the portfolio is plotted in Figures 1 and 2 for select values of the parameters. There is generally a region in which this optimal share is less than one, meaning the borrowing constraint is not binding. As one would expect, this region is in the northwest of the (p^S, q) plane. Comparing the two figures, it is apparent that the optimal annuity share is quite sensitive to the parameters and varies rather steeply near the curve on which the borrowing constraint is exactly met. While the perpetual youth model is far from realistic, it should provide useful insight into the life-cycle planning of someone around the retirement age, who may well live for another forty or so years and is facing slowly changing average survival probability for at least a decade. Under conditions represented in Figure 1, with quite plausible probability of shock of 2% and mortality rate in the sick state of 25%, only half the wealth would be annuitized.

4. The Computational Model

The models of the previous section provide a useful insight into the mechanism by which updating of mortality information reduces individuals' demand for annuities. They are, of course, too stylized to be useful for real-world quantitative estimates. Besides the obvious fact that realistic mortality and morbidity tables do not resemble a perpetual youth model, there are interesting, but complicated, situations to which analytical models do not apply. For example, expenses can be so high that they exhaust an individual's entire wealth; there are many people whose total assets do not exceed the cost of a year's stay in a nursing home, and many more whose assets are roughly of that order of magnitude.[11] Typically, some form of bankruptcy (or limited liability) constraint applies in those cases, often in the form of the state as the insurer of last resort. It has been observed that people behave rationally taking this into account, as evidenced by the "Medicaid spend-down". Almost half of the current nursing home residents admitted as privately paying eventually became Medicaid beneficiaries during their stay (U.S. House of Representatives 1998, p. 1062). Obviously, running out of private wealth is a possibility that any quantitatively useful model must include, but such a model is almost sure to be analytically intractable.

4.1. Individuals: Age and Health

The economy is populated by overlapping generations of individuals who live to a maximum of J periods (years), with one-period survival probabilities $p(j, \chi_j)$ at age j dependent on health status χ_j, which in turn is also uncertain. The state variable χ takes values on a discrete set. While the precise definition of "health" is not critical for the functioning of the model, the health states are generally distinguished and ordered by the respective survival probabilities. Assuming higher value denotes "worse" health:

$$\chi > \xi \Rightarrow p(j,\chi) \le p(j,\xi), \forall j, \forall \chi, \xi. \qquad (12)$$

[11] Annual cost of nursing home is roughly \$50,000 (e. g., McGarry and Schoeni 2001), about the same as the mean wealth of the 3rd decile in the Health and Retirement Study (HRS), excluding Social Security wealth which cannot be liquidated (Moore and Mitchell 2000). The median HRS family wealth, excluding Social Security, equals approximately the cost of four years in a nursing home.

Another feature of health is that states of bad health can be connected with financial losses, consistent with the definition of health shocks in Section 2.1.

Initially all individuals are "healthy," but in any period, one's health can change. Accordingly, χ follows an M-state Markov process with an age-dependent transition matrix $R_{mn}(j)$; m, $n = 1, \ldots, M$. For our present purposes, which focus on explaining annuity demand, $M = 3$ will suffice; more health states might be required in different contexts, such as pricing long-term care insurance or evaluating its money's worth, as in Brown and Finkelstein (2003).[12]

4.2. Savings: Bonds and Annuities

Individuals can hold the wealth they do not consume in two forms: noncontingent bonds and life annuities. The basic properties of these assets were introduced in Section 2.2; when there are no transaction costs (premiums are actuarially fair), we can rely fully on the concept, described in Section 3, of annuities as "actuarial notes" which mature in one period and pay $1 plus the amount equal to the next-year annuity premium for the same person. In this case it is sufficient to have one state variable for an individual's wealth because the portfolio weights (bonds vs. annuities) can be changed in each period at no cost. Optimal decisions do not depend on the portfolio composition at the beginning of the period.

Actuarially fair annuities are generally not available (Mitchell et al. 1999) and premium loading obviously can influence demand, so a realistic model ought to allow for

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[12] In fact, two health states are sufficient for most purposes. Three states add some additional flexibility and dynamics without significant extra computational cost. For example, with three states it is possible to treat the first year of illness differently from subsequent years in both its financial impact and its impact on mortality.

it. This is not a trivial adaptation since the problem essentially becomes a multi-period portfolio choice *with* transaction costs, which requires an additional state variable. Without transaction costs, purchasing a single-premium life annuity and simultaneously borrowing the amount equal to the annuity premium and buying life insurance with annual premium equal to the annuity payment is equivalent to no action at all. With transaction costs, it involves a "round-trip" transaction cost and is clearly inferior. Therefore, in every decision, one must consider not just the current total wealth, but also the initial portfolio composition. Therefore, the model must keep track of bonds and annuities as two separate state variables. To avoid slowing down the computation, it is best to use a model without transaction costs whenever those are not the object of attention. The precise way in which asset state(s) are modeled is described in the Appendix.

There is no formal restriction on borrowing or short sales of annuities. However, it is impossible for an individual, who might die before the loan repayment time, to borrow at the risk-free rate without carrying life insurance in the amount of the loan. Such a restriction is natural, and would arise in a rational equilibrium even if not modeled explicitly. Since borrowing plus buying life insurance is equivalent to selling an annuity on one's own life, we consider all transactions in the insurance market as annuity purchases or sales. Some formal restrictions do exist. One cannot trade annuities on other people's lives, consistent with prevalent laws that give life insurance companies the exclusive right to sell life annuities. An individual's net worth A must always remain nonnegative. This applies also to the situation when a health shock, and the associated expense, would otherwise make the individual's wealth negative. As the utility would become –∞ at zero consumption, this restriction is ensured the minimum level of consumption (bankruptcy/welfare protection) in each period.

4.3. Individuals' Preferences

Individuals are fully rational and have preferences for consumption and possibly for leaving bequests. Preferences for consumption are additively time-separable, with a constant relative risk aversion period utility (felicity). This choice is common in life-cycle literature as well as in multi-period finance, it characterizes risk aversion in a plausible way, and is easy to use and control in computations. To avoid problems with tractability and uniqueness that arise in OLG models with altruism, bequest motives are modeled as "joy of giving" and total felicity is additively separable in consumption and bequest. The full utility function is then:

$$U = \sum_{j=1}^{J} \beta^j u(c_j) = \sum_{j=1}^{J} \beta^j \left[\frac{c_j^{1-\sigma}}{1-\sigma} + \zeta D_j A_{j+1} \right] \tag{13}$$

Here, j denotes age, β is the rate of time preference, c_j is consumption at age j, σ is the curvature parameter, A_j is bequeathable wealth at age j, D_j is an indicator that equals 1 in the year of death and 0 otherwise, and ζ is a parameter that determines the strength of the bequest motive. As there are no preferences over leisure, the labor supply is inelastic. While this is a departure from realism in working years and especially around normal retirement age, which now must be exogenously determined, it does help focus on the problem of annuity demand, for several reasons. Firstly, annuity demand mainly exists due to uncertain consumption needs late in life; secondly, few people return to work at a very old age (which would be a significant issue if it were common) and it is unclear how utility for leisure should be described at those ages. Thirdly, and perhaps most importantly, it would be potentially difficult to sort out the general equilibrium effects of annuities if a change in savings level also caused a change in the labor supply.

When a "healthy" individual becomes "sick," his or her probability of survival decreases *and* a health-related expenditure may become necessary. This expenditure does not contribute to the individual's utility for consumption; it is assumed to be necessary for any chance of survival, and hence perfectly inelastic. In effect, individuals have lexicographic preferences in which one's own survival trumps any other form of consumption — probably a slight exaggeration, but a reasonably plausible one.[13]

4.4. Production and Labor Productivity

The output Y_t of the economy in period t is determined by the constant returns-to-scale technology with a Cobb-Douglas production function $Y = \theta_t K^{\alpha} L^{1-\alpha}$, where K denotes capital and L labor, in efficiency units as discussed below.

During the individuals' working years, their earning capacity (productivity) changes with age in a predictable pattern, as well as randomly due to individual (idiosyncratic) productivity shocks. This idiosyncratic productivity is quantified by a Markov state variable η with a transition matrix $Q_{kl}(j)$; $k, l = 1, \ldots, L$. While this is an additional, and realistic, source of financial risk, its main purpose in the present model is as a source of wealth heterogeneity. All individuals are a priori identical and all differences among them arise due to random events during their lifetimes (possibly

[13] Equivalently, preferences can be described using state-dependent utility, where sick individuals who do not receive adequate treatment derive no (or sufficiently little) utility from any consumption or bequests. Also, as we do not deal with moral hazard in the present paper, the fact that health expenses are exogenously fixed presents no significant restriction on the rational behavior of individuals.

including being born into different productivity and health states). Productivity is generally zero beyond normal retirement age and in the highest (sickest) health state, regardless of the value of η.[14] An individual's wage is a product of age-related productivity ε_j, individual productivity η, an indicator of the health status, and the market wage rate per unit of labor.

Individuals do not know what their productivity and health state will be in the next period, but know their respective distributions, so they make their decisions to maximize the expected utility over the set of next-period states. With a given consumption and saving decision, next-period wealth is uncertain, but depends only on η and χ, so it is possible to optimize over consumption, saving in bonds, and saving in annuities using standard constrained-optimization techniques.

Individuals spend the first part of their lives working and earning, and the later years in retirement. They can satisfy their consumption needs in retirement by accumulating some wealth through private savings during their working years, by relying on government transfers, or both. We model government-provided retirement scheme as unfunded (pay-as-you-go) transfer from workers to retirees in each period, so that it is effectively a mandatory life annuity to a retiree. The "Social Security" transfer tax is determined from the equilibrium distribution of individuals and the desired average income replacement rate. Government also acts as the insurer of last resort for long-term care, providing transfers to those individuals whose nursing home costs exceed their total available assets. As with Social Security, this transfer is financed through a balanced-budget tax.

[14] Age-related zero productivity reflects inelastic labor supply, as discussed later.

In the absence of Social Security and transaction costs, an individual is fully described by four state variables: age *j*, health χ, idiosyncratic productivity η, and wealth (assets) *A*. The economy, as a collection of individuals, is then described by the measure $\Phi(j, \chi, \eta, A)$ of individuals by state, and by the values of market wage *w*, interest rate *r*, capital stock *K*, and labor supply *L*. Presence of transaction costs requires an extra variable, so *A* must be replaced by two variables (*A* for annuities and *B* for bonds), and otherwise the structure is unchanged. Social Security has a peripheral role in this model, so it is simplified to avoid adding a state variable for earnings history.[15]

4.5. The Dynamic Programming Problem

To complete the description formally, an individual characterized by age *j*, wealth *A*, productivity state η and health state χ solves the following problem at time *t*, taking the prices *w*, *r*, ρ and initial conditions *j*, *A*, η, χ as given:

$$V_t(A,\eta,\chi,j) = \max_{a,b,c}\left\{u(c)+\beta p(j,\chi)\int_{\chi'}\int_{\eta'} V_{t+1}\left[A'(a,b;\chi'),\eta',\chi',j+1\right]Q(\eta,d\eta')R_j(\chi,d\chi')\right\} \tag{14}$$

subject to:

$$A' = a\left(1+\rho\right)+b\left(1+r\right)+\varepsilon_j\eta I\left(\chi{=}H\right)w\left(1-T\right)-c+B+Tr-L$$
$$b \geq 0$$
$$0 \leq c \leq A+\varepsilon_j\eta I\left(\chi{=}H\right)w\left(1-T\right)+B+Tr-L,$$

where ε_j is the (deterministic) average productivity of a worker of age *j*, *B* is the amount received from the distribution of unintended bequests, *L* is the financial loss in the sick state, *Tr* is the sum of all government transfers received (Social Security and welfare), and *T* is the total tax rate required to finance those transfers. Here $A \in \mathbf{R}_+$,

[15] More about modeling Social Security is discussed in Section 5, Calibration.

$\eta \in \mathbf{H} = \{\eta_1, \eta_2, ..., \eta_n\}$, $\chi \in \mathbf{X} = \{\chi_1, \chi_2, ..., \chi_m\}$, $j \in \mathbf{J} = \{1, 2, ..., J\}$, and the functions $\{V_t, a_t, b_t, c_t : \mathbf{S} \to \mathbf{R}_+\}_{t=1}^{\infty}$ are measurable with respect to F, where $\mathbf{S} = \mathbf{R}_+ \times \mathbf{H} \times \mathbf{X} \times \mathbf{J}$, $\mathrm{F} = \mathbf{B}(\mathbf{R}_+) \times \mathbf{P}(\mathbf{H}) \times \mathbf{P}(\mathbf{X}) \times \mathbf{P}(\mathbf{J})$, and $\mathbf{P}(\cdot)$ denote power sets and $\mathbf{B}(\mathbf{R}_+)$ is the Borel σ-algebra of $\mathbf{R}_+$.

5. Calibration

Mortality data for the population as a whole are based on the U. S. Social Security Administration tables (Faber and Wade 1982; Bell, Wade and Goss 1992). Since the "sick state" in the context of this paper corresponds most closely with long-term disability during working years and with a need for long-term care for retirees, the sources of mortality data for "sick" individuals are RP-2000 disabled life mortality tables (Society of Actuaries 2000) for ages 21–65 and nursing home discharge and mortality data from the 1998 Green Book and the National Nursing Home Survey (NNHS) for ages over 65. The latter data are in coarse age groups, so the rates by age by year were obtained using the relative rates from population mortality and from mortality in continuing-care retirement communities (Barney 1998). Survival probabilities by age and health state are shown in Table 3.

Three health states are generally enough to capture all information available from the existing data. Besides the healthy and sick states with by now obvious interpretations, an intermediate state is useful as representing impaired health with significantly higher mortality, but without actually being institutionalized and suffering the associated expense. For working years, we use the intermediate state to represent the first year of disability so that we can vary recovery rates with the duration of disability. Transition probabilities between health states are shown in Table 4. Probabilities for the working-age population are based on long-term disability morbidity tables (Society of Actuaries 1982) and Social Security data regarding the number of persons receiving benefits. For older ages we used nursing home population and admission data from the Census Bureau, National Nursing Home Survey, Medicare, and the 1998 Green Book; we refined these transition probabilities based on the actuarial model of Robinson (1996). The expenditure connected with poor health for retired population is based on average nursing home costs as reported in the Green Book and NNHS. The information in LTC data is necessarily incomplete: on one hand, some expenditures are higher than necessary and in fact represent voluntary (and hence elastic) consumption; on the other hand, expenditures from loss of income of a family member caring for a non-institutionalized person requiring LTC are not captured.

Individual productivity states and transition probabilities are taken from Nishiyama and Smetters (2003), with the original eight states combined into two for most computations. When the two states represent the bottom 60% and the top 40% of the workers for a given age, idiosyncratic productivity factors are given in Table 5 and the transition probabilities in each period are $R_{11} = 0.9422$, $R_{12} = 0.0578$, $R_{21} = 0.0867$, and $R_{22} = 0.9133$. Macroeconomic variables are also calibrated consistent with Nishiyama and Smetters (2002): the capital share of output is $\alpha = 0.32$, the depreciation rate of physical capital is $\delta = 0.046$, and the capital-to-output ratio is 2.8.

Individuals are followed from the beginning of their working lives, and it is assumed that all start out healthy. This obviously does not capture congenital or early-onset disabilities, but those would deprive the individual of earning power before he or she could make any decisions on saving; such cases are thus irrelevant for the present model. Disabled children would, however, have an effect on their parents, so it would perhaps be ideal to increase effective disability rates during working years (and introduce another state, since the parents' earning power would remain, perhaps diminished). Nevertheless, as the most affected ages precede significantly the ages typical for major annuity purchases, there is little harm in ignoring such cases at this stage.

The rate of population growth is assumed to be a constant 1%. This roughly approximates population growth in the United States, but the particular value is not critical. The rate of productivity increase in the production function is also not critical and, for the sake of simplicity, productivity is assumed to be unchanging.

Social Security benefits are modeled with the primary goal of matching the average level. A worker who earns the average wage (according to SSA) throughout his or her career and retires at the normal retirement age, will get a replacement ratio of approximately 45%.[16] Social Security formula is redistributive and hence the worker who remains in the high productivity state throughout his or her career will have only 2.14 times higher benefits than the worker who always remains in the low-productivity state, although the former worker will have 3.5 times the career earnings of the latter.[17] These ratios are based on extreme cases, comprising just the top 7.29% and the bottom 1.85% of the earnings distribution in the two-state model; most workers will not have spent their entire career in one productivity state.

[16] Author's calculation based on benefit information published on the SSA web site, http://www.ssa.gov.

[17] Based on Nishiyama productivity factors; using Conesa-Krueger factors, the Social Security ratio is 1.49 and the earnings ratio 1.74. These examples ignore any additional differences that may arise from spousal benefits.

When the accuracy of Social Security benefits is highly important in a dynamic programming model, a good approach is to use an additional state variable representing the accrued benefits. In our model, however, there are already so many state variables and states that this approach would be computationally very costly, while the benefits would be minimal for at least two reasons. First, we are primarily interested in the effect on aggregate demand for annuities, as that can be compared to observations. Individual differences in benefit levels, within realistic limits, are not likely to have much effect on the aggregate demand. Second, such differences in benefits as do exist are the most significant among relatively low earners, who are likely to have low wealth and little influence on the annuity market. On the other hand, the difference in benefits between average and high earners is small and that between high and extremely high earners is zero due to the earnings and benefit limits.[18] To avoid using an additional state variable while using as much information as possible with the existing set of state variables, we use the conditional expectation of benefits by final productivity state: an individual who reaches the last working year in state i is assigned the expected benefit amount conditional on this (last working year state) information only. Using a simple Monte Carlo simulation, the ratio of the two levels of benefits is calculated to be approximately 1.197.

Size and distribution of bequests can potentially have a significant impact on savings behavior, but no consensus seems to exist regarding the strength of bequest motives or even the fraction of aggregate wealth that is bequeathed. For this reason, the

[18] According to the examples on the SSA web site, http://www.ssa.gov/OACT/COLA/exampleAvg.html, for workers retiring in 2003 at age 65, one with average earnings throughout the career will receive a monthly benefit of $1,158, while one whose earnings equaled or exceeded the earnings limit in every year will receive $1,721 monthly, or 1.486 times more.

strength of bequest motives cannot be truly calibrated. Instead, several different levels and distribution modes are used for sensitivity analysis.[19]

6. Results

6.1. Basic model

In the absence of health shocks, we know that symmetric information, actuarially fair premiums, lack of bequest motives, zero transaction costs, and no exogenous obstacles to annuity market would be a sufficient set of conditions for full annuitization to be optimal for every individual. It is therefore the most natural to start by asking what the effect of health shocks will be if all other potential impediments to annuity markets are absent. At this point we also turn off any source of retirement income other than private savings. It is true that the mere existence of mandatory life annuities, holding everything else constant, would not alter the optimality of full annuitization, but would at most reduce the portion of the portfolio an individual can control. When other factors affect the dominance of annuities over bonds, however, existence of mandatory annuities can compound the effect. As we want to isolate the effect of health shocks, we consider the counterfactual world with no Social Security.

An example of individual choice is shown in Figure 3. We chose a healthy individual age 65 as perhaps the most typical potential purchaser of a life annuity. Here and in the rest of this section, the magnitude of a health shock (about $50,000) is used as a unit of wealth. In this diagram, the coefficient of relative risk aversion σ equals 2, and

[19] Perhaps the most reliable indicator of the strength of bequest motives is the observation that the assets of retirees do not tend to decrease with time, even at very old ages. PSID data show that households whose head was aged 75+ in 1999 had *higher* net worth than households whose head was aged 65-74 in 1989 (i.e., same cohort, 10 years earlier).

the interest rate and time preference parameter are set to 5% and 0.95, respectively. Optimal portfolio deviates significantly from full annuitization for a fairly wide range of wealth levels; the effect is the most pronounced for average to slightly below-average wealth. The poorest, who would exhaust all of their assets if ever hit by a health shock even for one period, annuitize fully; the wealthy, who can afford annuities with periodic payout that leaves sufficient funds even in the event of a shock, have little or no need to avoid annuities. Individuals most averse to holding annuities are those who are not poor, but would quickly become poor if hit by a health shock.[20] Individuals with assets between 1 and 4 (corresponding to approximately $50,000 to $200,000) want to annuitize, on average, less than 20% of their wealth. This fraction gradually increases with wealth, but it does not reach one until wealth is about 15 times the magnitude of the shock.[21] A majority of people of this age have assets in the range of 1–5 times the cost of a year's stay in a nursing home (Moore and Mitchell 2000), so the aggregate effect on annuity demand, in terms of the number of policies, is substantial.

One of the goals of this paper is to analyze the factors of aggregate demand for annuities, so the fraction of aggregate wealth held in annuity form is one of the most interesting results of each simulation of a steady-state economy. The aggregate departure from full annuitization is not as dramatic as Figure 3 might have suggested, but are certainly not negligible: 4.9% of total wealth will be held as bonds.[22] If we consider only the wealth of retirees, 6% of it will be in bequeathable form—despite a complete lack of

[20] Departures from the comparable result in Essay 2 include a somewhat smaller magnitude of the effect and a shift to lower wealth levels.

[21] The apparently non-smooth behavior of the portfolio composition as a function for wealth for wealth levels of the same order of magnitude as the shock is due to the discrete nature of the health states and to nonconcavity of the value function arising from limited liability.

[22] Besides the lack of significant reduction in annuity holding at high wealth levels, it also turns out that at most ages the effects are not as strong as at age 65.

desire to leave bequests. The aggregate annuitization percentage results are summarized in Tables 4 and 5.

Figure 4 shows the same decision, under equal conditions, for a more risk-averse individual ($\sigma = 4$). The effect is dramatic: many people — those whose assets are one to ten times the size of the shock — up to about half a million dollars — would not want to hold *any* life annuities. The aggregate effect is also much higher now: 17.3% of all wealth, and 13.4% of retirees' wealth, is held as bonds. On the other hand, we do not show the diagram for $\sigma = 1$ as it looks rather boring: there is no effect at all. Individuals with high risk tolerance will still want to annuitize fully, at any level of wealth.

6.2. Social Security

As mentioned in the previous subsection, Social Security cannot, by itself, render full annuitization suboptimal in a world in which annuities dominate bonds. It can, however, interact with other factors of annuity demand and can alter annuitization decisions of some individuals, depending on their individual circumstances. Besides the obvious direct effect of by imposing some existing annuitized fraction of wealth on the individual, there are also indirect, general equilibrium effects. We have seen in the last subsection that annuity demand is sensitive to the interest rate, which is in turn higher with pay-as-you-go Social Security than without it. It is thus appropriate to evaluate the effect of Social Security with constant factor prices as well as in the general equilibrium. We focus on the former in this paper.

When factor prices are kept constant, the aggregate effect of Social Security on the level of annuitization of wealth is small and ambiguous. At moderate levels of risk aversion ($\sigma = 2$), less wealth is annuitized with Social Security than without it, but the difference is less than 3% for retiree wealth and even less in total. It should be noted that the denominator (total wealth) in these results does not include the "Social Security wealth" (i.e., the expected present value of future Social Security benefits) so the relative reliance on annuity income actually increases. For higher levels of risk aversion, the additional effect of Social Security is even less significant, adding only about 2% to the bond portfolio weight at $\sigma = 4$. It does not appear that Social Security has a higher crowdout effect on annuities than on savings in general.

It is interesting, however, that an individual's optimal portfolio, as a function of wealth, is noticeably altered when Social Security exists. As Figures 5 and 6 show, there is no full-annuitization region at low wealth levels now. The overall effect on low-to-medium wealth 65-year-olds is more pronounced, leading to no annuity holding at all (actually, they prefer to hold negative private annuities to offset the Social Security annuity in part).

6.3. Bequest Motives and Transaction Costs

Bequest motives, if they are present, are certain to diminish the demand for annuities. We find indeed that, by varying the altruism parameter ζ (the strength of the bequest motive) in the utility function, we can reduce the annuitized fraction of wealth to an arbitrary degree. For example if $\zeta = 1.0$, only 34% of total wealth and only 19% of the retirees' wealth will be annuitized. However, this is an implausible value of altruism; it would mean that consumption, no matter how high one's wealth, would be limited to about $50,000 per year. In terms of macroeconomic variables, capital-to-output ratio would be at least 4.5 and interest rate by about three percentage points lower than calibrated. We would also find that intergenerational bequests exceed 10% of GDP and 14% of consumption; all this is incompatible with empirical data. A high but still somewhat plausible value of $\zeta = 0.5$ (resulting in $K/Y = 3.6$ and total bequests consistent with observation) would reduce total annuitization (at $\sigma = 2$) to 72.6%, and that of retirees to 66.5%. This is approximately a quadrupling of the bond portfolio weight, relative to the no-altruism case. On the other hand, lower values of altruism, which do not alter macroeconomic quantities significantly, have a very small effect on annuitization. With $\zeta = 0.25$, over 90% of wealth would still be held in annuity form.[23]

Premium loading is another obvious deterrent from annuity purchases, but its importance in deviations from Yaari have been questioned recently (Mitchell et al. 1999; Davidoff et al. 2002). In the framework of our model, we are not limited to evaluating the effect of the higher price on money's worth of an annuity; we can also look at the more subtle transaction-cost nature of the loading. Transaction costs limit the usefulness of the annuity as a short-term investment and, if they are high enough, can make annuities an inferior investment for those who cannot commit to them for a long term or indefinitely. In other words, they reduce the liquidity of annuities, compounding the problem of possible high cash need as caused by health shocks.[24]

[23] This is not a small value for altruism. In our model it means no individual ever spends more than about $100,000 per year, even with unlimited wealth.

[24] Our preliminary finding is that a premium loading of 10% reduces annuitization of retirees' wealth to 62%, having a greater effect than a moderately high bequest motive.

7. Conclusion

This paper provides the insight into the effect of mortality information updating and related changes in wealth on the demand for life annuities. It shows that, under realistic mortality and morbidity assumptions and expense levels, many individuals do not find it optimal to hold all of their wealth in life annuity form, even in the absence of premium loading, adverse selection, and bequest motives. Effectively, a life annuity ceases to be a dominant asset and becomes a risky asset with an expected, but not guaranteed, premium over the riskless return.

While this mechanism does not depend on the other, better known, factors affecting demand for annuities, it is not mutually exclusive with any of those. In fact, a typical individual will have a variety of factors influencing his or her decision to annuitize: transaction costs (premium loading and obstacles to undoing an annuity), a desire to bequeath some wealth, private information, presence of mandatory annuities, availability of family help or social welfare in case of outliving one's wealth, and so forth. The relative effects of each factor on annuity demand are quite sensitive to the precise choice of parameters, but the effect of health shocks is significant in most cases.

There are numerous potential applications of this model. Long-term care has been mentioned repeatedly in the paper as a source of wealth shocks of the type assumed in the model. It is thus natural to try to use this framework to study the joint effects of long-term care and mortality risks and optimal insurance approaches. In that respect this model can complement and extend the work of Murtaugh, Spillman and Warshawsky (2001). The issue of payout options from defined-contribution pensions or other individual account retirement schemes can also be made clearer with this framework. This paper shows that annuity payouts do not necessarily provide the most financial security to the beneficiary, especially if retirement programs are designed in isolation from health insurance programs. The issue of the optimal, welfare-maximizing, level of mandatory annuitization of retirement wealth is a nontrivial one and requires a sound analytical and computational framework. On the other hand, government transfers such as income floor protection and insurance of the last resort may provide incentives for some individuals to draw down their savings early. Mandatory annuitization can prevent that and help control the cost of such programs, but that there is no general guarantee of a welfare gain from such mandates.

APPENDIX: Discretization of State Space

A1. No Transaction Costs

Total wealth at age j, A_j, is represented as one of 101 points of the wealth grid, A_{jk}, $k = 0, 1, \ldots, 100$. We fix point $A_{j0} = 0$, A_{j100} equals the assumed maximum wealth, and the value of A_{jk} increases with k. Ideally, for best interpolation during optimization and evaluation, the spacing between adjacent grid points should be narrower where the value function's curvature is the most pronounced. We chose equidistant points at the low end of the wealth distribution, and geometrically increasing values for intermediate to high wealth. For example, if $A_{j100} = 100$ (in units where 1 is the average annual wage), then:

$$A_{j,k} = A_{j,k-1} + 0.05, \qquad k = 1, \ldots, 20 \qquad (A_{j20} = 1)$$

$$A_{j,k} = 1.05\, A_{j,k-1}, \qquad k = 21, \ldots, 75 \qquad (A_{j75} = 14.63563)$$

$$A_{j,k} = 1.0799\, A_{j,k-1}, \qquad k = 76, \ldots, 100.$$

As most people's wealth increases during the early part of life, the maximum wealth A_{j100} does not have to be the same for all ages; we also allow the grid to be expanded during the computation if the maximum wealth is actually reached by a positive measure of agents.

When the optimal policy (consumption, bond saving and annuity saving) is computed for an agent at the node (j, h, i, k), where the indices represent age, health, productivity, and wealth, respectively, the wealth A' next period (age j+1) is allowed to take any positive value, rather than be limited to the values of the grid points. The value function $V(j+1, h', i', A')$ corresponding to that wealth is determined by interpolation between the two grid points bracketing it, for the given final health and productivity state (h', i') and age j+1.

The simplest way to interpolate V is by piecewise linear base functions: V is assumed to be linear in A between A_k and A_{k+1} such that $A_k < A < A_{k+1}$. The disadvantage of this approach is that is alters the shape (curvature) of the value function. In optimization problems, concavity is often very important and it is desirable to keep concave functions concave not just globally but also locally, on any scale. We thus use interpolation by Schumaker's shape-preserving quadratic splines (Judd 1999, pp. 231–234). They preserve local concavity/convexity of V as a function of wealth, as well as its smoothness, avoiding artificial kinks at grid points.

With respect to the curvature of the value function, it is important to keep in mind that this is not a standard, well-behaved concave problem. Limited liability introduces nonconcavity in wealth, and there are further complications due to the discrete nature of health. Risk involved in annuity returns arises from the difference in the next-period present value of the annuity in healthy and sick states, as well as the transition probabilities for the respective states, so this is the main determinant of the optimal portfolio weights. The objective function can, and frequently does, have multiple local optima. For this reason, it is necessary to perform optimization at each node starting from several different initial guesses chosen by randomized search techniques.

In most cases we find insignificant difference between results of runs using linear and spline interpolation. Greater difference is normally a sign that the grid spacing is too wide, and it is then generally best to rerun the computation using a finer grid.

When the measure of agents is computed, it is also represented as having a discrete distribution, with probability masses at the nodes of the grid. For this purpose, a value from the continuum must be apportioned to the nearest two grid points. This process must preserve expected utility as well as the total measure, so the weights given to the two points will be inversely proportional to the distance to them (i.e., they will be based on a linear interpolation scheme).

The number of nodes in the full dynamic-programming tree is $J \times m \times n \times (k^{\max}+1)$, where J is the maximum age (actually the age span between the minimum and the maximum), m is the number of health states, n the number of productivity states, and $k^{\max}$ the highest index of the wealth grid. We use ages from 21 to 120, so $J = 100$; as defined above, $k^{\max} = 100$, and as discussed in the paper, $m = 3$ and $n = 8$ (or less in some versions). Therefore, $J \times m \times n \times (k^{\max}+1) = 100 \times 3 \times 8 \times 101 = 242{,}400$. (The smallest version, with $n = 2$, has 60,600 nodes.)

A1.2. With Transaction Costs

When transaction costs are present, portfolio composition is part of the description of the state, so there will not be one, but two continuous state variables, and each of them must be discretized in computation. We use liquid wealth (bond) holding and annual payout of the annuity policy owned as the two variables. Annual payout remains constant if the annuitant enters no annuity transactions during a period, and is hence a better modeling choice than the present value of the annuity, which changes with age and health. The resulting two-dimensional grid for each age-health-productivity triad has a large number of nodes, slowing the computation down considerably.[25] Second, two-dimensional interpolation is needed instead of one-dimensional as described in the previous section. While shape-preserving bivariate interpolation schemes do exist, they

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[25] With 40 grid points for bond holding and 40 for annuities — the bare minimum for a reasonable spacing — and 2 productivity states, there are 960,000 nodes to evaluate in each pass in this problem.

are more complicated to implement (and the problem is already computationally intensive), so we limit the interpolation to a simple bilinear procedure.

While the transaction cost model has an additional state variable, its optimization space is essentially the same as for the model without transaction costs, and the associated nonconcavity problems are similar. Interpolation leads to potential new problems, however. If an individual's optimum annuity holding (measured by annual payout) is constant over a number of years, but not exactly on a node of the annuity grid, this point may never be reached if the transaction costs are high enough. Since the initial state is always at a grid node, and it is advantageous to avoid annuity transactions, solutions will also have a tendency to "stick" to the nodes. This tendency may lead to understatement of the dynamics of annuity transactions over time. The safest way to minimize this problem is to use as dense a grid for annuities as possible, but of course, there is a tradeoff between accuracy and computational speed.

Table 1

Critical Magnitude of Shocks in the Three-Period Model

Relative Risk Aversion (σ)	Interest Rate (r)	Probability of Remaining Healthy (α)	Survival Probability (p_1)	Critical Value of Shock (s)
2	0.25	0.6	0.9	0.196
2	0.25	0.6	0.75	0.533
2	0.25	0.8	0.9	0.824
2	0.25	0.4	0.9	0.222
2	0.1	0.6	0.9	0.150
2	0.5	0.6	0.9	0.278
1.5	0.25	0.6	0.9	0.256
4	0.25	0.6	0.9	0.100

Note: The rightmost column shows the magnitude of second-period shock (s), as a fraction of first-period wealth, at which the borrowing constraint ceases to be binding, for some values of the parameters (shown in the first four columns) in the three-period model of Section 3.1. At higher levels of s, it is optimal to include bonds in the portfolio.

Table 2

Optimal Portfolio in the Three-Period Model

Shock Size as a Fraction of Period-1 Wealth (s)	Annuity Share of Portfolio for Various Probabilities of Period-1 Survival (p_1) and Favorable Period-2 State (α)				
	$p_1 = 0.95$, $\alpha = 0.6$	$p_1 = 0.9$, $\alpha = 0.4$	$p_1 = 0.9$, $\alpha = 0.6$	$p_1 = 0.9$, $\alpha = 0.8$	$p_1 = 0.75$, $\alpha = 0.6$
0.05	1.000	1.000	1.000	1.000	1.000
0.10	1.000	1.000	1.000	1.000	1.000
0.15	0.878	1.000	1.000	1.000	1.000
0.20	0.724	1.000	0.985	1.000	1.000
0.30	0.429	0.735	0.668	0.758	1.000
0.40	0.150	0.424	0.370	0.460	1.000
0.50	0.000	0.139	0.091	0.175	1.000
0.60	0.000	0.000	0.000	0.000	0.731
0.80	0.000	0.000	0.000	0.000	1.000

Note: Entries show proportion of annuities in the savings portfolio for the first period in the three-period model of Section 3.1., for selected values of the magnitude of shocks (rows) and mortality and morbidity parameters (columns). The interest rate is 0.25 per period throughout, and the coefficient of relative risk aversion equals 2.

Table 3

Survival Probabilities

Age	Healthy	Disabled
21	0.999643	0.998394
22	0.999634	0.998353
23	0.999627	0.998322
24	0.999624	0.998308
25	0.999624	0.998308
26	0.999622	0.998299
27	0.999618	0.998281
28	0.999607	0.998232
29	0.999588	0.998146
30	0.999556	0.998002
31	0.999501	0.997755
32	0.999438	0.997471
33	0.999369	0.997161
34	0.999298	0.996841
35	0.999227	0.996522
36	0.999159	0.996216
37	0.999096	0.995932
38	0.999036	0.995662
39	0.998979	0.995406
40	0.998921	0.995145
41	0.998858	0.994861
42	0.998785	0.994533
43	0.998701	0.994155
44	0.998603	0.993714
45	0.998492	0.993214
46	0.998384	0.992728
47	0.998266	0.992197
48	0.998140	0.991630
49	0.998005	0.991023
50	0.997862	0.990379
51	0.997551	0.988980
52	0.997333	0.987999
53	0.997084	0.986878
54	0.996804	0.985618
55	0.996376	0.983692
56	0.995800	0.981100
57	0.995307	0.978882
58	0.994727	0.976272
59	0.994055	0.973248
60	0.993253	0.969639
61	0.992324	0.965458
62	0.991243	0.960594
63	0.989988	0.954946
64	0.988720	0.949240

Age	Healthy	Impaired, Not in LTC	Impaired, In LTC
65	0.987263	0.948234	0.885833
66	0.985591	0.945612	0.884889
67	0.983925	0.942856	0.883937
68	0.982129	0.939960	0.882977
69	0.980198	0.936918	0.882009
70	0.977794	0.933722	0.881033
71	0.975430	0.930364	0.880049
72	0.972719	0.926835	0.879057
73	0.969613	0.923128	0.878056
74	0.966100	0.919233	0.877047
75	0.962166	0.915141	0.876030
76	0.957831	0.910841	0.875005
77	0.953094	0.906324	0.873971
78	0.947877	0.901577	0.872929
79	0.942073	0.898280	0.868672
80	0.935632	0.894872	0.864272
81	0.927959	0.891351	0.859725
82	0.919514	0.887711	0.855026
83	0.910282	0.883949	0.850169
84	0.900221	0.880062	0.845150
85	0.889243	0.876044	0.839963
86	0.877203	0.870000	0.829061
87	0.863957	0.855000	0.817416
88	0.849410	0.840000	0.804978
89	0.833580	0.825000	0.791692
90	0.816592	0.810000	0.777502
91	0.800231	0.790000	0.762345
92	0.783395	0.770000	0.746155
93	0.766338	0.750000	0.728863
94	0.749307	0.740000	0.710392
95	0.732509	0.732509	0.690664
96	0.716095	0.716095	0.669591
97	0.700148	0.700148	0.647083
98	0.684704	0.684704	0.623041
99	0.669793	0.669793	0.597362
100	0.655444	0.655444	0.569933
101	0.641372	0.641372	0.540000
102	0.628315	0.628315	0.520000
103	0.616960	0.616960	0.500000
104	0.607997	0.607997	0.480000
105	0.602114	0.602114	0.460000
106	0.600000	0.600000	0.440000
107	0.600000	0.600000	0.420000
108–119	0.600000	0.600000	0.400000
120	0.000000	0.000000	0.000000

Source: Author’s calculations based on references cited in Section 3.

Table 4

Health Transition Probabilities

Age	Q_{11}	Q_{12}	Q_{13}	Q_{21}	Q_{22}	Q_{23}	Q_{31}	Q_{32}	Q_{33}
21	0.999000	0.001000	0	0.600000	0	0.400000	0.300000	0	0.700000
22	0.998500	0.001500	0	0.596000	0	0.404000	0.295000	0	0.705000
23	0.998000	0.002000	0	0.592000	0	0.408000	0.290000	0	0.710000
24	0.997500	0.002500	0	0.588000	0	0.412000	0.285000	0	0.715000
25	0.997500	0.002500	0	0.584000	0	0.416000	0.280000	0	0.720000
26	0.997000	0.003000	0	0.580000	0	0.420000	0.275000	0	0.725000
27	0.997000	0.003000	0	0.576000	0	0.424000	0.270000	0	0.730000
28	0.996500	0.003500	0	0.572000	0	0.428000	0.265000	0	0.735000
29	0.996000	0.004000	0	0.568000	0	0.432000	0.260000	0	0.740000
30	0.996000	0.004000	0	0.564000	0	0.436000	0.255000	0	0.745000
31	0.995500	0.004500	0	0.560000	0	0.440000	0.250000	0	0.750000
32	0.995000	0.005000	0	0.556000	0	0.444000	0.245000	0	0.755000
33	0.994500	0.005500	0	0.552000	0	0.448000	0.240000	0	0.760000
34	0.994000	0.006000	0	0.548000	0	0.452000	0.235000	0	0.765000
35	0.993500	0.006500	0	0.544000	0	0.456000	0.230000	0	0.770000
36	0.993000	0.007000	0	0.540000	0	0.460000	0.225000	0	0.775000
37	0.992500	0.007500	0	0.536000	0	0.464000	0.220000	0	0.780000
38	0.992000	0.008000	0	0.532000	0	0.468000	0.215000	0	0.785000
39	0.991500	0.008500	0	0.528000	0	0.472000	0.210000	0	0.790000
40	0.991000	0.009000	0	0.524000	0	0.476000	0.205000	0	0.795000
41	0.990500	0.009500	0	0.520000	0	0.480000	0.200000	0	0.800000
42	0.990000	0.010000	0	0.516000	0	0.484000	0.195000	0	0.805000
43	0.989500	0.010500	0	0.512000	0	0.488000	0.190000	0	0.810000
44	0.989000	0.011000	0	0.508000	0	0.492000	0.185000	0	0.815000
45	0.989000	0.011000	0	0.504000	0	0.496000	0.180000	0	0.820000
46	0.988500	0.011500	0	0.500000	0	0.500000	0.175000	0	0.825000
47	0.988000	0.012000	0	0.496000	0	0.504000	0.170000	0	0.830000
48	0.987500	0.012500	0	0.492000	0	0.508000	0.165000	0	0.835000
49	0.987000	0.013000	0	0.488000	0	0.512000	0.160000	0	0.840000
50	0.986000	0.014000	0	0.484000	0	0.516000	0.155000	0	0.845000
51	0.985000	0.015000	0	0.480000	0	0.520000	0.150000	0	0.850000
52	0.984000	0.016000	0	0.476000	0	0.524000	0.145000	0	0.855000
53	0.982500	0.017500	0	0.472000	0	0.528000	0.140000	0	0.860000
54	0.981000	0.019000	0	0.468000	0	0.532000	0.135000	0	0.865000
55	0.979500	0.020500	0	0.464000	0	0.536000	0.130000	0	0.870000
56	0.978000	0.022000	0	0.460000	0	0.540000	0.125000	0	0.875000
57	0.976500	0.023500	0	0.456000	0	0.544000	0.120000	0	0.880000
58	0.975000	0.025000	0	0.452000	0	0.548000	0.115000	0	0.885000
59	0.975000	0.025000	0	0.448000	0	0.552000	0.110000	0	0.890000
60	0.975000	0.025000	0	0.444000	0	0.556000	0.105000	0	0.895000
61	0.975000	0.025000	0	0.440000	0	0.560000	0.100000	0	0.900000
62	0.975000	0.025000	0	0.436000	0	0.564000	0.095000	0	0.905000
63	0.972500	0.027500	0	0.432000	0	0.568000	0.090000	0	0.910000
64	0.970000	0.030000	0	0.428000	0	0.572000	0.085000	0	0.915000
65	0.965038	0.034621	0.000341	0.060312	0.929934	0.009754	0.147410	0.399893	0.452697
66	0.962417	0.037170	0.000413	0.059067	0.929877	0.011056	0.128059	0.381839	0.490102
67	0.959573	0.039928	0.000499	0.057856	0.929611	0.012534	0.111249	0.364603	0.524148
68	0.956480	0.042915	0.000604	0.056679	0.929111	0.014211	0.096647	0.348149	0.555204
69	0.953112	0.046157	0.000731	0.055535	0.928351	0.016115	0.083962	0.332441	0.583597

70	0.949435	0.049679	0.000885	0.054423	0.927300	0.018277	0.072943	0.317445	0.609612
71	0.945414	0.053514	0.001072	0.053344	0.925922	0.020734	0.063370	0.303129	0.633501
72	0.941006	0.057697	0.001297	0.052296	0.924179	0.023525	0.055055	0.289461	0.655484
73	0.936160	0.062269	0.001570	0.051280	0.922022	0.026698	0.047831	0.276412	0.675757
74	0.930820	0.067279	0.001901	0.050295	0.919400	0.030305	0.041555	0.263955	0.694490
75	0.924916	0.072782	0.002302	0.049340	0.916253	0.034408	0.036104	0.252061	0.711835
76	0.918371	0.078843	0.002787	0.048415	0.912510	0.039075	0.031367	0.240707	0.727926
77	0.911089	0.085536	0.003374	0.047520	0.908092	0.044388	0.027253	0.229866	0.742881
78	0.902962	0.092952	0.004086	0.046654	0.902909	0.050437	0.023678	0.219516	0.756806
79	0.893155	0.102346	0.004499	0.044664	0.903140	0.052197	0.022696	0.210408	0.766896
80	0.882349	0.112698	0.004954	0.042764	0.903211	0.054025	0.021758	0.201716	0.776526
81	0.870441	0.124104	0.005455	0.040951	0.903124	0.055925	0.020863	0.193421	0.785715
82	0.857318	0.136674	0.006008	0.039221	0.902879	0.057901	0.020010	0.185506	0.794485
83	0.842854	0.150529	0.006617	0.037569	0.902475	0.059956	0.019195	0.177953	0.802853
84	0.826910	0.165802	0.007288	0.035993	0.901914	0.062093	0.018417	0.170745	0.810837
85	0.809333	0.182639	0.008028	0.034489	0.901193	0.064318	0.017676	0.163869	0.818456
86	0.789952	0.201204	0.008844	0.033053	0.900313	0.066634	0.017081	0.158359	0.824560
87	0.768580	0.221676	0.009744	0.031683	0.899270	0.069046	0.016525	0.153200	0.830275
88	0.745007	0.244256	0.010737	0.030376	0.898065	0.071559	0.016006	0.148385	0.835609
89	0.740209	0.247960	0.011831	0.029128	0.896693	0.074179	0.015523	0.143910	0.840567
90	0.734934	0.252027	0.013039	0.029290	0.896120	0.074591	0.015076	0.139772	0.845152
91	0.734350	0.252582	0.013068	0.029459	0.895520	0.075021	0.015017	0.139221	0.845762
92	0.733731	0.253170	0.013098	0.029636	0.894894	0.075471	0.014985	0.138920	0.846096
93	0.733077	0.253793	0.013131	0.029820	0.894238	0.075941	0.014982	0.138894	0.846124
94	0.732383	0.254452	0.013165	0.030014	0.893552	0.076434	0.015012	0.139177	0.845810
95	0.731649	0.255150	0.013201	0.030216	0.892834	0.076950	0.015081	0.139809	0.845110
96	0.730871	0.255890	0.013239	0.030429	0.892081	0.077490	0.015192	0.140841	0.843967
97	0.730046	0.256675	0.013280	0.030651	0.891292	0.078057	0.015353	0.142336	0.842310
98	0.729171	0.257506	0.013323	0.030885	0.890464	0.078651	0.015573	0.144376	0.840051
99	0.728243	0.258389	0.013368	0.031130	0.889595	0.079275	0.015863	0.147066	0.837071
100	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
101	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
102	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
103	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
104	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
105	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
106	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
107	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
108	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
109	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
110	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
111	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
112	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
113	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
114	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
115	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
116	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
117	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
118	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
119	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218
120	0.727258	0.259325	0.013417	0.031387	0.888683	0.079931	0.016238	0.150543	0.833218

Source: Author’s calculations based on references cited in Section 3.

Table 5

Productivity Factors by Age and State: 2-State Version

Age	Age-related (deterministic) productivity	Idiosyncratic productivity, low state	Idiosyncratic productivity, high state
21–24	9.8703	0.6612	1.5081
25–29	12.4385	0.6692	1.4963
30–34	17.5824	0.5430	1.6856
35–39	18.7831	0.5895	1.6157
40–44	21.8091	0.5054	1.7418
45–49	23.7994	0.5114	1.7330
50–54	22.9768	0.4812	1.7782
55–59	22.3280	0.4156	1.8766
60–64	21.3683	0.3001	2.0498

Source: Authors' calculations based on Nishiyama and Smetters (2003).

Table 6

Annuitization of Aggregate Wealth

Relative risk aversion σ	1	2	4
Annuitized fraction of total wealth (no OASI)	99.9%	95.1%	82.7%
Annuitized fraction of total wealth (with OASI)	99.9%	93.7%	81.6%
Annuitized fraction of retiree wealth (no OASI)	99.9%	94.0%	86.6%
Annuitized fraction of retiree wealth (with OASI)	99.9%	91.3%	84.1%

Note: Entries show annuitized proportion of aggregate wealth, for various levels of risk aversion, in partial equilibrium with bond interest rate and the subjective discount rate equal to 5%. Annuity premiums are assumed to be actuarially fair and individuals completely selfish, so the existence of health shocks is the only deviation from Yaari's assumptions.

Table 7

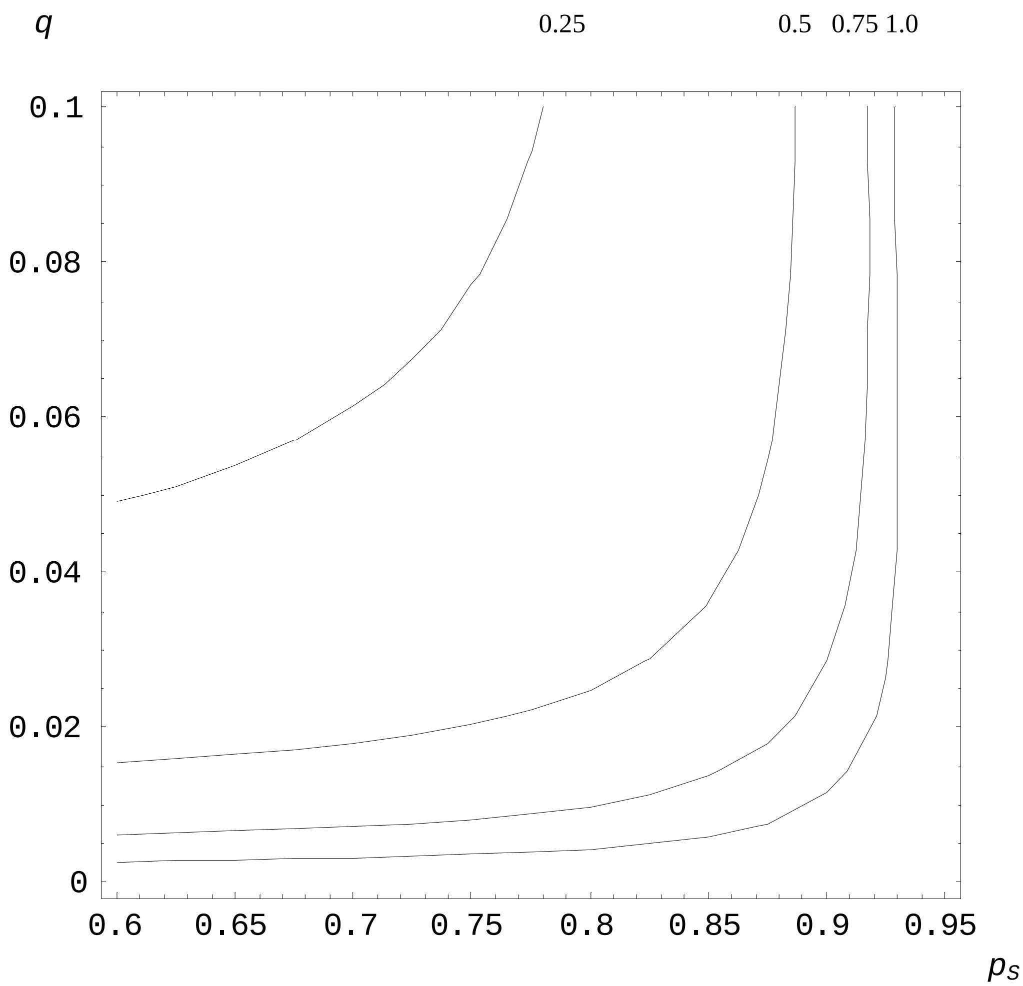
Effect of Bequest Motives on Annuitization

Altruism parameter ζ	Annuitized fraction of aggregate wealth		*K/Y*
	Total	Retirees only	
0.0	93.7%	91.3%	3.3
0.1	93.3%	90.9%	3.4
0.25	92.1%	89.8%	3.5
0.5	72.6%	66.5%	3.6
1.0	34.1%	19.2%	4.5

Note: Entries in the middle two columns show annuitized proportion of aggregate wealth, for various levels of altruism, in partial equilibrium with bond interest rate and the subjective discount rate equal to 5%. Annuity premiums are assumed to be actuarially fair and Social Security is present. Coefficient of relative risk aversion is assumed to be 2. Capital/output (*K/Y*) ratio is given to illustrate the overall effect on savings of the given value of ζ.

Figure 1

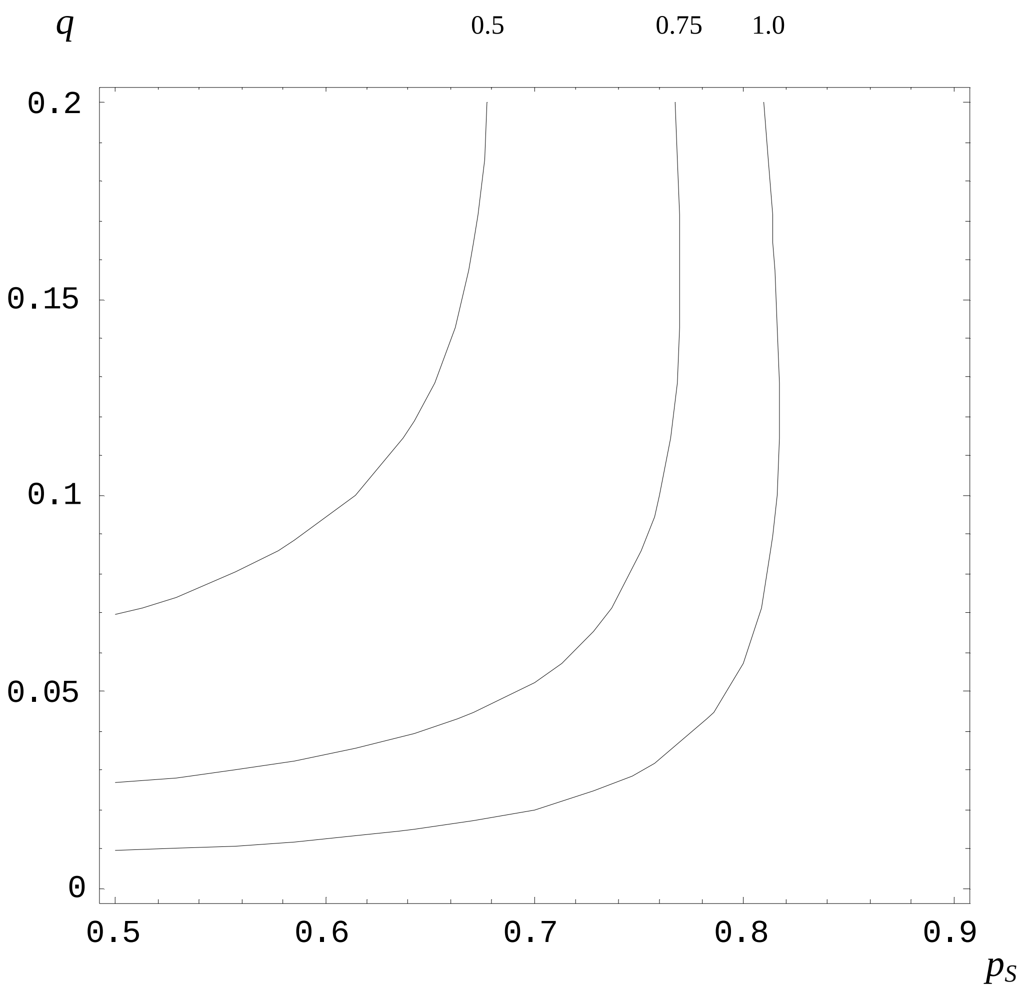
Optimal Portfolio in the Infinitely-Lived Model



Note: Contour plot of the annuity share of the portfolio in the infinite-period model. Vertical axis is the probability of becoming sick next period, and the horizontal axis is the probability of one-period survival in the sick state. The lowest curve is the locus of points where the borrowing constraint becomes binding; the full annuitization region is below and to the right of it. $\beta = 0.97$, $p_H = 0.99$, $r = 0.04$ for this figure.

Figure 2

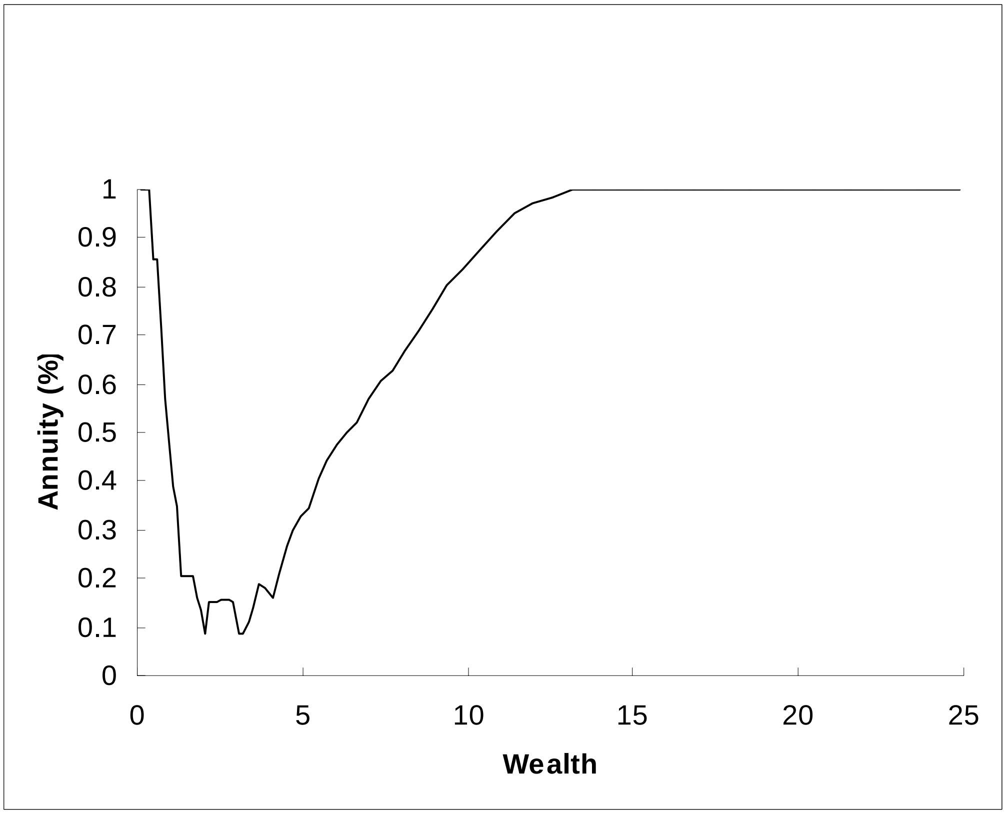
Optimal Portfolio in the Infinitely-Lived Model



Note: Contour plot of the annuity share of the portfolio in the infinite-period model for $\beta = 0.97$, $p_H = 0.97$, $r = 0.07$.

Figure 3

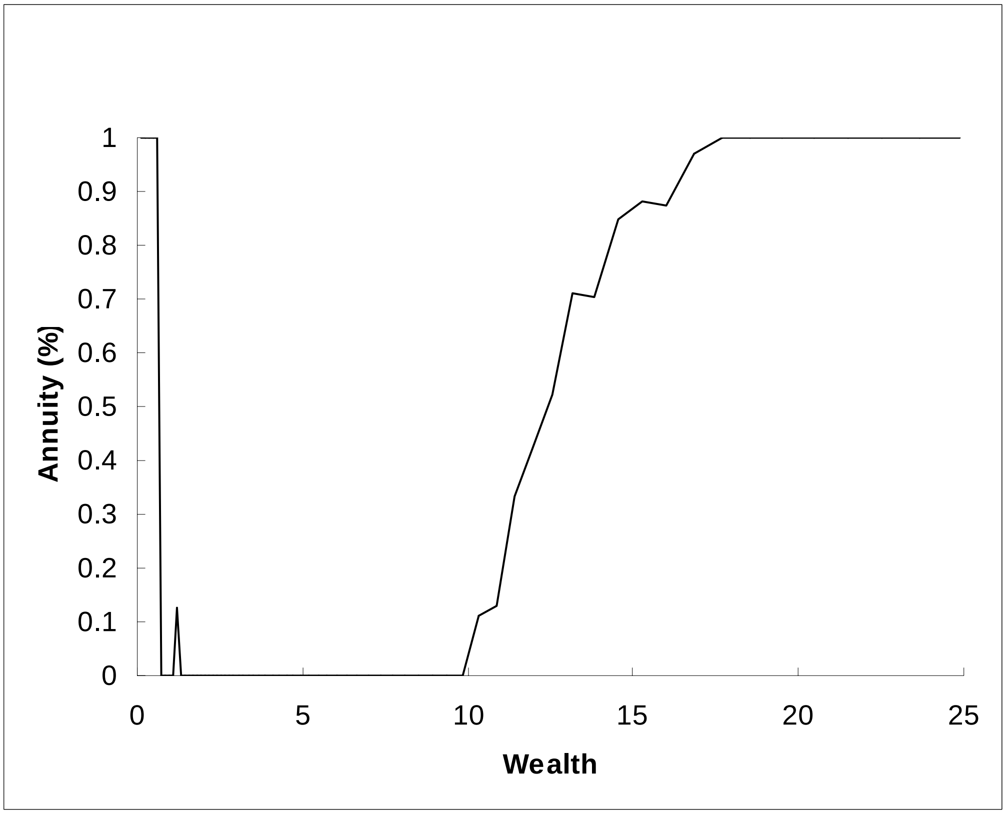
Annuitized Fraction of Wealth at Age 65:
No Social Security, CRRA = 2



Note: Optimal fraction of wealth held in life annuity form by a healthy person age 65 with a coefficient of relative risk aversion (CRRA) equal to 2.0, plotted as a function of total wealth. The unit of wealth is average loss in the sick state. Social Security, transaction costs, and bequest motives are absent. Bond interest rate and the subjective discount rate are both 5%.

Figure 4

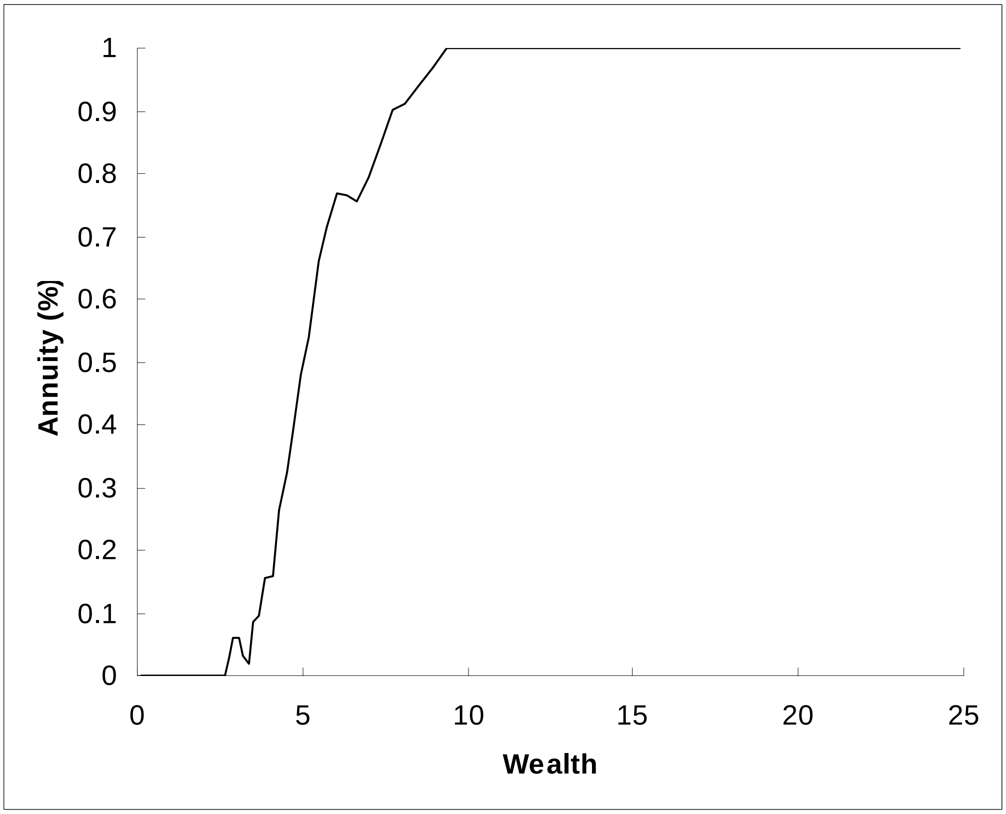
**Annuitized Fraction of Wealth at Age 65:
No Social Security, CRRA = 4**



Note: Optimal fraction of wealth held in life annuity form by a healthy person age 65 with a coefficient of relative risk aversion (CRRA) equal to 4.0, plotted as a function of total wealth. The unit of wealth is average loss in the sick state. Social Security, transaction costs, and bequest motives are absent. Bond interest rate and the subjective discount rate are both 5%.

Figure 5

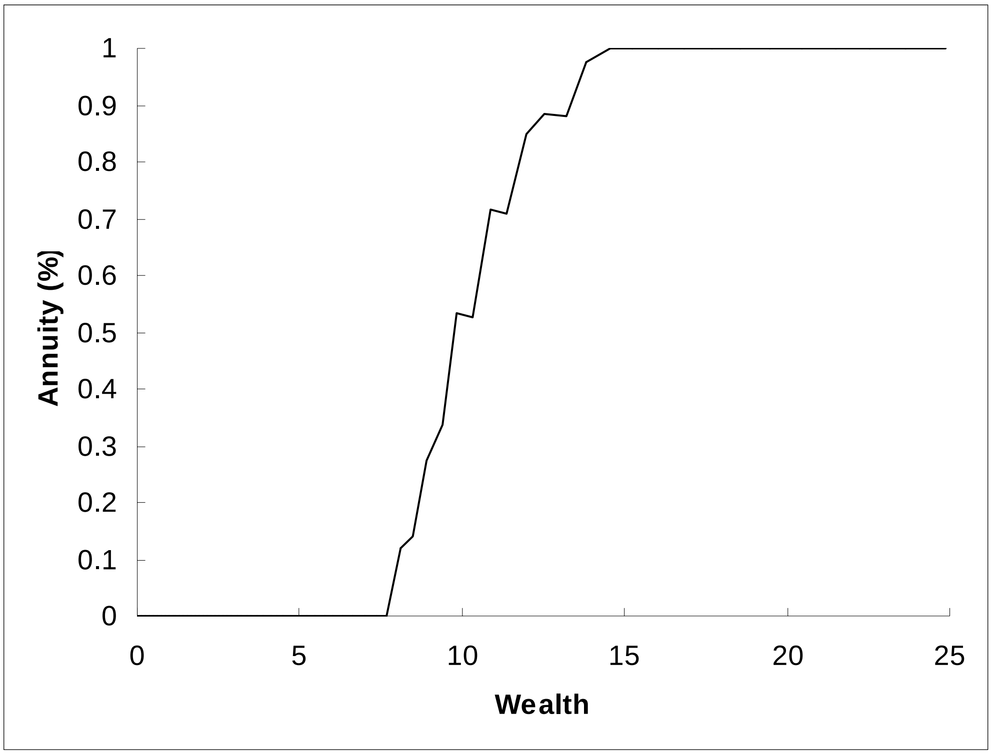
Annuitized Fraction of Wealth at Age 65:
With Social Security, CRRA = 2



Note: Optimal fraction of wealth held in life annuity form by a healthy person age 65 with a coefficient of relative risk aversion (CRRA) equal to 2.0, plotted as a function of total wealth. The unit of wealth is average loss in the sick state. Social Security exists, but transaction costs and bequest motives are absent. Bond interest rate and the subjective discount rate are both 5%.

Figure 6

Annuitized Fraction of Wealth at Age 65:
With Social Security, CRRA = 4



Note: Optimal fraction of wealth held in life annuity form by a healthy person age 65 with a coefficient of relative risk aversion (CRRA) equal to 4.0, plotted as a function of total wealth. The unit of wealth is average loss in the sick state. Social Security exists, but transaction costs and bequest motives are absent. Bond interest rate and the subjective discount rate are both 5%.

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