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Fiscal Policy Effects in a Heterogeneous-Agent Overlapping-Generations Economy With an Aging Population

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Abstract

Among the models that the Congressional Budget Office uses to analyze the economic effects of changes in federal fiscal policy is a life-cycle growth model with overlapping generations of heterogeneous households. In this paper, we extend a similar dynamic model by incorporating a population that ages in a manner consistent with projections for the United States. Under that extension, gross domestic product per capita is projected to be about 7 percent lower in 2040, and more than 11 percent lower in the long run, if the government cuts transfer rates and increases income taxes to finance the budgetary cost of an aging population, compared to results from a similar model without an aging population. In addition, the paper shows that considering an aging population is important in analyzing long-term policy changes that involve intergenerational transfers.

1 Introduction

The Congressional Budget Office (CBO, 2013b) projects that the federal budget deficit will shrink from 4 percent of gross domestic product (GDP) in 2013 to about 2 percent of GDP by 2015 if fiscal policy remains unchanged from that specified in current law. (The analysis in this paper was conducted prior to the recent release of revised historical GDP data by the Bureau of Economic Analysis; therefore, the budget figures reported in this paper and the calibration of the models used in the analysis all refer to data prior to the BEA revision.) However, budget shortfalls are projected to increase later in the coming decade, reaching 3.5 percent of GDP in 2023, because of the pressures of an aging population, rising health care costs, an expansion of federal subsidies for health insurance, and growing interest payments on federal debt. With such deficits, federal debt held by the public is projected to remain above 70 percent of GDP—far higher than the 39 percent average seen over the past four decades. (As recently as the end of 2007, federal debt equaled 36 percent of GDP.)

We analyze the effect of an aging population on the U.S. economy and the federal budget, using a life-cycle growth model that is similar to one of the models that CBO uses to analyze the economic effects of changes in federal fiscal policy.[1] The model incorporates a large number of households in different age cohorts (also known as overlapping generations) that are forward-looking in their behavior, have different levels of wealth and work abilities, and are subject to uncertainty about future wages and life spans. The model is considered a general-equilibrium model because households make decisions in response to prices (such as wages and rates of return on saving), and prices are determined by households' decisions.

The paper compares the results from two model economies—one without an aging population and one with an aging population. The paper uses those economies to analyze how households change their optimal behavior to consume, supply labor, and save in the presence of the aging population, and how they react to the future fiscal policy changes—spending cuts and tax increases—required to finance the cost of the aging population. The economy without an aging population has a balanced-growth path aligned to the U.S. economy in 2013 with constant productivity and population growth rates.[2] The economy with an aging population incorporates historical data and projections of the population distributions by age and

[1]For a discussion of the models that CBO uses to estimate the economic effects of fiscal policies, see CBO (2012b), Appendix, pp. 13–18. For a detailed description of the life-cycle model, see Nishiyama (2003).

[2]To focus on the budgetary effect of an aging population, we abstract from the short-term budgetary problem caused, in part, by the recent recession. In the economy without an aging population, the paper assumes higher marginal income tax rates and a budget deficit for 2013 calibrated to keep the debt-to-GDP ratio at 75 percent. That deficit is 2.1 percent of GDP, smaller than the deficit in CBO (2013b).

corresponding mortality rates by age. Comparing the simulation results from the two model economies shows the impact of the aging population on the aggregate economy and on the federal budget.

Key implications of the aging population are that federal revenue from income taxes will likely decrease and expenditures on Social Security and major health care programs will increase compared with the outcomes with a stationary population. Specifically, in the future the government will not be able to sustain its budgetary policies—or equivalently, will not be able to stabilize the debt-to-GDP ratio—under the current tax and spending system. The paper consequently examines the implications of two different fiscal policies to finance the federal budgetary shortfall arising from aging: to reduce government purchases in ways that have no effects on households' behavior (hereafter, Option 1), or to increase marginal income tax rates and reduce transfers to households in such a way that total federal tax revenues and transfers were changed by roughly equal amounts (Option 2).[3]

After 2013, the growth rate of GDP per capita in the economy with the aging population is lower, with a growing gap, than in the economy without aging.[4] If the change in federal fiscal policy had no marginal effects on behavior (as under Option 1), GDP per capita would be lower by 6.1 percent in 2040 and by 7.8 percent in the long term, compared with estimates that do not incorporate an aging population.[5] If, instead, the federal government cut its transfer spending and increased income tax rates roughly equally (as under Option 2), GDP per capita would be lower by 6.8 percent in 2040 and by 11.4 percent in the long run, compared with estimates that do not incorporate an aging population.[6]

The effect of the aging population on the federal budget is significant. If the government cut its purchases to finance the cost of the aging population (as under Option 1), the government's tax revenue per capita would be lower by 1.2 percent of GDP in 2040 and by 1.5 percent of GDP in the long term, and the government's transfer expenditure per capita would be 2.3 percent higher in 2040 and 3.5 percent higher in the long run, because of the aging of the population. To keep the debt per capita at the same level as in the economy without the aging population, the government would have to cut purchases by 3.7 percent of

[3]In the model economy of this paper, government purchases are not in the households' utility function, thus the change will not directly affect their behavior (Option 1). Government transfers and income tax rates are in the households' budget constraint, thus these changes will directly affect the households' decision (Option 2).

[4]The model abstracts from individuals of ages 0 to 20 and does not assume any changes in the average size of a household. Thus, in this paper, the growth rate of GDP per capita is equal to the growth rate of GDP per household.

[5]The long-run real growth rate of GDP per capita is assumed to be 1.8 percent in the model economies. Therefore, between 2013 and 2040, real GDP per capita grows 61.9 percent ($1.018^{27} - 1 = 0.619$) in the stationary-population economy, whereas it grows 52.0 percent ($1.619 \times 0.939 - 1 = 0.520$) in the aging-population economy under Option 1.

[6]Generally speaking, cuts in the government's transfers to households will increase labor supply, capital accumulation, and GDP. However, increases in the marginal income tax rates will decrease labor supply, capital accumulation, and GDP. Thus, if the cost of the aging population is solely financed by the increase in income tax rates, its negative macroeconomic effect will be larger.

GDP in the economy without aging in 2040 and by 5.2 percent in the long run.[7] If the government instead cut transfer spending and increased income tax rates (as under Option 2), the government would have to cut the transfers by 1.9 percent of GDP in 2040 and 3.0 percent in the long term, and would have to increase the marginal income tax rates by 19.6 percent in 2040 and by 31.7 percent in the long term, compared with outcomes in an economy without the aging population.[8] Such changes would significantly reduce the welfare of future households.

We also study the extent to which the aging population is important when analyzing short-term and long-term fiscal policy changes, such as tax reform and Social Security reform. Because the model with aging takes a long time to generate results, using the model with an aging population may only be practical in limited circumstances—in particular, to analyze policies that affect the intergenerational distribution of income. For fiscal policies whose economic effects are not importantly determined by the age structure of the population—such as a temporary tax cut or increase—incorporating demographic changes generally has small impacts on the results. For policies that involve intergenerational income redistribution—such as changing a tax rate on labor income or transfer spending to elderly households—the effects of the aging population tend to be more important, although the short-term and medium-term effects (that is, for the next 10 or 20 years) may not be very different. For changes to Social Security and major health care programs, the ages of households often have more important quantitative effects. However, the qualitative implications can be similar with and without the aging population.

2 The Model Economy

The model economy consists of many overlapping-generations households, a perfectly competitive representative firm with constant-returns-to-scale technology, and a government that can credibly commit to a fiscal policy.[9] The time is discrete, and one model period is a year, which is denoted by t. In a stationary (steady-state) equilibrium of the model, the economy is assumed to be on a balanced-growth path with a constant labor-augmenting productivity growth rate, μ, and a constant population growth rate, ν.[10] In the

[7] In the stationary-population baseline economy, federal government purchases are assumed to be 6.0 percent of GDP. The number is roughly consistent with the 2013 U.S. economy. See Table 3 in Section 3.

[8] These changes in the marginal income tax rates would increase income tax revenue per capita by 1.8 percent in 2040 and 2.7 percent in the long run, as a percentage of GDP, relative to Option 1.

[9] The model economy without an aging population in this paper is a simplified version of those in Nishiyama (2010, 2011).

[10] When the economy is on the balanced-growth path with labor-augmenting productivity and population growth rates, μ and ν, respectively, individual variables other than working hours, on average, grow at $1+\mu$, and aggregate variables other than total working hours grow at $(1+\mu)(1+\nu)$. In the economy with an aging population, the population growth rate is not constant over time,

following model description, individual variables other than working hours are growth-adjusted by $(1+\mu)^{-t}$, and aggregate variables are adjusted by $[\,(1+\mu)(1+\nu)\,]^{-t}$.

2.1 The Households

Households in the model economy are heterogeneous with respect to their age, $i = 21, \ldots, I$, beginning-of-year wealth (net worth), $a \in A = [\,a_{\min}, a_{\max}]$, average historical earnings used to calculate Social Security Old-Age and Survivors Insurance (OASI) benefits, $b \in B = [\,0, b_{\max}]$, and labor productivity, $e \in E = [\,0, e_{\max}]$. The households enter the economy and start working at age $i = 21$, and they can live to age $i = I = 100$.[11] In each year, t, the households receive idiosyncratic and age-dependent working ability shocks, e, and they each choose their consumption, c, working hours, h, and thus wealth at the beginning of the next year, a', to maximize their expected (remaining) lifetime utility.

We assume, for simplicity, that the households start receiving OASI benefits at their own full retirement age (FRA), $i = I_{\tilde{R}}$, which is between ages 65 and 67, depending on their birth year, although they can continue working after their FRA if they find it optimal.[12] This assumption is probably inconsequential because (i) the early retirement benefit reductions (6.7 percent per year for 3 years before FRA and 5 percent per year before the 3-year period) and the delayed retirement credit (8 percent) make the benefits approximately actuarially fair, and (ii) the benefits reduced by the retirement earnings test (RET) before the FRA are mostly recaptured by the households later with the recalculation of the primary insurance amount (PIA) when they reach their FRA.[13] The average historical earnings, b, are used to approximate the average indexed monthly earnings (AIME) and determine OASI benefits. The individual working ability, e, is equivalent to an hourly wage, and it follows a first-order Markov process.

but we assume that the growth rate will eventually stabilize to its long-run growth rate, ν. This assumption is necessary because, to solve the model for an equilibrium-transition path, the economy must be stationary in the long run, thus the population distribution by age must be stationary with a constant population growth rate, ν, in the long run.

[11]The full retirement age (FRA) of people who were born in 1937 or earlier is 65 years old, the FRA of those who were born between 1943 and 1954 is 66 years old, and the FRA of those who were born in 1960 or later is 67. See Table 2.A17.1 in Social Security Administration (SSA, 2013) for detailed information. The share of people age 100 or older is 0.0176 percent in 2010, and the same share is projected to be 0.2336 percent in 2100, according to the population estimate and intermediate projection (alternative II) used in the 2012 Old-Age, Survivors, and Disability Insurance (OASDI) Trustees Report (SSA, 2012).

[12]Social Security OASI benefits are partially taxable when the sum of adjusted gross income and 50 percent of benefits is above \$25,000 (single) or \$32,000 (married filing jointly). Thus, when the income tax on Social Security benefits is explicitly modeled, it is optimal for the workers to delay the Social Security benefit claim until they stop working. However, we exclude this possibility in the model economy.

[13]According to Nuschler and Shelton (2012), the cost of removing RET on the Social Security OASI trust fund would be \$81 billion in the short run from 2012 to 2018, but it would have no major effect on Social Security's projected long-range financial outlook.

2.1.1 The State Variables

Let s and $\mathbf{S}_t$ denote the individual state of the household and the aggregate state of the economy in year t, respectively,

$$\mathbf{s} = (i, a, b, e), \qquad \mathbf{S}_t = (x_t(\mathbf{s}), d_{G,t}),$$

where $x_t(\mathbf{s})$ is the growth-adjusted population distribution (density) function of households, and $d_{G,t}$ is the government's debt (held by the public) per household at the beginning of year t. Let $p_{i,t}$ be the growth-adjusted population of age-i households in year t. The population distribution function, $x_t(\mathbf{s})$, and the corresponding cumulative distribution function, $X_t(\mathbf{s})$, satisfy

$$\int_{A\times B\times E} x_t(i,a,b,e)da\,db\,de = \int_{A\times B\times E} dX_t(i,a,b,e) = p_{i,t}.$$

Let $\mathbf{\Psi}_t$ be the government's policy schedule at the beginning of year t,

$$\mathbf{\Psi}_t = \left\{c_{G,s}, tr_{LS,s}, \tau_{I,s}(\cdot), \tau_{P,s}(\cdot), tr_{SS,s}(\cdot), \tau_{C,s}, d_{G,s+1}\right\}_{s=t}^{\infty},$$

where $c_{G,t}$ is the government's consumption per household, $tr_{LS,t}$ is a lump-sum transfer per household, $\tau_{I,t}(\cdot)$ is a progressive income tax function, $\tau_{P,t}(\cdot)$ is a Social Security Old-Age, Survivors, Disability, and Hospital Insurance(OASDHI) payroll tax function, $tr_{SS,t}(\cdot)$ is an OASDHI benefit function, $\tau_{C,s}$ is a flat consumption tax rate, and $d_{G,t+1}$ is the government's debt per household at the beginning of the next year. The government's consumption is not in the household's utility function and thus does not affect the household's decision on private consumption and labor supply, while lump-sum transfers directly affect the household's decision through the intertemporal budget constraint.[14] The flat consumption tax, $\tau_{C,s}$, is assumed to approximate other federal revenues, such as those from excise taxes and customs duties.

Finally, let $\mathbf{\Phi}_t$ be the population projection at the beginning of year t,

$$\mathbf{\Phi}_t = \left\{(p_{i,s})_{i=0}^{I}, (\phi_{i,s})_{i=0}^{I}\right\}_{s=t}^{\infty},$$

where $\phi_{i,t}$ is the conditional survival rate at the end of age i in year t, given that the household is alive at the

[14] In a policy experiment, therefore, lump-sum transfers can be used to approximate the increase in government-provided goods and services—such as public education and public health care—that are the strong substitutes of private goods and services.

beginning of age i in year t. For simplicity, the population distribution and the corresponding mortality rates are assumed to be deterministic, and households have perfect foresight about the future population distribution.[15] The population projection, $\mathbf{\Phi}_t$, affects the model economy in the following ways: First, the projection of the household's own survival rates and corresponding longevity have an influence on the household's consumption, working, and saving decisions though their retirement planning. Second, projected changes in the population distribution by age as well as the share of the working-age population directly affect macroeconomic variables, factor prices, and the government's budget and fiscal policies. Finally, these changes in future factor prices and the government's fiscal policies generate feedback effects on the household's optimal decisions—both the current decision and future decisions—in a dynamic general-equilibrium economy like the one in this paper.

2.1.2 The Optimization Problem

Let $v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)$ be the value function of a household at the beginning of year t. Then, the household's optimization problem is

$$(1) \qquad v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) = \max_{c,h,a'} \Big\{ u(c,h) + \tilde{\beta}\phi_{i,t} E\big[\, v(\mathbf{s}', \mathbf{S}_{t+1}; \mathbf{\Psi}_{t+1}, \mathbf{\Phi}_{t+1}) \,|\, \mathbf{s} \,\big] \Big\}$$

subject to the constraints for the decision variables,

$$(2) \qquad c > 0, \qquad 0 \le h < h_{\max}, \qquad a' \ge a'_{\min}(\mathbf{s}),$$

and the law of motion of the individual state,

$$(3) \qquad \mathbf{s}' = (i+1, a', b', e'),$$

$$(4) \qquad a' = \frac{1}{1+\mu}\Big[(1+\tilde{r}_t)a + w_t eh + tr_{SS,t}(i,b) + tr_{LS,t} + q_t(i) - \tau_{I,t}(w_t eh, \tilde{r}_t a, tr_{SS,t}(i,b)) - \tau_{P,t}(w_t eh) - (1+\tau_{C,t})c \Big],$$

$$(5) \qquad b' = \mathbf{1}_{\{i<I_R\}} \frac{1}{i-20}\Big[(i-21)\, b\, \frac{w_t}{w_{t-1}} + \min(\eta w_t eh, \vartheta_{\max}) \Big] + \mathbf{1}_{\{i \ge I_R\}} b,$$

[15] Introducing aggregate uncertainty, such as total factor productivity shocks and mortality/fertility shocks, to a heterogeneous-agent model causes the "curse of dimensionality" problem. When there are aggregate shocks, under the rational expectation assumption, we need to solve the household's optimization problem for all possible paths (sample paths) of the aggregate economy, which is computationally infeasible in a heterogeneous-agent economy.

where $u(\cdot)$ is a period utility function, a combination of Cobb–Douglas and constant relative risk aversion (CRRA),

$$(6) \qquad u(c,h) = \frac{\left[c^{\alpha}(h_{\max} - h)^{1-\alpha}\right]^{1-\gamma}}{1-\gamma},$$

$\tilde{\beta}$ is a growth-adjusted discount factor explained below, and $E[\,\cdot \mid \mathbf{s}\,]$ denotes the conditional expected value given the household's current state. The household's decision variables are constrained: consumption, c, is strictly positive; working hours, h, are nonnegative and are less than a time endowment, $h_{\max}$; and wealth at the beginning of the next year, a', satisfies a borrowing constraint, $a' \geq a'_{\min}(\mathbf{s})$.[16] In the law of motion, $\tilde{r}_t$ is the interest rate, w_t is the wage rate per efficiency unit of labor, q_t is the distribution of accidental bequests explained below, $\mathbf{1}_{\{\cdot\}}$ is an indicator function that returns 1 if the condition in $\{\ \}$ holds and 0 otherwise, I_R is set at 65 so that the household's OASI benefits are calculated based on its growth-adjusted earnings between ages 21 and 64, η is the ratio of taxable labor income to total labor income, and $\vartheta_{\max}$ is the maximum taxable earnings for the OASI program.[17] The household's wealth at the beginning of the next year, a', is adjusted by the productivity growth rate, $1+\mu$. The average historical earnings for a household at the beginning of the following year, b', are calculated as wage-indexed, and the price indexation of AIME after age 60 is reflected in the Social Security benefit function.[18]

[16] We assume, for simplicity, that the borrowing limit (the lowest-possible wealth level) depends only on the household's age i. See Section 3.2 for the assumption of $a'_{\min}(\mathbf{s})$. The borrowing limit is set at 0 for elderly households with ages $i \geq 75$. Although some short-term loans are available to those who receive Social Security benefits, as the U.S.C. Title 42 Section 407 states, lenders are not allowed to garnish Social Security benefits, and thus the size of loans is expected to be small.

[17] In the current Social Security system, AIME is calculated based on the highest 35 years of growth-adjusted earnings. However, keeping the previous 35 highest earnings as the household's state variables would make the household problem computationally intractable. In the model economy, therefore, AIME is approximated by the growth-adjusted earnings of all ages before $I_R = 65$.

[18] The calculation of the average historical earnings in equation (5) imitates the calculation of AIME. Suppose that a household starts working at age 21. Then, the household's average historical earnings at the beginning of age 26 is calculated as $\sum_{i=21}^{25} y_i/5$, where y_i is a taxable earnings at age i or the maximum taxable earnings, $\vartheta_{\max}$, whichever is smaller, adjusted upward (wage-indexed) by using the aggregate wage growth rate from the year at age i to the year at age 26. See Appendix D in SSA (2013) for a detailed explanation on wage indexing.

2.1.3 The Distribution of Households

Solving the household's problem for c, h, and a' for all possible states,[19] we obtain the household's decision rules and the average historical earnings in the next year as $c(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)$, $h(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)$, and

$$\begin{aligned} a'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) &= \frac{1}{1+\mu}\Big[(1+\tilde{r}_t)a + w_t e h(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) + tr_{SS,t}(i,b) + tr_{LS,t} + q_t(i) \\ &\quad - \tau_{I,t}(w_t e h(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t), \tilde{r}_t a, tr_{SS,t}(i,b)) - \tau_{P,t}(w_t e h(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)) \\ &\quad - (1+\tau_{C,t}) c(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) \Big], \\ b'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) &= \mathbf{1}_{\{i<I_R\}} \frac{1}{i-20}\Big[(i-21)\, b \,\frac{w_t}{w_{t-1}} + \min(\eta w_t e h(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t), \vartheta_{\max}) \Big] + \mathbf{1}_{\{i \geq I_R\}} b. \end{aligned}$$

Households are assumed to enter the economy at age 21 without any assets and working histories, thus,

$$\int_{A\times B\times E} dX_t(21,a,b,e) = \int_E dX_t(21,0,0,e) = p_{21,t},$$

where $p_{21,t}$ is the population of age 21 households in year t. In a stationary population economy, the growth-adjusted population of age 21 households is normalized to unity, that is, $p_{21,t} = 1$. The law of motion of the growth-adjusted population distribution is, for $i = 21, \ldots, I$,

$$\begin{aligned} x_{t+1}(\mathbf{s}') &= x_{t+1}(i+1, a', b', e') \\ &= \frac{\phi_{i,t}}{1+\nu} \int_{A\times B\times E} \mathbf{1}_{\{a'=a'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t),\, b'=b'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)\}} \pi_i(e' \,|\, e)\, dX_t(\mathbf{s}), \end{aligned}$$

where ν is the population growth rate, and $\pi_i(e' \mid e)$ is a probability density function of working ability e' at age $i+1$ given that the working ability is e at age i.[20]

[19] Because the household's individual state vector, $\mathbf{s}$, includes three continuous variables, a, b, and e, the number of all possible states is infinite. Therefore, we discretize the state space and solve the optimization for all nodes in each year. The aggregate state of the economy, $\mathbf{S}_t$, consists of a continuous function, $x_t(\mathbf{s})$, and thus the aggregate state space is infinitely dimensional. When there are no aggregate shocks in the model economy, however, there is a way to avoid this dimensionality problem. See Appendix A for the computational algorithm.

[20] The integrand on the right-hand side is the conditional density function of the household's state at age $i+1$ given the state at age i—that is, $f(i+1, a', b', e' \mid i, a, b, e) = \mathbf{1}_{\{a'=a'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t),\, b'=b'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)\}} \pi_i(e' | e)$. Multiplying the density function, $x(i,a,b,e)$, we have the two-year joint density function of the state, $f(i,a,b,e,i+1,a',b',e')$. Integrating the joint density function with respect to a, b, and e and adjusting the population with the survival rate, $\phi_{i,t}$, and with the long-run growth rate, ν, we get the growth-adjusted marginal density function, $x(i+1,a',b',e')$, at age $i+1$.

2.2 The Firm

Total private wealth, $W_{P,t}$, the government's debt held by the public, $D_{G,t}$, national wealth, W_t, domestic capital stock, K_t, and labor supply in efficiency units, L_t, are

$$(7) \quad W_{P,t} = \sum_{i=21}^{I} \int_{A\times B\times E} a \, dX_t(\mathbf{s}),$$

$$(8) \quad D_{G,t} = \sum_{i=21}^{I} \int_{A\times B\times E} d_{G,t} \, dX_t(\mathbf{s}) = d_{G,t} \sum_{i=21}^{I} p_{i,t},$$

$$(9) \quad W_t = W_{P,t} - D_{G,t},$$

$$(10) \quad K_t = W_t + W_{F,t},$$

$$(11) \quad L_t = \sum_{i=21}^{I} \int_{A\times B\times E} eh(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) \, dX_t(\mathbf{s}),$$

where $W_{F,t}$ is net foreign wealth, which is assumed to be constant after growth adjustments. We assume that net foreign wealth, $W_{F,t}$, stays at the same level before and after a change in the government policy or the economic environment even if part of the domestic capital stock, K_t, is owned by foreign residents in the baseline economy.[21]

In each year, the representative firm chooses the capital input, $\tilde{K}_t$, and efficiency labor input, $\tilde{L}_t$, to maximize its profit, taking factor prices, r_t and w_t, as given, where r_t is the rate of return to capital. The firm's optimization problem is

$$(12) \quad \max_{\tilde{K}_t, \tilde{L}_t} F(\tilde{K}_t, \tilde{L}_t) - (r_t + \delta)\tilde{K}_t - w_t \tilde{L}_t,$$

where $F(\cdot)$ is a constant-returns-to-scale production function, $F(\tilde{K}_t, \tilde{L}_t) = A\tilde{K}_t^{\theta}\tilde{L}_t^{1-\theta}$, with total factor productivity A, and δ is the depreciation rate of capital. The firm's profit-maximizing conditions are

$$(13) \quad F_K(\tilde{K}_t, \tilde{L}_t) = r_t + \delta, \qquad F_L(\tilde{K}_t, \tilde{L}_t) = w_t,$$

[21] Under this assumption of net foreign wealth, if the household's savings did not change, an increase in government debt would fully crowd out domestic capital investment without causing an international capital inflow. Thus, fiscal policy effects in this model economy are similar to those in a closed economy.

and the factor markets are cleared when[22] [23]

$$K_t = \tilde{K}_t, \qquad L_t = \tilde{L}_t. \tag{14}$$

2.3 The Government

We assume the government policy schedule, $\boldsymbol{\Psi}_t$, as of year t is credible. The government collects taxes and makes its consumption and transfer spendings as scheduled in $\boldsymbol{\Psi}_t$. In addition, the government collects wealth left by deceased households (accidental bequests) and distributes it uniformly to working-age households.

2.3.1 The Government's Revenue and Expenditure

The government's income tax revenue, $T_{I,t}$, payroll tax revenue for Social Security, $T_{P,t}$, and consumption (or other) tax revenue, $T_{C,t}$, are

$$T_{I,t}(\varphi_t) = \sum_{i=21}^{I} \int_{A\times B\times E} \tau_{I,t}(w_t e h(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t), \tilde{r}_t a, tr_{SS,t}(i,b); \varphi_t)\, dX_t(\mathbf{s}), \tag{15}$$

$$T_{P,t}(\tau_{O,t}) = \sum_{i=21}^{I} \int_{A\times B\times E} \tau_{P,t}(w_t e h(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t); \tau_{O,t})\, dX_t(\mathbf{s}), \tag{16}$$

$$T_{C,t}(\tau_{C,t}) = \sum_{i=21}^{I} \int_{A\times B\times E} \tau_{C,t}\, c(\mathbf{s}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t)\, dX_t(\mathbf{s}), \tag{17}$$

where φ_t is one of the parameters of the income tax function, and $\tau_{O,t}$ is an OASI payroll tax rate. The government's consumption spending, $C_{G,t}$, non-Social Security transfer spending, $TR_{LS,t}$, and Social Security transfer spending, $TR_{SS,t}$, are

$$C_{G,t}(c_{G,t}) = \sum_{i=21}^{I} \int_{A\times B\times E} c_{G,t}\, dX_t(\mathbf{s}) = c_{G,t} \sum_{i=21}^{I} p_{i,t}, \tag{18}$$

[22] In an equilibrium, the factor markets are always cleared. Private wealth, $W_{P,t}$, and labor supply in efficiency units, L_t, are determined by the household's optimization problem; the government's debt, $D_{G,t}$, is determined by the government; and domestic capital stock, K_t, is determined by total wealth, $W_{P,t} - D_{G,t} + W_{F,t}$. Equilibrium factor prices, r_t and w_t, are then determined by capital stock and labor supply in the economy.

[23] Relaxing the assumption of fixed net foreign wealth, we can alternatively assume a large open economy with smaller changes in market-clearing factor prices or a small open economy with no changes in factor prices. Smaller changes in factor prices would affect not only the household's labor supply and saving decisions but also the government budget through the debt-service cost.

$$TR_{LS,t}(tr_{LS,t}) = \sum_{i=21}^{I} \int_{A\times B\times E} tr_{LS,t}\, dX_t(\mathbf{s}) = tr_{LS,t} \sum_{i=21}^{I} p_{i,t}, \tag{19}$$

$$TR_{SS,t}(\psi_{O,t}) = \sum_{i=21}^{I} \int_{A\times B\times E} tr_{SS,t}(i, b; \psi_{O,t})\, dX_t(\mathbf{s}), \tag{20}$$

where $\psi_{O,t}$ is a parameter of the OASI benefit function.

For simplicity, we assume that the government collects remaining wealth held by deceased households at the end of year t and distributes it in a lump-sum manner in the same year.[24] Because there are no aggregate shocks in the model economy, the government can perfectly predict the sum of accidental bequests during the year. The government revenue from these accidental bequests, Q_t, is

$$Q_t = \sum_{i=21}^{I} \int_{A\times B\times E} (1-\phi_{i,t})(1+\mu)a'(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)\, dX_t(\mathbf{s}). \tag{21}$$

Suppose that the government redistributes the accidental bequests uniformly to all working-age households. Then, for $i = 21, \dots, I_R - 1$, the bequest received by each working-age household is

$$q_t(i) = \left(\sum_{i=21}^{I_R-1} \int_{A\times B\times E} dX_t(\mathbf{s}) \right)^{-1} Q_t. \tag{22}$$

The law of motion of the government's debt held by the public, $D_{G,t}$, is

$$\begin{aligned} D_{G,t+1}(d_{G,t+1}) = \frac{1}{(1+\mu)(1+\nu)} \big[& (1+r_{D,t})D_{G,t}(w_{G,t}) - T_{I,t}(\varphi_t) - T_{P,t}(\tau_{O,t}) - T_{C,t}(\tau_{C,t}) \\ & + C_{G,t}(c_{G,t}) + TR_{LS,t}(tr_{LS,t}) + TR_{SS,t}(\psi_{O,t}) \big], \end{aligned} \tag{23}$$

where $r_{D,t}$ is the average government bond yield. Although the government's debt at the beginning of the next year, $D_{G,t+1}$, is shown on the left-hand side in equation (23), it is endogenous only if a policy change is assumed to be deficit-financing. In most other cases, the government's debt per household is fixed after the productivity growth adjustment, $d_{G,t+1} = d_{G,t}$, and one or two policy parameters on the right-hand side are adjusted to satisfy the government's intertemporal budget constraint.[25]

The government bond yields are on average significantly lower than the average rate of return to capital.

[24] In reality, the distribution of bequests is strongly skewed to the right (very unequal), and it is possibly important for the intragenerational distribution of wealth. See Nishiyama (2002) and De Nardi (2004) for further discussions.

[25] We assume that the economy is on a balanced-growth path in the long run. A government policy is sustainable when the growth-adjusted debt per household, $d_{G,t}$, is stabilized over time and constant in the long run.

The debt-service cost for the government will be inaccurately large in the model economy if the market interest rate is used for the government's debt. We assume that the average government bond yield, $r_{D,t}$, is a fraction of the rate of return to capital,

$$r_{D,t} = (1-\xi)r_t,$$

where $\xi r_t \geq 0$ is a risk premium. We also assume that the government bonds are proportionally held by domestic households and foreign investors.[26] Then, the average interest rate on household wealth, $\tilde{r}_t$, is the weighted average of the market rate of return and the government bond yield,

$$\tilde{r}_t = \frac{K_t}{W_{P,t}+W_{F,t}}\, r_t + \frac{D_{G,t}}{W_{P,t}+W_{F,t}}\, r_{D,t} = \left(1 - \frac{D_{G,t}}{W_{P,t}+W_{F,t}}\,\xi\right) r_t,$$

which is lower than the market rate of return, r_t, when the government debt exists, or $D_{G,t} > 0$.

2.4 Recursive Competitive Equilibrium

This section defines a recursive competitive equilibrium for the model economy.

Definition of Recursive Competitive Equilibrium: Let $\mathbf{s} = (i, a, b, e)$ be the individual state of households, let $\mathbf{S}_t = (x(\mathbf{s}), d_{G,t})$ be the state of the economy, let $\mathbf{\Psi}_t$ be the government policy schedule committed at the beginning of year t,

$$\mathbf{\Psi}_t = \left\{c_{G,s}, tr_{LS,s}, \tau_{I,s}(\cdot), \tau_{P,s}(\cdot), tr_{SS,s}(\cdot), \tau_{C,s}, d_{G,s+1}\right\}_{s=t}^{\infty},$$

and let $\mathbf{\Phi}_t = \left\{(p_{i,s})_{i=0}^{I}, (\phi_{i,s})_{i=0}^{I}\right\}_{s=t}^{\infty}$ be the population projection and corresponding survival rates. A time series of factor prices and the government policy variables,

$$\mathbf{\Omega}_t = \left\{r_s, w_s, c_{G,s}, tr_{LS,s}, \varphi_s, \tau_{O,s}, \psi_s, \tau_{C,s}, d_{G,s+1}\right\}_{s=t}^{\infty},$$

[26] Suppose that the share of net foreign wealth in total private wealth, $W_{F,t}/(W_{P,t}+W_{F,t})$, is 10 percent. If the government debt increased by $10 billion but private wealth stayed at the same level, then foreign investors would hold 10 percent, or $1 billion, of new government bonds and reduce other U.S. assets by $1 billion, leaving net foreign wealth unchanged. So, the crowding out effect of the debt increase is $10 billion, which is same as the effect under a closed-economy assumption.

the value functions of households, $\{v(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s)\}_{s=t}^{\infty}$, the decision rules of households,

$$\mathbf{d}(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s) = \left\{c(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s), h(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s), a'(\mathbf{s}, \mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s)\right\}_{s=t}^{\infty},$$

and the distribution of households, $\{x_s(\mathbf{s})\}_{s=t}^{\infty}$, are in a recursive competitive equilibrium if, for all $s = t, \dots, \infty$, each household solves the optimization problem of equations (1) through (5), taking $\mathbf{S}_s$, $\mathbf{\Psi}_s$, and $\mathbf{\Phi}_s$ as given; the firm solves its profit maximization problem of equations (12) and (13); the government policy schedule satisfies equations (15) through (23); and the goods and factor markets clear, thus these satisfy equations (7) through (11) and equation (14). The economy is in a stationary (steady-state) equilibrium—thus, on the balanced-growth path—if in addition, $\mathbf{S}_s = \mathbf{S}_{s+1}$, $\mathbf{\Psi}_{s+1} = \mathbf{\Psi}_s$, and $\mathbf{\Phi}_{s+1} = \mathbf{\Phi}_s$ for all $s = t, \dots, \infty$.[27]

2.5 Social Welfare Measures

Suppose that the economy is in the initial equilibrium in year $t = 0$ and that the government introduces a new policy at the beginning of year 1. Then, the rest-of-the-lifetime value of a household of state $\mathbf{s} = (i, a, b, e)$ is denoted by $v(\mathbf{s}, \mathbf{S}_0; \mathbf{\Psi}_0, \mathbf{\Phi}_0)$ before the policy change and $v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)$ for $t = 1, \dots, \infty$ after the policy change.

2.5.1 The Veil of Ignorance

Under the veil-of-ignorance welfare measure, welfare gains or losses of newborn households (age $i = 21$) at the beginning of $t = 1, \dots, \infty$ are calculated by uniform percent changes, $\lambda_{21,t}$, in the baseline consumption path that would make their expected lifetime utility equivalent to the expected utility after the policy change, that is,[28]

$$\lambda_{21,t} = \left[\left(\frac{E\, v(\mathbf{s}_{21}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)}{E\, v(\mathbf{s}_{21}, \mathbf{S}_0; \mathbf{\Psi}_0, \mathbf{\Phi}_0)} \right)^{\frac{1}{\alpha(1-\gamma)}} - 1 \right] \times 100.$$

Similarly, the average welfare changes of households of age $i = 22, \dots, I$ at the time of the policy

[27] To solve the model for a stationary (steady-state) equilibrium, the population distribution needs to be constant, after growth adjustment, in the long run. Even if we assume the fertility rates to be constant after 2100 (because the number of newborn babies depends on the population distribution ages 15–50 for women), it takes several hundred years before the population distribution stabilizes.

[28] The calculation of $\lambda_{i,t}$ depends on the period utility function. With the Cobb–Douglas and CRRA utility function, equation (6), when consumption proportionally increases by $\lambda_{i,t}/100$, the household value increases by $(1 + \lambda_{i,t}/100)^{\alpha(1-\gamma)} - 1$.

change ($t = 1$) are calculated by the uniform percent changes, $\lambda_{i,1}$, required in the baseline consumption path so that the rest of the lifetime value would be equal to the rest of the lifetime value after the policy change, that is,

$$\lambda_{i,1} = \left[\left(\frac{E\, v(\mathbf{s}_i, \mathbf{S}_1; \boldsymbol{\Psi}_1, \boldsymbol{\Phi}_1)}{E\, v(\mathbf{s}_i, \mathbf{S}_0; \boldsymbol{\Psi}_0, \boldsymbol{\Phi}_0)} \right)^{\frac{1}{\alpha(1-\gamma)}} - 1 \right] \times 100.$$

Note that $\lambda_{i,1}$ for $i = I, \ldots, 21$ shows the cohort-average welfare changes of all current households alive at the time of the policy change, and $\lambda_{21,t}$ for $t = 2, \ldots, \infty$ shows the cohort-average welfare changes of all future households.

This welfare measure is referred to as the veil-of-ignorance measure because households evaluate policy changes under the assumption that they do not know their own state, (a, b, e), in the economy.

2.5.2 Equivalent Variations

The equivalent variation of a household with individual state $\mathbf{s} = (i, a, b, e)$ is a one-time wealth transfer, $ev(\mathbf{s})$, that generates as much welfare gain in the baseline economy as the policy change does. The equivalent variations of newborn households (age $i = 21$) at the beginning of $t = 1, \ldots, \infty$ are calculated as $ev(\mathbf{s}_{21}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t)$ such that

$$v(21, a, b, e, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t) = v\big(21, a + ev(\mathbf{s}_{21}, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t), b, e, \mathbf{S}_0; \boldsymbol{\Psi}_0, \boldsymbol{\Phi}_0\big),$$

and the equivalent variations of age $i = 22, \ldots, I$ at the time of the policy change ($t = 1$) are calculated as $ev(\mathbf{s}_i, \mathbf{S}_1; \boldsymbol{\Psi}_1, \boldsymbol{\Phi}_1)$ such that

$$v(i, a, b, e, \mathbf{S}_1; \boldsymbol{\Psi}_1, \boldsymbol{\Phi}_1) = v\big(i, a + ev(\mathbf{s}_i, \mathbf{S}_1; \boldsymbol{\Psi}_1, \boldsymbol{\Phi}_1), b, e, \mathbf{S}_0; \boldsymbol{\Psi}_0, \boldsymbol{\Phi}_0\big).$$

The average growth-adjusted equivalent variations in wealth by age cohort are calculated as

$$\begin{aligned}
\lambda_{21,t}^{EV} &= \int_{A \times B \times E} ev(21, a, b, e, \mathbf{S}_t; \boldsymbol{\Psi}_t, \boldsymbol{\Phi}_t) dX_t(\mathbf{s}_{21}) \times \frac{1}{p_{21,t}}\ , \\
\lambda_{i,1}^{EV} &= \int_{A \times B \times E} ev(i, a, b, e, \mathbf{S}_1; \boldsymbol{\Psi}_1, \boldsymbol{\Phi}_1) dX_1(\mathbf{s}_i) \times \frac{1}{p_{i,1}}\ ,
\end{aligned}$$

for $t = 1, \ldots, \infty$ and $i = 22, \ldots, I$.

2.5.3 Compensating Variations

The compensating variation of a household with individual state $\mathbf{s} = (i, a, b, e)$ is a one-time negative wealth transfer, $cv(\mathbf{s})$, that restores the baseline welfare level in the alternative economy after the policy change. The compensating variations of newborn households (age $i = 21$) at the beginning of $t = 1, \dots, \infty$ are calculated as $cv(\mathbf{s}_{21}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)$ such that

$$v(21, a - cv(\mathbf{s}_{21}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t), b, e, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) = v\big(21, a, b, e, \mathbf{S}_0; \mathbf{\Psi}_0, \mathbf{\Phi}_0\big),$$

and the compensating variations of age $i = 22, \dots, I$ at the time of the policy change ($t = 1$) are calculated as $cv(\mathbf{s}_i, \mathbf{S}_1; \mathbf{\Psi}_1, \mathbf{\Phi}_1)$ such that

$$v(i, a - cv(\mathbf{s}_i, \mathbf{S}_1; \mathbf{\Psi}_1, \mathbf{\Phi}_1), b, e, \mathbf{S}_1; \mathbf{\Psi}_1, \mathbf{\Phi}_1) = v\big(i, a, b, e, \mathbf{S}_0; \mathbf{\Psi}_0, \mathbf{\Phi}_0\big).$$

The average growth-adjusted compensating variations in wealth by age cohort are calculated as

$$\begin{aligned}
\lambda_{21,t}^{CV} &= \int_{A \times B \times E} cv(21, a, b, e, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t) dX_t(\mathbf{s}_{21}) \times \frac{1}{p_{21,t}} \,, \\
\lambda_{i,1}^{CV} &= \int_{A \times B \times E} cv(i, a, b, e, \mathbf{S}_1; \mathbf{\Psi}_1, \mathbf{\Phi}_1) dX_1(\mathbf{s}_i) \times \frac{1}{p_{i,1}} \,,
\end{aligned}$$

for $t = 1, \dots, \infty$ and $i = 22, \dots, I$.

3 Calibration: Stationary-Population Economy

We construct one stationary-population baseline economy and two aging-population baseline economies. The stationary-population baseline economy is in stationary (steady-state) equilibrium—that is, the baseline economy is on a balanced-growth path. By contrast, because the projected population distribution is not stationary, the aging-population baseline economies are obtained as equilibrium-transition paths between 1961 and 2200.

Table 1 shows the values of main preference and technology parameters in the stationary-population economy, and Table 2 (on page 26) shows the baseline values of the government policy parameters in the stationary-population economy. All parameter values in Table 1 are fixed all the time, but some of the policy parameter values in Table 2 are changed in policy experiments discussed below.

Table 1: Main Preference and Technology Parameter Values in the Stationary-Population Economy

Parameter		Value	Comment
Demographics			
Maximum age	I	100	
Maximum age households can work		75	
Minimum age of elderly households	I_R	65	Medicare eligible age
Productivity growth rate	μ	0.0180	Real GDP per capita growth in 1971-2011
Population growth rate	ν	0.0100	Population growth rate in 1971-2011
Total population		43.8252	When $p_{21,t} = 1.0$ in the baseline
Working-age population (ages 21-64)		34.4777	
Preferences			
Discount factor	β	1.0313	$K_t/Y_t = 2.4$ in the baseline
Growth-adjusted discount factor	$\tilde{\beta}$	1.0063	$\tilde{\beta} = \beta(1+\mu)^{\alpha(1-\gamma)}$
Coefficient of relative risk aversion	γ	3.0000	
Share parameter of consumption	α	0.6881	$\bar{h}_t = 1.0$ in the baseline
Maximum working hours	$h_{\max}$	1.6313	Frisch elasticity $= 0.5$
Production technology and wage process			
Share parameter of capital stock	θ	0.3840	Labor income share $= 0.616$ in 2007-2011
Depreciation rate of capital stock	δ	0.1100	$r_t = 0.05$ in the baseline
Total factor productivity	A	0.9630	$w_t = 1.0$ in the baseline
Autocorrelation parameter of log wage	ρ	0.9500	
Standard deviation of log wage shocks	σ	0.2440	Variance of log earnings in ages 21-64 in 2007
Median working ability	$\bar{e}_i$		Estimated by ordinary least squares

GDP = gross domestic product.

3.1 Demographics

The maximum possible age of households in the model economy, I, is 100. In the model economy, households of ages 21 to 64 are called working-age households, and households of age $I_R = 65$ or older are called elderly households, even though they can possibly work until age 75. In the stationary-population economy, for simplicity, all households start receiving Social Security (OASI) benefits at the current full retirement age, $I_{\tilde{R}} = 66$, which is the FRA of those who were born between 1943 and 1954.

The labor-augmenting productivity growth rate, μ, is set at 1.8 percent, which is close to the average growth rate, 1.82 percent, of real GDP per capita in 1971–2011. The population growth rate, ν, is set at 1.0 percent, which is also close to the average population growth rate in 1971–2011. The conditional survival rate, $\phi_{i,t}$, at the end of age i, given that households are alive at the beginning of age i, is calculated from the 2007 Period Life Table (Table 4.C6) in Social Security Administration (SSA, 2013). We use population-weighted averages of male and female survival rates; the survival rate at the end of age $I = 100$ is replaced with zero. When the population (the number of households) for age 21 is normalized to unity, and the population growth rate is 1.0 percent, the total population of households becomes 43.83 and the population of working-age households (ages 21–64) becomes 34.48 in the stationary-population economy.

3.2 Preferences

Households in the model economy are assumed to be a mixture of married (60 percent) and single (40 percent) households.[29] The household's period utility function is a combination of Cobb–Douglas and constant relative risk aversion,

$$u(c,h) = \frac{\left[c^{\alpha}(h_{\max} - h)^{1-\alpha}\right]^{1-\gamma}}{1-\gamma},$$

which is consistent with a growth economy.[30] Let c_i and h_i be the household's consumption and working hours at age i after growth adjustment, and $\hat{c}_i = (1+\mu)^{i-21} c_i$ and $\hat{h}_i = h_i$ are those before growth adjustment. In the absence of wage and lifetime uncertainties, the household's lifetime utility is shown as

$$\begin{aligned}\sum_{i=21}^{I} \beta^{i-21} u(\hat{c}_i, \hat{h}_i) &= \sum_{i=21}^{I} \beta^{i-21} \frac{\left[\left((1+\mu)^{i-21} c_i\right)^{\alpha} (h_{\max} - h_i)^{1-\alpha}\right]^{1-\gamma}}{1-\gamma} \\ &= \sum_{i=21}^{I} \beta^{i-21} \left((1+\mu)^{i-21}\right)^{\alpha(1-\gamma)} \frac{\left[c_i^{\alpha}(h_{\max} - h_i)^{1-\alpha}\right]^{1-\gamma}}{1-\gamma} = \sum_{i=21}^{I} \tilde{\beta}^{i-21} u\big(c_i, h_i\big),\end{aligned}$$

where $\tilde{\beta} = \beta(1+\mu)^{\alpha(1-\gamma)}$ is the growth-adjusted discount factor.

The coefficient of relative risk aversion, γ, for the combination of consumption and leisure is set at 3.0, which is roughly in the middle of the range assumed in macroeconomic public finance literature.[31] The share parameter of consumption, α, and the maximum working hours, $h_{\max}$, are set at 0.6881 and 1.6313, respectively, so that the baseline economy satisfies the following two conditions. First, the average working

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[29] According to the population estimate used in the 2012 OASDI Trustees Report (SSA, 2012), 2010 data show that 16 percent of people ages 26 to 65 are single, 67 percent are married, 13 percent are divorced, and 3 percent are widowed. In the 2007 Survey of Consumer Finances (SCF) sponsored by the Board of Governors of the Federal Reserve System (FRB, 2009), 62 percent of households for ages 21–65 and 59 percent of all households are married.

[30] When the period utility function is additively separable, as in $u(c,h) = f(c) - g(h)$, then the utility function is consistent with a balanced-growth path—hours worked are stable in a steady state even though productivity is rising—only if $f(c) = \alpha \ln c$ (that is, only if the coefficient of relative risk aversion with respect to consumption is one). Compared to a separable utility function, the combined Cobb–Douglas and CRRA utility function has a property that labor supply depends not only on the after-tax wage rate but also on household wealth.

[31] For example, Domeij and Heathcote (2004) use 1.0, İmrohoroğlu, İmrohoroğlu, and Joines (1995) use 2.0, and Auerbach and Kotlikoff (1987) and Conesa, Kitao, and Krueger (2009) use 4.0.

hours of the working-age households (households of ages 21–64), $\bar{h}_t$, is normalized to unity.[32] Second, the Frisch elasticity of working hours of the average household (a household whose working hours are the average of those of all working-age households) is 0.5, where the elasticity is calculated as

$$\frac{h_{\max} - \bar{h}}{\bar{h}} \frac{1 - \alpha(1 - \gamma)}{\gamma} = 0.5.$$

In a heterogeneous-agent overlapping-generations economy with uninsurable wage shocks, because of its heterogeneity and precautionary labor supply effects, the Frisch elasticities of average working hours and aggregate labor supply are lowered from 0.5 to around 0.4, which is consistent with the empirical labor literature.[33][34] When $\gamma = 3.0$ and $\alpha = 0.6881$, the coefficient of relative risk aversion with respect to consumption is calculated as

$$-\frac{c\, u_{cc}(c, h)}{u_c(c, h)} = 1 - \alpha(1 - \gamma) = 1 - 0.6881(1 - 3.0) = 2.3762.$$

The ratio of capital stock to GDP, K_t/Y_t, in the baseline economy is targeted at 2.4, which is close to the average ratio of private fixed assets to GDP, 2.38, in 2007–2011 in the national income and product account (NIPA) data produced by the Bureau of Economic Analysis (BEA, 2012).[35] We do not consider the government's fixed assets in the model economy because most of the government capital income is not counted in GDP or government revenue.[36] The ratio of the government's debt held by the public to GDP, $D_{G,t}/Y_t$, in the baseline economy is set at 0.75, which is close to the level, 0.744, at the end of the first quarter of 2013. Net foreign wealth, $W_{F,t}$, is set to be 30 percent of baseline GDP, which is close to the ratio

[32]The normalization of average hours to one does not affect the model's projections of the baseline or of the effect of policy experiments. Only the ratio of maximum to average hours is important for those outcomes. Therefore the "average hours" in the model can be thought of as corresponding to any real-world quantity of hours.

[33]For a discussion of estimates of the Frisch elasticity of labor supply, see Reichling and Whalen (2012). Under the current assumptions, based on an experiment of a temporary marginal wage rate change without direct income effects, the Frisch elasticity of working hours (ages 21–64) is estimated as 0.44, and the Frisch elasticity of labor supply is estimated as 0.31.

[34]The wage elasticity of labor supply can be lowered by increasing the coefficient of relative risk aversion, γ; or by increasing the ratio of the average working hours, $\bar{h}$, to the maximum working hours, $h_{\max}$. We first fix the average working hours at 1.0 then obtain the maximum working hours, 1.6313, so that the Frisch elasticity of the average household is 0.5. In this way, the average labor income in model units is kept at about the same level even if the Frisch elasticity target is changed in an alternative baseline economy.

[35]We acknowledge that BEA released the 13th comprehensive (benchmark) revision of NIPAs in July 2013, but that the analysis here was largely completed before that revision was available. The average ratio of private fixed assets to GDP in 2007–2011 is 2.30 after the revision.

[36]In NIPA accounting, returns generated by government capital (net of depreciation) accrue to the private sector, as for example government spending on infrastructure leads to higher private-sector wages (for people who work for trucking companies) and higher profits (for people who own the trucks). We could alternatively define capital stock, K_t, as the sum of private capital stock and government capital stock. Then, the effect of government capital stock will be captured as higher labor income.

of the U.S. net investment position to GDP, -0.276, at the end of 2012.[37]

Under this setting, the ratio of national wealth to GDP, W_t/Y_t, in the baseline economy is $2.4 - 0.3 = 2.1$, and the ratio of private wealth (held by the U.S. residents) to GDP, $W_{P,t}/Y_t$, in the baseline economy is $2.1 + 0.75 = 2.85$. The discount factor, β, of households is set at 1.0313 so that the ratio of private wealth to GDP in the baseline economy is 2.85, or equivalently, the capital–GDP ratio is 2.4. Then, the growth-adjusted discount factor is calculated as

$$\tilde{\beta} = \beta(1+\mu)^{\alpha(1-\gamma)} = 1.0063.$$

Because the discount rate, $1/\beta - 1$, is negative and smaller than the after-tax interest rate in the model economy, the average household consumption increases with age when the households are age 65 or younger and their mortality rates are low.[38]

We assume that the household's minimum wealth level (≤ 0) depends only on its age, as $a' \geq a'_{\min}(i,a,b,e) = a_{\min}(i+1)$, and

$$a_{\min}(i) = \begin{cases} 0 & \text{if} \quad i = 21 \text{ or } I+1, \\ \left[(1+\mu)a_{\min}(i+1) - 0.1 \times \bar{w}e_{\min}(i)\bar{h}\right]/(1+\bar{r}) & \text{if} \quad i = 22, \ldots, I, \end{cases}$$

where $e_{\min}(i)$ is the lowest working ability for age i, and where $\bar{r}$, $\bar{w}$, and $\bar{h}$ are the interest rate, the wage rate, and the average working hours, respectively, in the baseline economy. One might expect minimum assets to be set at what a household could repay under the worst possible realizations for working ability. However, labor income with the lowest working ability and with average working hours is instead multiplied by 0.1 to align household debt to the U.S. economy. According to the 2010 Survey of Consumer Finances (SCF) sponsored by the Board of Governors of the Federal Reserve System (FRB, 2012), 11.0 percent of households have negative net worth, and their average net worth is -\$31,447 (in other words, their average debt is \$31,447). In the model economy, 23.0 percent of households have negative wealth, and their average debt corresponds to \$11,934.

[37] The U.S. government debt held by the public was \$11,917 billion at the end of the first quarter of 2013, according to the Treasury department, and the U.S. net international investment position was -\$4,416 billion at the end of the fourth quarter of 2012, according to BEA. The GDP estimate for the first quarter of 2013 was used as the denominator to calculate the ratios 0.744 and -0.276.

[38] This increasing age–consumption profile is not counterintuitive, because consumption in the model economy includes consumption on behalf of children in the household, imputed rents from owner-occupied housing, and imputed medical expenditures paid by Medicare.

3.3 The Production Technology and the Wage Process

The production function of the representative firm is one of Cobb–Douglas,

$$Y = F(K, L) = AK^{\theta}L^{1-\theta}.$$

The share parameter of capital stock, θ, is set equal to 0.384, which reflects 1 minus the average labor income share, $w_t L_t / Y_t = 61.6$ percent, in 2007–2011 in the NIPA data. In the same period, the average capital income share, $r_{N,t} K_t / Y_t$, is calculated as 16.3 percent. When the capital–GDP ratio, K_t / Y_t, is 2.4, the average nominal rate of return to capital is $r_{N,t} = 16.3/2.4 = 6.8$ percent. Subtracting the inflation rate, the real rate of return to capital, r_t, in the baseline economy is set at 5.0 percent.[39] Because gross capital income (including depreciation) is equal to $(r_t + \delta)K_t = \theta Y_t$, the depreciation rate of capital stock, δ, is set at $\theta(Y_t/K_t) - r_t = 38.4/2.4 - 5.0 = 11.0$ percent.[40] Total factor productivity, A, is fixed at 0.9630 so that the average wage rate, w_t, is normalized to unity in the baseline economy.

The working ability, e_i, of an age i household in the model economy is assumed to satisfy

$$\ln e_i = \ln \bar{e}_i + \ln z_i$$

for $i = 21, \ldots, 75$, where $\bar{e}_i$ is the median wage rate at age i, and z_i is the persistent shock that follows an AR(1) process,

$$\ln z_i = \rho \ln z_{i-1} + \epsilon_i$$

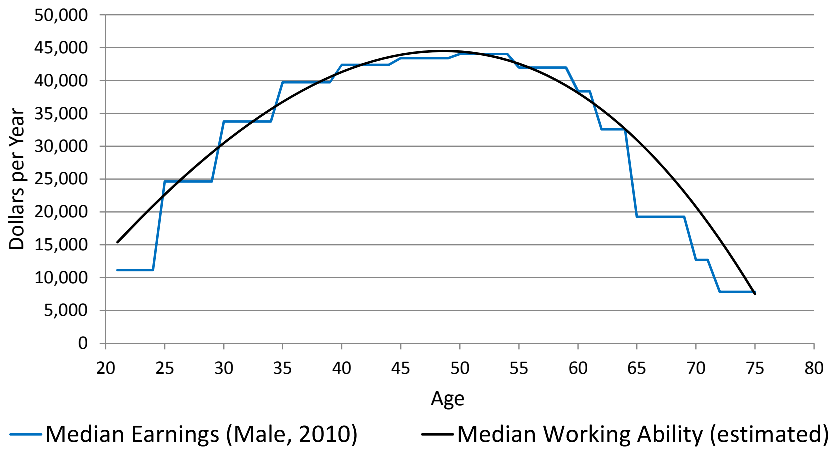
for $i = 21, \ldots, 75$. The temporary shock, ϵ_i, is normally distributed, $\epsilon_i \sim N(0, \sigma^2)$, and the initial distribution of the log-persistent shock satisfies $\ln z_{20} \sim N(0, \sigma^2_{\ln z_{20}})$.

The median working ability, $\bar{e}_i$, for ages 21 to 75 is constructed from the 2010 Median Earnings of Workers by Age (Table 4.B6, male workers) in SSA (2013). The median working ability profile is estimated by using ordinary least squares (OLS) to regress the median earnings on age for ages 25 to 64 and is extrap-

[39]The inflation rate of 1.8 percent is consistent with the recent inflation rates measured by the consumer price index, private consumption expenditure deflater, and GDP deflater. However, money is a veil in this model economy, thus the inflation rate itself is not important in this paper.

[40]The depreciation rate is less than 5 percent in the recent U.S. economy if we calculate it as the ratio of consumption of fixed capital to total fixed capital. We assume a much higher depreciation rate because, in the model economy, capital stock does not include the government's capital stock and GDP does not include taxes on production and imports. The higher depreciation rate is used, in part, to fill the gap between GDP in the model and GDP in the data.

Figure 1: The Median Earnings of Male Workers by Age



Sources: Author's calculations and Social Security Administration (2013).

olated for ages 21 to 24 and 65 to 75. We use the male median earnings data because a larger proportion of female workers choose not to work full time. We extrapolate rather than using the actual data on individuals ages 21 to 24 and 65 to 75 because they are more likely to be in school or voluntarily retired.[41] Figure 1 shows the original data and estimated values. The population-weighted average of the median wage rate, $w\bar{e}$, for ages 21 to 64 is also normalized to unity in the baseline economy.

The autocorrelation parameter, ρ, of the log-persistent shock is set at 0.95, which is approximately in the middle of the range in the previous literature.[42] Given the initial variance, $\sigma^2_{\ln z_{20}}$, the variance of log-persistent shock, $\ln z_i$, is calculated recursively as

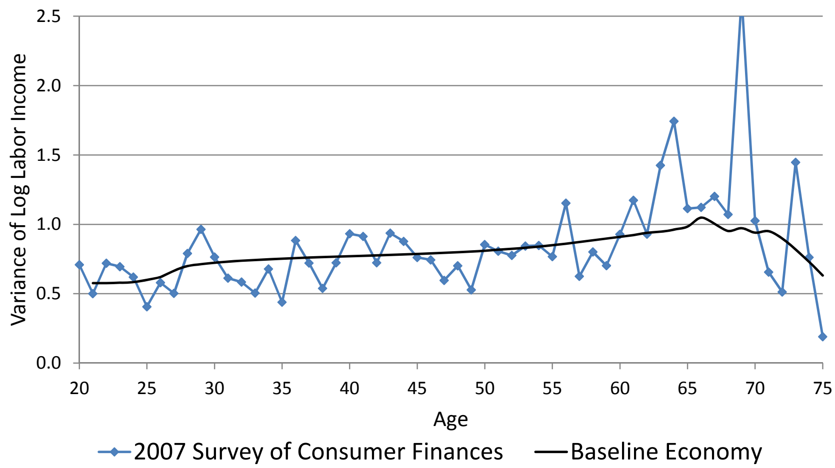
$$\sigma^2_{\ln z_i} = \rho^2 \sigma^2_{\ln z_{i-1}} + \sigma^2$$

for $i = 21, \ldots, 75$. To align the wage process to the U.S. data, the standard deviation, σ, of the transitory shock, ϵ_i, is set at 0.244, and the initial variance, $\sigma^2_{\ln z_{20}}$, of the log-persistent shock is set at a fraction of its

[41] We do not assume that all men ages 25 to 64 are full-time workers but calibrate the median full-time-equivalent earnings in the model to the median earnings by age of those individuals in SSA (2013). (In other words, we assume that most men with potential wages higher than the median wage in this age group do not choose to work part time.) Then, the paper can assume that the median working abilities (hourly wage) by age for ages 25 to 64 are also proportional to their median earnings by age because most full-time workers work about the same number of hours.

[42] For example, Domeij and Heathcote (2004) use 0.90, Huggett (1996) uses 0.96, and Conesa, Kitao, and Krueger (2009) use 0.98.

Figure 2: The Variance of Log Labor Income by Age



Sources: Author's calculations and 2007 Survey of Consumer Finances (FRB, 2009).

limiting variance,

$$\sigma^2_{\ln z_{20}} = 0.50 \lim_{i \to \infty} \sigma^2_{\ln z_i} = 0.50 \times \frac{\sigma^2}{1-\rho^2} = 0.50 \times 0.6106 = 0.2748.$$

The number $0.50 < 1$ is chosen so that the variance of log earnings increases with the age of the household. Figure 2 shows that the variances of log labor income calculated using the 2007 SCF data (FRB, 2009) and those in the baseline economy are quite similar except for households ages 71 to 75.[43]

The log-persistent shock is first discretized into 13 nodes for each age by using Gauss–Hermite quadrature; then the number of nodes for $\ln z_i$ is reduced to 7 by combining 4 nodes in each tail distribution into one node. The unconditional probability distribution of the seven nodes is

$$\pi_i = \begin{pmatrix} 0.0125 & 0.0792 & 0.2379 & 0.3410 & 0.2379 & 0.0792 & 0.0125 \end{pmatrix}$$

for $i = 21, \ldots, 75$. The Markov transition matrix of an age i household, $\Pi_i = [\,\pi(e^{j'}_{i+1} \,|\, e^j_i)\,]$, that corre-

[43] Although the variance of log working ability, $\sigma^2_{\ln z_i}$, is assumed to strictly increase with age, the variance of log labor income is not necessarily monotone in the model economy, because working hours are endogenous.

sponds to $\rho = 0.95$ is calculated by using the bivariate normal distribution function as

$$\Pi_i = \begin{pmatrix} 0.8460 & 0.1540 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0243 & 0.8517 & 0.1240 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0413 & 0.8678 & 0.0909 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0634 & 0.8732 & 0.0634 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0909 & 0.8678 & 0.0413 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1240 & 0.8517 & 0.0243 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.1540 & 0.8460 \end{pmatrix}$$

for $i = 21, \ldots, 74$.

3.4 The Progressive Income Tax Function

The average household labor income for ages 21 to 64 is \$64,162 in the 2010 SCF (FRB, 2012). Because labor income per capita increased by 3.3 percent between 2010 and 2011, the average labor income is estimated at \$66,279 in 2011. In the units employed in the model economy, the average labor income of households ages 21 to 64 is 1.4517. We assume that a fraction, $\eta < 1$, of labor income (compensation and part of proprietors' income) in the NIPA data is taxable for individual income tax and Social Security payroll tax. This parameter, η, is set at 0.7659 so that the ratio of OASDHI payroll tax revenue to GDP is 0.065 in the baseline economy. Under this assumption, one model unit corresponds to $\$66,279/1.4517/0.7659 \approx \$59,610$ in the 2011 U.S. economy. The model uses this number to convert some policy variables into the model parameters.

The income tax function in the model economy includes both the individual income tax and the corporate income tax. We assume it is a combination of a smooth progressive labor income tax function, a flat capital income tax function, and a lump-sum tax (constant),

$$\begin{aligned} \tau_{I,t}(r_t a, w_t eh, tr_{SS,t}) &= \tau_{L,t}(w_t eh) + \tau_{K,t} r_t a + \tau_{LS,t} \\ &= \varphi_t \big\{ \varphi_0 \big[(y_L - d) - \big((y_L - d)^{-\varphi_1} + \varphi_2 \big)^{-1/\varphi_1} \big] + \tau_{K,t} y_K \big\} + \tau_{LS,t}, \end{aligned}$$

where $y_L - d = \eta \cdot w_t eh - d$ is the household's taxable labor income after approximated deductions and exemptions, and $y_K = r_{N,t} a = (r_t + \pi_e) a$ is capital income that includes corporate income and imputed

Table 2: Baseline Government Policy Parameter Values in the Stationary-Population Economy

Parameter		Value	Comment
Model units			
Taxable labor income ratio	η	0.7659	$T_{P,t}/Y_t = 0.065$ in the baseline
Scale adjustment[a]		59.6099	Average earnings = \$66,279 in 2011
Progressive income tax			
Income tax adjustment factor	φ_t	0.9822	Average marginal rate $= 21.8\%$
Labor income tax: tax rate limit	φ_0	0.3640	Estimated by ordinary least squares
: curvature	φ_1	0.5016	Estimated by ordinary least squares
: scale	φ_2	0.3124	Estimated by ordinary least squares
: deduction & exemptions	d	0.1302	Estimated by ordinary least squares
Capital income tax rate	τ_K	0.3041	Average effective marginal rate $= 20.7\%$
Lump-sum tax portion of income tax	τ_{LS}	0.0008	$T_{I,t}/Y_t = 0.112$
Social Security system			
Social Security payroll tax rate: OASI	$\tau_{O,t}$	0.1060	Current-law tax rate
: DI	$\tau_{D,t}$	0.0180	
: HI	$\tau_{H,t}$	0.0290	
Maximum taxable earnings[b]	$\vartheta_{\max}$	2.5083	$1.4\times$\$106,800 = \$149,520 in 2011
OASI benefit adjustment factor	$\psi_{O,t}$	1.3390	$TR_{SS,t} = T_{P,t}$ in the baseline
Replacement rate threshold: 0.90 - 0.32[b]	ϑ_1	0.2111	$1.4\times$\$749$\times 12$ = \$12,583 in 2011
: 0.32 - 0.15[b]	ϑ_2	1.2730	$1.4\times$\$4,517$\times 12$ = \$75,886 in 2011
Social Security benefit: DI	$\psi_{D,t}$	0.0185	Budget balanced given the DI tax rate
: HI	$\psi_{H,t}$	0.1256	Budget balanced given the HI tax rate
Other policy variables			
Government consumption per household	$c_{G,t}$	0.1182	48% of non-Social Security spending
Lump-sum transfers per household	$tr_{LS,t}$	0.1280	52% of non-Social Security spending
Consumption tax rate	$\tau_{C,t}$	0.0250	$T_{C,t}/Y_t = 0.015$ in the baseline
Government's debt per household	$d_{G,t}$	1.4688	$D_{G,t}/Y_t = 0.75$ in the baseline
Accidental bequests per household	q_t	0.0927	Q divided by working-age population
Wealth held by foreigners	$W_{F,t}$	25.7490	$W_{F,t}/Y_t = 0.30$ in the baseline
Ratio of risk premium to interest rate	ξ	0.4000	$r_t(1-\xi) = 0.03$ in the baseline

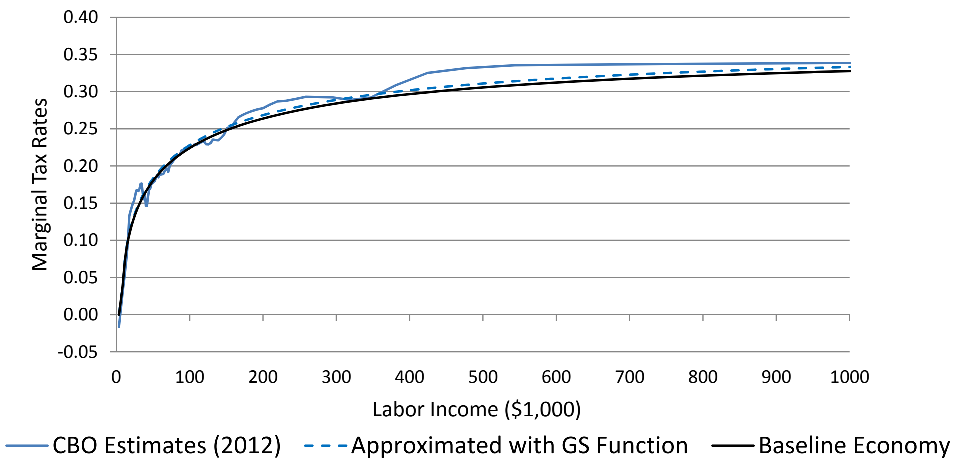
[a] A unit in the model economy corresponds to \$59,610 in 2011 dollars. [b] 40% of all households (67% of married households) are assumed to be two-earner households, and 60% are one-earner households. The thresholds are multiplied by the average number of workers, $0.40(2) + 0.60(1) = 1.4$, in a working-age household.
OASI = Old-Age and Survivors Insurance; DI = Disability Insurance; HI = Hospital Insurance.

rent from owner-occupied housing. The expected inflation rate, π_e, is set at 2.0 percent. The functional form of a smooth progressive labor income tax is taken from Gouveia and Strauss (1994).

We obtain the parameters, φ_0, φ_1, and φ_2, of the Gouveia–Strauss function as well as deductions and exemptions, d, by OLS with the 2013 effective labor income tax schedule estimated by CBO (2012a).[44]

[44]In this paper, therefore, we do not incorporate the changes from the American Taxpayer Relief Act of 2012, which was enacted

Figure 3: The Marginal Labor Income Tax Rate Schedule of Households



Sources: Author's calculations and Congressional Budget Office (2012a).

The first parameter, $\varphi_0 = 0.3640$, shows the limit of the effective marginal labor income tax rate as taxable income goes to infinity; the second parameter, $\varphi_1 = 0.5016$, shows the curvature of the tax function; and the third parameter, $\varphi_2 = 0.3124$, is used to adjust the scale of the tax function.[45] The additional parameter, φ_t, is set at 0.9822 so that the effective marginal labor income tax rate is, on average, 21.8 percent in the baseline economy as estimated by CBO. Figure 3 shows the marginal labor income tax rates estimated by CBO, approximated by using the Gouveia–Strauss function, and used in the baseline economy (after the adjustment factor, φ_t, is applied). The flat capital income tax rate, $\tau_{K,t}$, is set so that the effective capital income tax rate is 20.7 percent.[46] The lump-sum income tax parameter, $\tau_{LS,t}$, is set to make income tax revenue (the sum of individual income tax and corporate income tax) be 11.2 percent of GDP in the baseline economy. These three values are estimated by CBO (2012a) for 2013.[47]

in January 2013.

[45]The third parameter, φ_2, is initially estimated as 0.0402 by using labor income in thousands of dollars. Then, the Gouveia–Strauss tax function is rescaled to the model unit as $\varphi_2 = 0.0402 \times 56.610^{\varphi_1} = 0.3124$.

[46]The effective capital income tax rate is slightly lower than the effective marginal labor income tax, even though the tax function includes the corporate income tax. This is because capital stock in the model economy includes residential fixed assets. In recent years, almost 40 percent of fixed assets in the U.S. have been residential assets, and about three-quarters of those assets have been owner-occupied houses. With the mortgage interest deduction, the effective tax rate on imputed rents from owner-occupied housing is negative, and it reduces the effective capital income tax rate. Also, there are the various exemptions of capital income for 401(k) accounts, IRAs, pension funds, and similar items.

[47]CBO (2013a) revised down the projected tax rates in 2013, reflecting the policy changes made in January 2013. The current income tax revenue assumption, 11.2 percent of GDP, in the model economy is equal to that in 2015 in CBO's new projection.

3.5 The Social Security System

The Social Security OASDHI payroll tax function is

$$\tau_{P,t}(w_t eh) = (\tau_{O,t} + \tau_{D,t}) \min(\eta \cdot w_t eh, \vartheta_{\max}) + \tau_{H,t}\, \eta \cdot w_t eh,$$

where $\tau_{O,t}$ is the flat Old-Age and Survivors Insurance tax rate, $\tau_{D,t}$ is the flat Disability Insurance (DI) tax rate, and $\tau_{H,t}$ is a flat Hospital Insurance (HI, Medicare Part A) tax rate.[48] All three tax rates include the portion paid by employers. When labor income is below the maximum taxable earnings, the statutory OASI tax rate is 10.6 percent, including 5.3 percent paid by employers; thus, $\tau_{O,t} = 0.106$. The effective DI and HI tax rates are similarly set at $\tau_{D,t} = 0.018$ and $\tau_{H,t} = 0.029$, respectively. The maximum taxable earnings per worker for the OASDI payroll tax were \$106,800 in 2009 to 2011 (SSA, 2013). We assume that 60 percent of households are married households, of which two-thirds are two-earner households—in other words, 40 percent of all households are two-earner households. Thus, the maximum taxable earnings, $\vartheta_{\max}$, are the weighted average of those of a two-earner household and a one-earner household. In the model economy, the maximum taxable earnings for a household are set at 2.5083 in model units, which correspond to $1.4 \times$ \$106,800 $=$ \$149,520 in 2011.

The Social Security OASDHI benefit function is

$$\begin{aligned} tr_{SS,t}(i,b) = \mathbf{1}_{\{i \geq I_{\tilde{R}}\}} \psi_{O,t} \frac{1}{(1+\mu)^{i-60}} \big\{ & 0.90 \min(b, \vartheta_1) + 0.32 \max\left[\min(b, \vartheta_2) - \vartheta_1, 0\right] \\ & + 0.15 \max(b - \vartheta_2, 0) \big\} + \mathbf{1}_{\{i < I_R\}} \psi_{D,t} + \mathbf{1}_{\{i \geq I_R\}} \psi_{H,t}, \end{aligned}$$

where the full retirement age, $I_{\tilde{R}}$ is set at 66 in the stationary-population economy,[49] ϑ_1 and ϑ_2 are the thresholds for the three replacement rate brackets, 90 percent, 32 percent, and 15 percent, that calculate the OASI benefit from the average historical earnings, $\psi_{O,t}$ is an adjustment factor to balance the OASI budget, $\psi_{D,t}$ is the DI benefit, and $\psi_{H,t}$ is the HI benefit.[50] In the current U.S. Social Security system, the thresholds

[48] In this paper, we do not model the 0.9 percent Medicare surtax, applied to wages and compensation above a threshold amount of \$125,000–\$250,000, that went into effect in 2013.

[49] As explained in Section 2, households are assumed to start claiming their OASI benefits at their full retirement age (FRA), $I_{\tilde{R}}$, although they can continue working if they like. FRAs in the aging-population economy are set between ages 65 and 67 in two-month increments, depending on the household's birth year.

[50] AIMEs for those ages 60 or older are not wage-indexed but price-indexed, and the bracket thresholds are also price-indexed for each age cohort. To simplify the computation in the growth economy, the model first assumes all of the above variables are wage-indexed. Then, the model converts the Social Security benefits to be price-indexed by dividing the benefits by $(1+\mu)^{i-60}$.

to calculate primary insurance amounts are set for each age cohort when a worker reaches age 62. In the model economy, the growth-adjusted thresholds are fixed for all age cohorts, and the PIA is adjusted later by using the long-term productivity growth rate and years after age 60. Thus, the model simply uses the thresholds for the age 62 cohort in 2011 after scale adjustment.

We assume that the Social Security program is pay-as-you-go and that its budget is balanced in the baseline economy. The OASI benefit parameter, $\psi_{O,t}$, is set at 1.3390 to balance the OASI budget, which is roughly consistent with the data when benefits include survivors and spousal benefits. We also assume, for simplicity, that DI benefits are received only by working-age households (ages 21 to 64) and that HI benefits are received only by elderly households (ages 65 to 100). The benefit parameters, $\psi_{D,t}$ and $\psi_{H,t}$, are set at 0.0185 and 0.1256, respectively, to balance the DI and HI budgets in the baseline economy.[51]

3.6 The Other Policy Variables

We assume that 48 percent of the government's non-Social Security spending is government consumption and 52 percent is lump-sum transfers. This allocation is consistent with the average allocation in the NIPA data for 2001–2011 and for 2007–2008.[52] The government consumption per household, $c_{G,t}$, and the lump-sum transfer per household, $tr_{LS,t}$, are set at 0.1182 and 0.1280, respectively, in the baseline economy.

The rate of the national consumption tax, which approximates the excise tax and other taxes, is set at $\tau_{C,t} = 0.025$ so that the consumption tax revenue is 1.5 percent of GDP in the baseline economy. As explained above, the ratio of the government's debt to GDP is assumed to be 0.75 and, thus, the government's debt per household, $d_{G,t}$ is 1.4688 in the baseline economy. Also, the net foreign wealth, $W_{F,t}$, is set at 25.7490 so that the ratio of net foreign wealth (which is equivalent to the negative of the U.S. investment position) to GDP is 0.30. Accidental bequest per working-age household is calculated as 0.0927. Finally, the ratio of risk premium to interest rate, ξ, is assumed to be 0.4 so that the average government bond yield is 3.0 percent in the baseline economy.

Table 3 shows the government revenue and spending in the stationary-population baseline economy. The government revenue is roughly matched with the 2013 U.S. tax revenue projected in CBO (2012a).

[51] There is no disability risk or health risk in the model economy. DI benefits and HI benefits are uniformly distributed to working-age households and elderly households, respectively. The model implicitly assumes that the government transfers the actuarially fair insurance premium values of DI and HI to those households.

[52] The average share of transfer spending in the government's non-Social Security spending for 2009–2011 is 56 percent, which is significantly higher than the above assumption. However, the recent increase in transfer spending is mostly due to the fiscal stimulus policy after the most recent recession.

Table 3: Government Budget in the Stationary-Population Baseline Economy (Percentage of GDP)

Revenue	Total	Expenditure	Working Age (ages 21-64)	Elderly (ages 65+)	Total
Individual and corporate income tax	11.2	Government transfers	5.9	7.1	13.0
		(Transfer share %)	45.2	54.8	
Social Security payroll tax	6.5	Social Security	0.7	5.8	6.5
OASI	4.4	OASI	0.0	4.4	4.4
DI	0.7	DI	0.7	0.0	0.7
HI	1.4	HI	0.0	1.4	1.4
Consumption (other) tax	1.5	Other transfers	5.1	1.4	6.5
		Government consumption			6.0
		Interest payments			2.3
Total revenue	19.2	Total expenditures			21.3

Note: When the government debt-to-GDP ratio is 0.75, the productivity growth rate is 1.8%, and the population growth rate is 1.0%, then the sustainable budget deficit is $0.75 \times (1.8 + 1.0) = 2.1\%$ in the baseline economy.
GDP = gross domestic product; OASI = Old-Age and Survivors Insurance; DI = Disability Insurance; HI = Hospital Insurance.

The government spending in the baseline economy is adjusted so that the government budget is sustainable with the debt-to-GDP ratio at 75 percent. The share of transfers to elderly households is 54.8 percent in the model economy, which is very close to the share, 55.2 percent, calculated from the data in CBO (2013c). In addition, total government transfers are 13.0 percent as a percentage of GDP in the model economy, which is also very close to the size of mandatory government spending as a percentage of GDP in the CBO projection.

4 Calibration: Aging-Population Economies

In the aging-population baseline economies, the current-law fiscal policy is not sustainable. So we assume two different government financing rules in the aging-population economies. In the first economy, the government cuts its consumption spending to finance the cost of the aging population. In the second economy, the government uniformly cuts its non-Social Security transfer spending and proportionally increases marginal income tax rates.

4.1 Demographics

For the aging-population baseline economies, we use the population projection estimated by CBO (2012c) for 1961 to 2086.[53] CBO's population projection is based on the intermediate population projection (alternative II) estimated by the Social Security Administration, with some revisions in the projected immigration.

We extrapolate the population projection through 2200. In this extrapolation, the conditional survival rates of men and women, $\phi_{m,i}$ and $\phi_{f,i}$ for $i = 0, \ldots, 99$, are assumed to stay at the same levels from 2086 to 2200 as the levels at the end of 2085. Using these survival rates, the population distributions by age (for ages 1 to 100) of men and women in 2087 are calculated. The population of newborn (age 0) babies in 2087 is estimated by using the 2006 U.S. age-specific female fertility rates in United Nations (2008). These fertility rates in 2006 are adjusted proportionally to match the population of newborn babies and the age–population distribution of women ages 15 to 49 in 2086, and those rates are assumed to stay at the same levels for the years from 2087 to 2200. The age–population distributions of men and women from 2088 to 2200 are obtained recursively by using the age-specific female fertility rates and the conditional mortality rates. The population distribution will be roughly stationary in 2200, with a long-run growth rate $\nu = 0.37$ percent.[54]

4.2 Calibration Strategy

Because the U.S. population distribution is aging and thus not stationary, it is impossible to construct the 2013 baseline economy as a stationary equilibrium. Therefore, following Nishiyama (2004), we construct the aging-population baseline economies by the procedure explained below.

1. We first calibrate and solve the model for a 2013 stationary economy by using the age–population distribution in 2013. In this step, the households in the model economy falsely believe that the population distribution is unchanged and the economy is stationary. However, aggregate variables are calculated by using the non-stationary 2013 population distribution.

2. Using the parameters obtained in Step 1, we next solve the model for a 1961 stationary economy by

[53] We acknowledge that CBO has the *2013 Long-Term Budget Outlook* but that this analysis was largely completed before that outlook was published.

[54] Although the age-specific female fertility rates and the conditional survival rates are kept at the same level after 2086, the population distribution will not be stationary in 2200. This is because the population of newborns is dependent on the age distribution of women, which takes many years to stabilize.

using the age–population distribution in 1961. Again, in this step, the households in the model economy falsely believe that the population distribution is unchanged and that the economy is stationary. However, aggregate variables are calculated by using the non-stationary 1961 population distribution.

3. We then solve the model for an equilibrium-transition path between 1961 and 2200.[55] At the beginning of 1961, the households in the model economy realize that the population distribution is not stationary. They precisely project the changes in fertility rates and mortality rates through 2200 and choose their consumption, labor supply, and saving to maximize their lifetime utility.

In Step 1, the calibration of the model to the 2013 stationary economy, we assume the same macroeconomic targets for 2013, rather than the same parameter values, in the aging-population baseline economy as those in the stationary-population baseline economy. For example, the capital-to-GDP ratio, K_t/Y_t, is 2.4, the Frisch elasticity is 0.5, income tax revenue is 11.2 percent of GDP, payroll tax revenue is 6.5 percent of GDP, and the government's debt-to-GDP ratio is 0.75. Table 4 shows the revised values of main parameters and the 2013 steady-state values of the government policy variables in the aging-population economies. Similar to the stationary-population economy, all preference and technology parameter values are fixed all the time, but some of the policy parameter values are changed in the aging-population baseline economies in the years from 1961 to 2200 and before and after the government's policy changes.

Step 3 in the above procedure is counterfactual for two reasons. First, the households realize in 1961 that the population is aging and start adjusting their savings to prepare for the aging population. So, any macroeconomic implications for the first 20 years, 1961 to 1980, are probably not reliable. Second, the model economy does not consider the changes in fiscal policy between 1961 and 2013. Until the 1980s, for example, the marginal income tax rates in the high income brackets were much higher, and the Social Security pension system was much smaller. We also observed a steadily increasing trend of female labor participation through the 1990s. However, the aging-population baseline economies can show how the demographic changes would have affected the overall economy, other things being equal.

4.3 The Government's Financing Assumptions

The aging population is costly for the government for two reasons. First, as the population ages, the proportion of working-age households decreases. So, labor supply per capita will decline and the govern-

[55] The procedure to find an equilibrium-transition path includes solving the model for a stationary equilibrium in 2200. See Appendix A for a detailed explanation.

Table 4: Main Parameter Values and Baseline Policy Variables in the Aging-Population Economies

Parameter		Value	Comment
Demographics			
Long-run population growth rate	ν	0.0037	Population growth projected in 2200
Total population		50.9839	When $p_{21,t} = 1.0$ in 2013
Working-age population (ages 21-64)		41.3136	
Preferences			
Discount factor	β	1.0320	$K_t/Y_t = 2.4$ in the baseline
Growth-adjusted discount factor	$\tilde{\beta}$	1.0073	$\tilde{\beta} = \beta(1+\mu)^{\alpha(1-\gamma)}$
Share parameter of consumption	α	0.6796	$\bar{h}_t = 1.0$ in the baseline
Maximum working hours	$h_{\max}$	1.6358	Frisch elasticity $= 0.5$
Model units			
Taxable labor income ratio	η	0.7654	$T_{P,t}/Y_t = 0.065$ in the baseline
Scale adjustment[a]		58.4226	Average earnings = \$66,279 in 2011
Progressive income tax			
Income tax adjustment factor	φ_t	0.9825	Average marginal rate $= 21.8\%$
Labor income tax: tax rate limit	φ_0	0.3640	Estimated by ordinary least squares
: curvature	φ_1	0.5016	Estimated by ordinary least squares
: scale	φ_2	0.3093	Estimated by ordinary least squares
: deduction & exemptions	d	0.1328	Estimated by ordinary least squares
Capital income tax rate	τ_K	0.3040	Average marginal rate $= 20.7\%$
Lump-sum tax portion of income tax	τ_{LS}	0.0010	$T_{I,t}/Y_t = 0.112$
Social Security system			
Maximum taxable earnings[b]	$\vartheta_{\max}$	2.5593	1.4×\$106,800 = \$149,520 in 2011
OASI benefit adjustment factor	$\psi_{O,t}$	1.5809	$TR_{SS,t} = T_{P,t}$ in the baseline
Replacement rate threshold: 0.90 - 0.32[b]	ϑ_1	0.2154	1.4×\$749×12 = \$12,583 in 2011
: 0.32 - 0.15[b]	ϑ_2	1.2989	1.4×\$4,517×12 = \$75,886 in 2011
Social Security benefit: DI	$\psi_{D,t}$	0.0188	Budget balanced given the DI tax rate
: HI	$\psi_{H,t}$	0.1477	Budget balanced given the HI tax rate
Other policy variables			
Government consumption per household	$c_{G,t}$	0.1186	48% of non-Social Security spending
Lump-sum transfers per household	$tr_{LS,t}$	0.1285	52% of non-Social Security spending
Government's debt per household	$d_{G,t}$	1.5371	$D_{G,t}/Y_t = 0.75$ in the baseline
Accidental bequests per household	q_t	0.0591	Q divided by working-age population
Wealth held by foreigners	$W_{F,t}$	31.3465	$W_{F,t}/Y_t = 0.30$ in the baseline

[a] A unit in the model economy corresponds to \$58,423 in 2011 dollars. [b] 40% of all households (67% of married households) are assumed to be two-earner households, and 60% are one-earner households.
OASI = Old-Age and Survivors Insurance; DI = Disability Insurance; HI = Hospital Insurance.

ment tax revenue from labor income, such as individual income tax revenue and Social Security payroll tax revenue, will also decline over time. Second, the relative number and absolute number of elderly households will increase. A significant proportion of government transfer spending is age-dependent and is addressed to elderly households. Therefore, the government spending per capita will increase over time.

To finance the budgetary cost of the aging population in each year between 1961 and 2200, we compare results under the following two government budget rules:

- The government decreases its consumption spending, which is not in the household's utility function, to balance the budget each year (Option 1); or
- The government decreases its transfers to households and also raises the marginal income tax rates to balance the budget each year (Option 2).

Because government consumption is not in the utility function of the households, the change in government consumption will not directly affect the households' behavior and the aggregate economy. Therefore, the first budget assumption considers the effect of the aging population only in the absence of any budgetary feedbacks. The second economy, however, shows both the direct and indirect effects of the aging population including those from future fiscal policy changes.[56] [57]

4.4 The Aging Baseline Economies

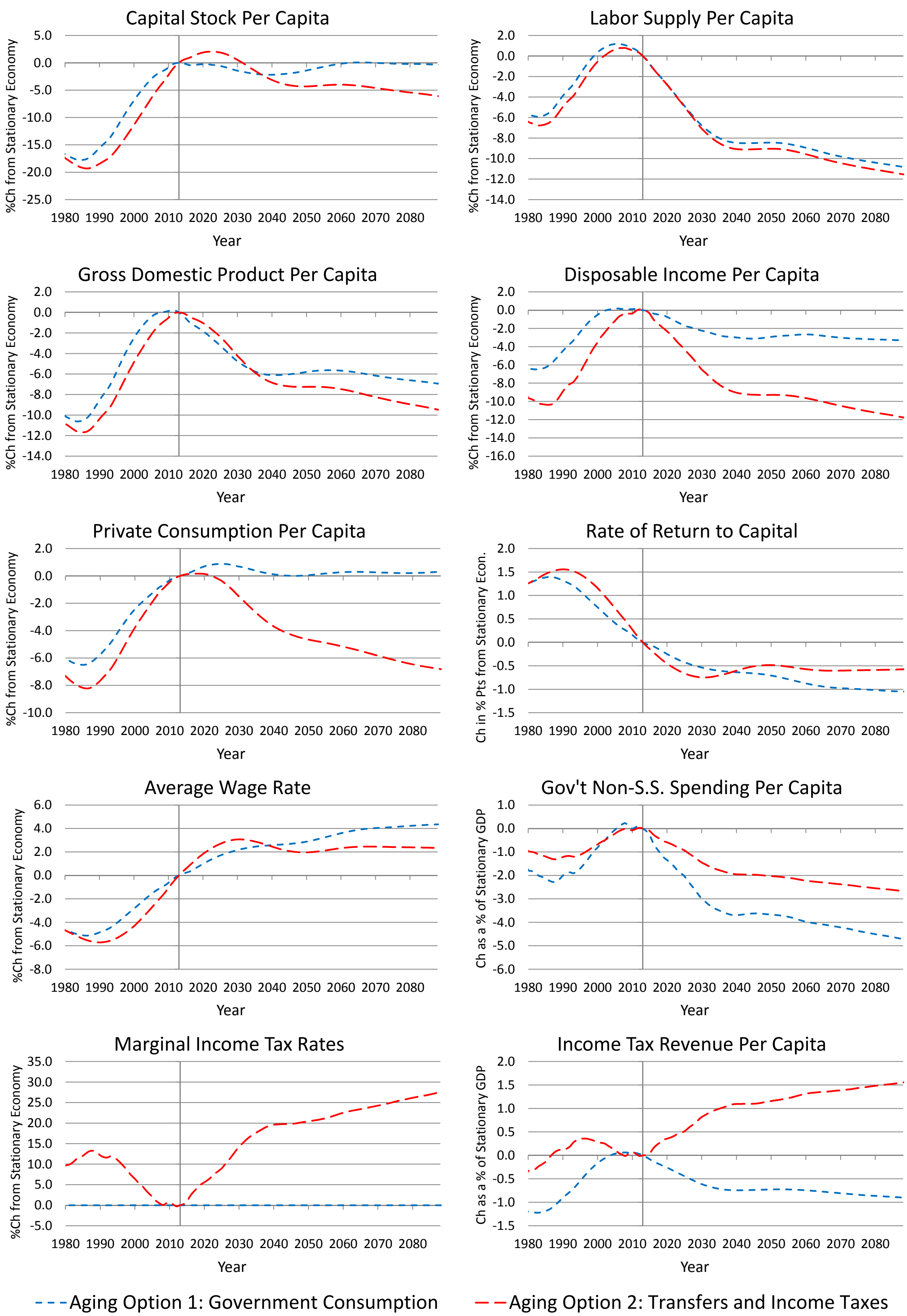
Figure 4 shows the two aging-population baseline economies for the years from 1980 to 2087. The horizontal line at 0.0 ($y = 0$) in each panel of the figure shows the level in the stationary-population economy, which is the balanced-growth path aligned to the economy in 2013 with constant productivity and population growth rates. The vertical line at 2013 ($x = 2013$) in each figure indicates year 2013. The graphs before 2013 show how the economy would have been affected by the aging population, other conditions being equal to those in 2013.[58]

[56] Many other ways of closing the government's budget are possible. If the government finances the cost of the aging population solely by cutting lump-sum transfers (equivalently, raising lump-sum taxes), the policy change will be more regressive. If the government finances the cost solely by increasing the income tax rates proportionally, the policy change will be more progressive. The government can further adjust the progressivity of the income tax schedule and the weights of labor income and capital income taxes. The government can also change the payroll tax rate, consumption and other tax rates, and Social Security benefits as well as increase its debt in the short run to satisfy the budget constraint. Any combination of those ways could lead to a different distribution of the net tax burden and different economic outcomes.

[57] For the government budget to be sustainable, the government does not necessarily have to reduce its budget deficit to zero. The government just has to keep the debt-to-GDP ratio stabilized. Thus, for a "balanced budget," we assume that the government's growth-adjusted debt per capita, $d_{G,t}$, is kept at the same level. See Section 5.

[58] It is not appropriate to directly compare the stationary-population baseline economy calibrated in Section 3 and two aging-population baseline economies calibrated in this section, because the two sets of economies use different parameter values (see Tables 1, 2, and 4). So, this section shows the effects of the aging population in the two baseline economies as the deviations of each macroeconomic variable from its 2013 growth-adjusted value in the corresponding baseline economies.

Figure 4: The Aging-Population Baseline Economies Relative to the Stationary Economy through 2013



According to the overlapping-generations model with an aging population, if fiscal policies and other economic factors had been unchanged, labor supply per capita would have increased since the mid-1980s and peaked around 2006–2008. The years from 1980 to 2006 coincide with the period when the baby boomers had a central role in the U.S. labor market. The baby boomers started retiring in 2008, and the proportion of working-age households and labor supply per capita will continue declining over time. This is the direct effect of the aging population on the U.S. economy. Labor supply per capita will be 8.5 percent lower in 2040 and 11.8 percent lower in the long run relative to the economy without the aging population.[59] If the government finances the budgetary cost of the aging population by transfer cuts and tax increases, labor supply per capita will be 9.1 percent lower in 2040 and 12.7 percent lower in the long run.[60]

It is, in general, difficult to predict the economy's capital stock, because it depends not only on households' saving decision but also on the government's debt in the future. We assume that the debt-to-GDP ratio will be stable at the 2013 level.[61] If the government cuts its consumption spending—in other words, if no additional budgetary feedbacks to the household's budget are considered—capital stock per capita will be 2.2 percent lower than the stationary-population baseline economy in 2040 but only 1.0 percent lower in the long run. The capital stock will not be significantly lower because the households will have to accumulate larger savings for their retirement when life expectancy is higher. However, if the government cuts its transfer spending and increases tax rates to finance the cost of aging, capital stock per capita will increase for a while, will peak in the early 2020s, and will fall gradually after that. Under the second financing assumption, capital stock per capita will be 3.1 percent lower in 2040 and 9.3 percent lower in the long run. Future tax increases on capital income will discourage households from saving, and transfer cuts will accelerate the dis-saving of retired households.

In the baseline economies with an aging population, other things being equal, GDP per capita has increased since the early 1980s compared with the stationary-population economy and has peaked in 2012–2013; it will likely decrease in later years. When no changes in productivity growth are assumed, the potential GDP growth rate has been higher than the productivity growth rate for the past 30 years and will

[59]Outcomes in the long run are those in the final stationary (steady-state) economies. In Section 4.4, the final stationary economies are equal to the economies in 2200, but these long-run outcomes are not shown in Figure 4.

[60]The effect of the future transfer cuts and tax increases on labor supply is not very large in the model economy. This is because these two policy changes will have two opposite effects on labor supply: The marginal tax rate cuts will discourage households from working, but transfer cuts will likely force households to stay in the labor market longer.

[61]Because we construct the 2013 aging-population economy by solving the model for years 1961 to 2200, it cannot exactly match the 2013 debt-to-GDP ratio to its target, which is 75 percent. Also, because GDP per capita will decline in the aging-population economies, the debt-to-GDP ratio (as a percentage of the current GDP) will increase gradually from 75 percent to 80 to 83 percent.

be lower than the productivity growth rate at least until 2040. GDP per capita will be 6.1 percent lower in 2040 and 7.8 percent lower in the long run with the aging population if the costs of aging are financed by cuts in government consumption, and it will be 6.8 percent lower in 2040 and 11.4 percent lower in the long run if the government cuts transfers and increases income tax rates.

In the first aging-population economy, the government will have to decrease its consumption spending as a share of stationary GDP by 3.7 percentage points (from 6.8 percent to 3.1 percent) in 2040 and by 5.2 percentage points (to 1.6 percent) in the long run.[62] It may be unrealistic for the government to reduce its consumption by 55 percent by 2040 and by 78 percent in the long run without hurting future households. In the second aging-population economy, the government will have to decrease its transfer spending as a share of stationary GDP by 1.9 percentage points (from 6.8 percent to 4.9 percent) in 2040 and by 3.0 percentage points (to 3.8 percent) of stationary GDP in the long run and also increase marginal income tax rates by 19.6 percent in 2040 and by 31.7 percent in the long run. The changes in marginal income tax rates will increase the government's income tax revenue by 1.8 percent of stationary GDP in 2040 and by 2.7 percent of stationary GDP in the long run. The fiscal policy change in the second aging-population economy is probably feasible. However, the sizable transfer cuts, together with the increase in marginal income tax rates, will significantly hurt future households in the economy, compared with the situation under the first aging-population economy.

5 Stylized Policy Experiments

This section examines the policies for which and the degree to which it is important to incorporate the aging population in the baseline economy when analyzing the effects of possible fiscal policy changes. As explained above, estimation incorporating the aging population is computationally costly. Because the aging-population economy is not stationary, a baseline economy for policy experiments has to be constructed, under certain government policy assumptions, as an equilibrium-transition path over many years, such as 1961 to 2200. The baseline economy also depends on the government's financing assumption, and thus the effects of fiscal policy changes are more difficult to interpret. This section introduces three stylized fiscal policy changes and shows to what extent considering the aging population affects the implication of

[62]In this paper, the aging population changes both fiscal policy variables and GDP. To show the effect of the aging population on the government budget clearer, the changes in fiscal policy variables are shown as a percentage of the stationary GDP in the economy without aging rather than the realized GDP in the aging-population economy. The GDP used as the denominator is also considered as the 2013 growth-adjusted GDP, since the stationary economy goes through the 2013 economy.

each policy change.

The stylized policy changes considered in this section are the following:

1. Tax increase followed by a tax cut: The government increases marginal income tax rates proportionally by 20 percent (as a percentage of 2013 tax rates) and reduces the government debt temporarily for the first 10 years, and the government reduces marginal income tax rates after 10 years so that the government debt per capita stays at the same level after the growth adjustment.

2. Cut in non-Social Security transfers and income taxes: The government permanently decreases its non-Social Security transfer spending to households uniformly by 20 percent (as a percentage of 2013 total non-Social Security spending per capita) and reduces the marginal income tax rates proportionally so that the government debt per capita stays at the same level after the growth adjustment.

3. Cut in Social Security benefits and payroll taxes: The government decreases the OASI benefits proportionally by 33 percent (as a percentage of the 2013 OASI benefit schedule) in a 40-year phased-in manner and also decreases the OASI payroll tax rate uniformly so that the government debt per capita stays at the same level after the growth adjustment.

All three policy experiments assume that the government announces and changes its policy at the beginning of 2013, which is model year 1, and that the debt-to-GDP ratio will stabilize as the GDP growth rate approaches its long-run growth rate.

5.1 A Surplus-Financing 10-Year Temporary Income Tax Increase

In the first policy experiment, the government increases marginal income tax rates proportionally by 20 percent and reduces the government debt temporarily for the first 10 years, and the government reduces marginal income tax rates after 10 years so that the government's growth-adjusted debt per capita stays at the same level thereafter. This policy experiment is conducted in all three baseline economies: one stationary-population baseline economy and two aging-population baseline economies. Because future marginal income tax rates are higher in the second aging-population baseline economy, the government increases the tax rates by 20 percent as a percentage of the 2013 baseline levels in all three economies.[63]

[63]Marginal income tax rates for 2013 in the three baseline economies—one stationary-population economy and two aging-population economies—are calibrated to be at about the same levels.

Let $t = 0$ be year 2013 before the fiscal policy change, and let $t = 1, 2, \ldots$ be years $2013, 2014, \ldots$ after the policy change. Then, the marginal income tax rate parameter is changed as

$$\varphi_t = \tilde{\varphi}_t + 0.2\varphi_0 \quad \text{for } t = 1, \ldots, 10,$$

where $\tilde{\varphi}_t$ is the marginal income tax rate parameter in the baseline economy, and the government intertemporal budget constraint is modified to

$$\begin{aligned} D_{G,t+1}(d_{G,t+1}) &= \frac{1}{(1+\mu)(1+\nu)} \big[\big(1 + (1-\xi)r_t\big) D_{G,t}(d_{G,t}) - T_{I,t}(\varphi_t) - T_{P,t}(\tau_{O,t}) - T_{C,t}(\tau_{C,t}) \\ &\quad + C_{G,t}(c_{G,t}) + TR_{LS,t}(tr_{LS,t}) + TR_{SS,t}(\psi_{O,t}) \big] \qquad \text{for } t = 1, \ldots, 10, \\ T_{I,t}(\varphi_t) &= \big(1 + (1-\xi)r_t\big) D_{G,t}(d_{G,t}) - (1+\mu)(1+\nu) D_{G,t+1}(d_{G,t}) - T_{P,t}(\tau_{O,t}) - T_{C,t}(\tau_{C,t}) \\ &\quad + C_{G,t}(c_{G,t}) + TR_{LS,t}(tr_{LS,t}) + TR_{SS,t}(\psi_{O,t}) \qquad \text{for } t = 11, \ldots, \infty. \end{aligned}$$

The government's growth-adjusted debt per capita, $d_{G,t}$, is endogenous for the first 10 years, and it is fixed at $d_{G,t+1} = d_{G,t} = d_{G,11}$ thereafter. The marginal income tax rate parameter, φ_t, is endogenous after 10 years.

Figure 5 shows the results of this policy experiment. The 20 percent 10-year temporary income tax increase is "financed" by income tax cuts in the future. Thus, this income tax increase is partially compensated. Relative to the baseline economies, labor supply will decrease by 1.3–1.7 percent for the first 10 years, and it will increase by 0.2–0.6 percent after 10 years. Because of the increase in the income tax and the decrease in labor income, disposable income will decrease for the first 10 years, and private consumption will decrease in the same period, even though the saving rate will also decline. Private wealth will decrease for the first 10 years, but government debt will decrease in the same period. The economy's capital stock will decrease by 0.4 percent by year 7, it will increase gradually for the next three years, then it will increase rapidly after 10 years to its long-run level, which is 5.0 to 5.8 percent higher than the baseline capital stock.[64] Gross domestic product will decrease by 0.9–1.1 percent for the first 10 years, but it will increase relative to the baseline economies by 2.2–2.6 percent in the long run.[65]

[64] Although the change in labor supply falls by decreasing amounts during the first 10 years, the changes in disposable income and private consumption both fall by increasing amounts. This is because the households' wealth and capital income are declining in this period.

[65] Outcomes in the long run are those in the final stationary (steady-state) economies. In Section 5, the final stationary economies after a policy change are equal to the economies in model year 120 when the population is stationary or 188 (real year 2200) when

Figure 5: A Surplus-Financing 10-Year Temporary Income Tax Increase

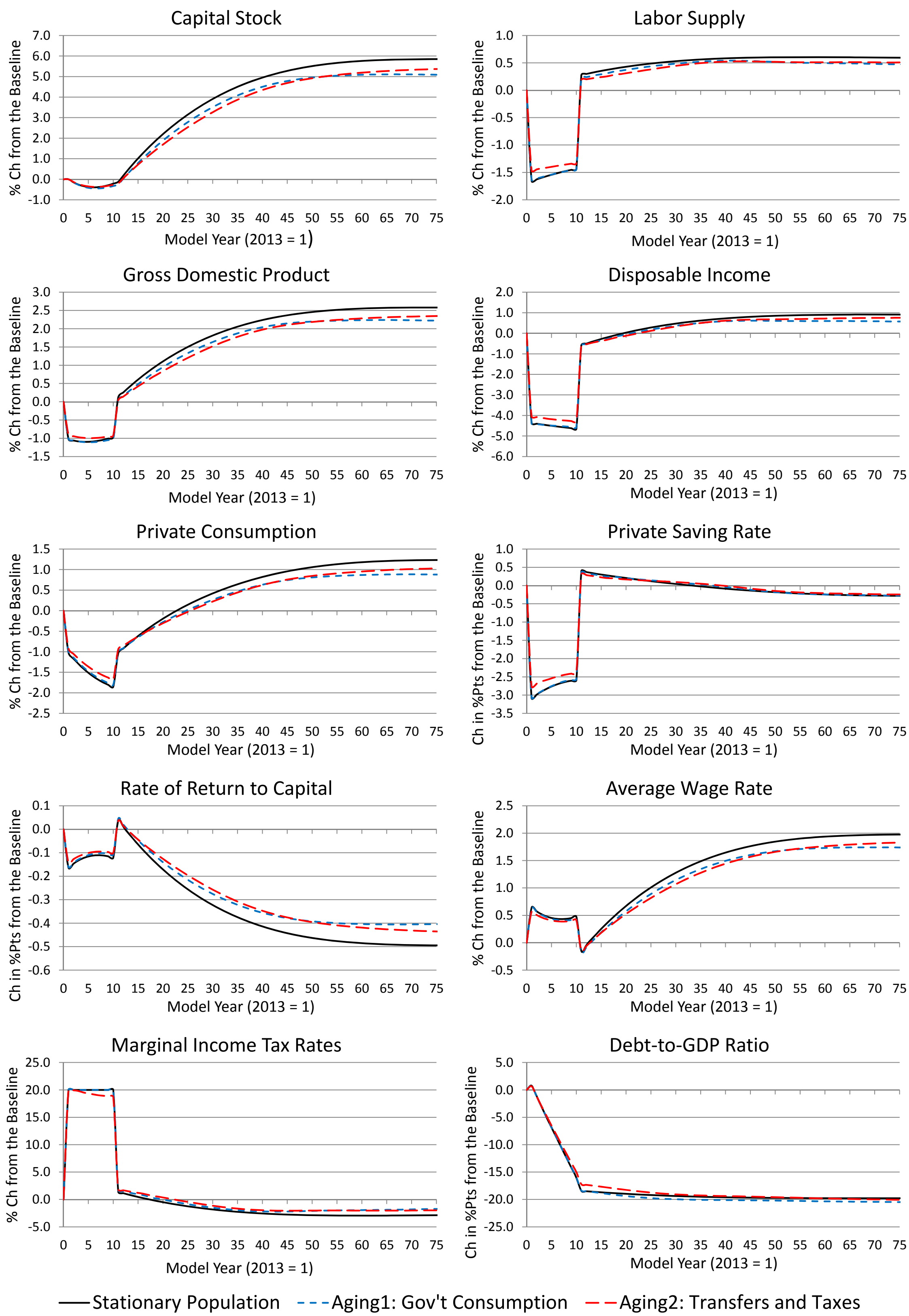


Figure 6: A Surplus-Financing 10-Year Temporary Income Tax Increase: Average Welfare Change

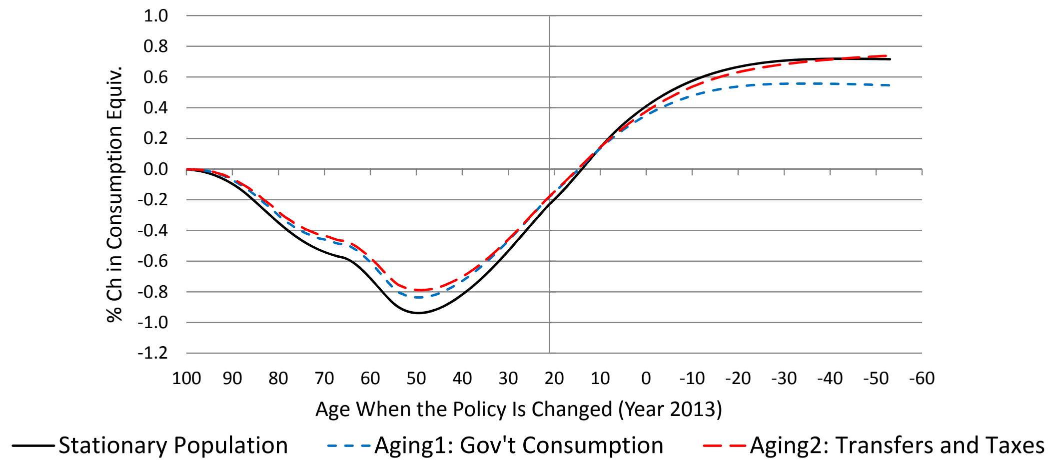


Figure 6 shows the average welfare change by age cohort. On average, current households—those ages 21 to 100 in year 1 (year 2013)—will be worse off under this policy change. However, most future households will be, on average, better off because they will be in an economy with lower government debt and lower income tax rates.

The effects of the policy change look very similar in the three baseline economies: one stationary-population economy (black solid line) and two aging-population economies (blue dashed line and red long-dashed line), especially for the first 20 years. In a short-run and medium-run tax policy experiment, thus, consideration of the aging population is not very important. In the second aging-population economy, in which the government decreases transfers and increases income tax rates to finance the cost of the aging population, the welfare improvement of the future households is noticeably larger than that in the first aging-population economy. This is because the average marginal tax rates in this baseline economy are higher than in the other economies, and thus efficiency gains from the future tax cuts will be larger.

5.2 A Permanent Lump-Sum Transfer Cut With an Income Tax Cut

In the second policy experiment, the government permanently decreases its transfers uniformly by 20 percent as a percentage of non-Social Security government spending, and it reduces marginal income tax rates proportionally so that the growth-adjusted government debt per capita stays at the same level.[66] Again,

the population is aging. These long-run outcomes are not shown in Figures 5, 7, and 9.

[66]Non-Social Security transfers are transfers other than OASI, DI, and HI (Medicare Part A) benefits, and these are assumed to be lump-sum and uniform for all households in the model economy. Non-Social Security government spending includes both lump-sum transfers and government consumption.

this policy experiment is conducted in all three baseline economies. Because non-Social Security government spending changes over time in the two aging-population baseline economies, the government is assumed to decrease its transfers by 20 percent as a percentage of the 2013 levels (with growth adjustment) to make the experiment similar in all three economies.

In the model economies, the government lump-sum transfer per capita is changed as

$$tr_{LS,t} = \tilde{tr}_{LS,t} - 0.2(c_{G,0} + tr_{LS,0}) \quad \text{for } t = 1, \ldots, \infty,$$

where $\tilde{tr}_{LS,t}$ is the lump-sum transfer per capita in the baseline economy, and the government intertemporal budget constraint is modified to

$$\begin{aligned} T_{I,t}(\varphi_t) = &\big(1 + (1-\xi)r_t\big)D_{G,t}(d_{G,t}) - (1+\mu)(1+\nu)D_{G,t+1}(d_{G,t}) - T_{P,t}(\tau_{O,t}) - T_{C,t}(\tau_{C,t}) \\ &+ C_{G,t}(c_{G,t}) + TR_{LS,t}(tr_{LS,t}) + TR_{SS,t}(\psi_{O,t}) \qquad \text{for } t = 1, \ldots, \infty. \end{aligned}$$

The government's debt per capita, $d_{G,t}$, is fixed at the 2013 baseline level—that is, $d_{G,t+1} = d_{G,t} = d_{G,0}$—and the marginal income tax rate parameter, φ_t, is endogenous throughout the transition path.

Figure 7 shows the result of this policy experiment. The 20 percent transfer cuts are financed by income tax cuts each year. In the model economies, the government's non-Social Security transfers are lump-sum and uniform to all households, and income taxes are progressive and are mainly paid by the working-age households. Therefore, this policy change has two income redistribution effects: intragenerational redistribution from poor households to rich households, and intergenerational redistribution from retired households to working-age households.

Relative to the corresponding baseline economies, labor supply will increase by 2.7–2.9 percent in the short run and by 3.5–4.1 percent in the long run. The government cuts transfers not only to working-age households but also to retired households, but the retired households do no benefit from the income tax cuts very much. Therefore, households will have to accumulate larger wealth for their retirement. The private saving rate will go up by 3.2–3.4 percentage points in the short run and by 0.8 percentage points in the long run.[67] Capital stock will increase by 15.7–19.6 percent in the long run. A large part of this increase is

[67] At the time of the policy change, elderly households need a larger adjustment in their saving rate than young households, because the former households have shorter or no time to adjust their wealth to the optimal level after the policy change. The increase in the economy-wide private saving rate will gradually be smaller as the initial elderly households are replaced by newborn households.

Figure 7: A Permanent Lump-Sum Transfer Cut With an Income Tax Cut

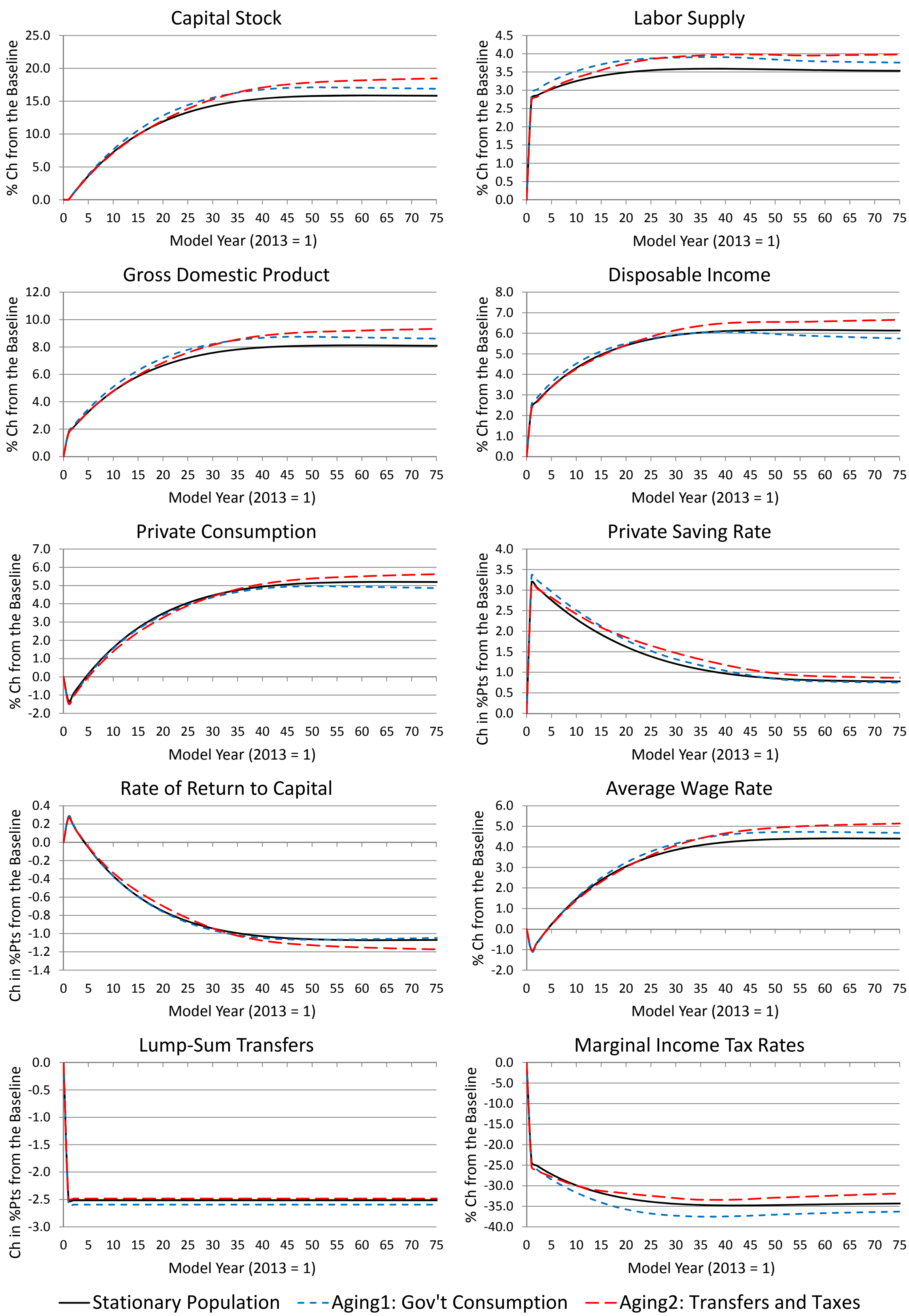
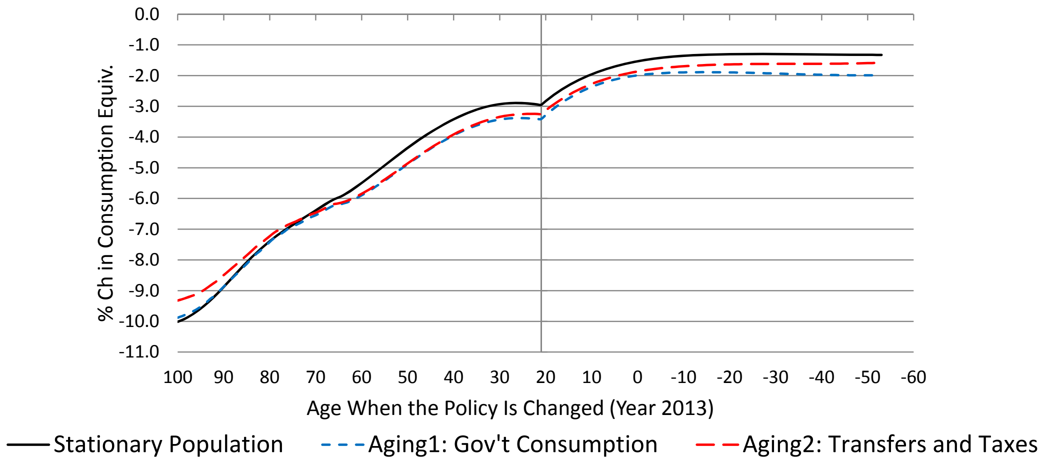


Figure 8: A Permanent Lump-Sum Transfer Cut With an Income Tax Cut: Average Welfare Change



caused by the life-cycle saving motive—the households need larger retirement wealth when the government reduces its transfers to all households. However, the increase is also caused by the precautionary saving motive—the households need a little more wealth because the risk-sharing effect of the progressive income tax becomes smaller after the policy change.

Gross domestic product will increase by 1.7–1.8 percent in the short run and by 8.0–9.8 percent in the long run. Although disposable income will also increase by 2.3–2.5 percent in the short run and by 5.7–7.1 percent in the long run, the large increase in rates of saving leads to a decline in private consumption for the first three years, relative to the baseline economies.

Figure 8 shows the average welfare change by age cohort. In this policy experiment, both the current households and the households entering the economy in the future will be, on average, worse off by the policy change. Cutting the distortionary marginal income tax rates, in general, improves the economic efficiency and increases GDP, but reducing lump-sum transfers at the same time decreases income redistribution and increases after-tax income inequality. In the three baseline economies, the latter effect on social welfare turns out to be stronger than the former effect. This welfare implication depends on the size of idiosyncratic shocks assumed in the model economies. For example, if idiosyncratic wage shocks are 50 percent smaller than those assumed in the baseline economies (in terms of standard deviation), the current young households and the future households will be, on average, better off under this policy of a cut in lump-sum transfers coupled with a cut in the income tax.

The effects of the policy change look very similar for the first 20 years in the three baseline economies with and without the aging population, although the macroeconomic effects differ noticeably in the long run. The difference in macroeconomic effects is larger in this policy experiment because the policy change includes a larger intergenerational redistribution, from elderly households to young households, than the first policy experiment includes. However, the welfare implications by age cohort look similar in all three baseline economies.

5.3 A 40-Year Phased-In Reduction of OASI Benefits With a Payroll Tax Cut

In the third policy experiment, the government decreases the OASI benefits proportionally by 33 percent as a percentage of the baseline schedule in a cohort-by-cohort phased-in manner, and it also decreases the OASI payroll tax rate uniformly so that the growth-adjusted government debt per capita stays at the same level. In the aging-population baseline economies, a sizable portion of future Social Security benefits has been financed by transfers from the government's general account—non-Social Security spending cuts or income tax increases or both. In this policy experiment, therefore, the long-run percentage change in the OASI payroll tax will be much larger than 33 percent.

In the model economies, for households ages 61 or older in year 1, the OASI benefit adjustment factor is unchanged':

$$\psi^i_{O,t} = \psi_{O,0} \quad \text{for} \quad i-(t-1) \geq 61, \quad t \geq 1.$$

For households ages 21 or younger in year 1, the benefit adjustment factor is reduced by 33 percent:

$$\psi^i_{O,t} = \psi_{O,T} = 0.67\,\psi_{O,0} \quad \text{for} \quad i-(t-1) \leq 21, \quad t \geq 1.$$

Finally, for households ages 22 to 60, the benefit adjustment factor is the weighted average of the above two parameter values:

$$\psi^i_{O,t} = \frac{(i-21)-t}{40}\,\psi_{O,0} + \left(1 - \frac{(i-21)-t}{40}\right)\psi_{O,T} \quad \text{for} \quad 22 \leq i-(t-1) \leq 60, \quad t \geq 1.$$

The government intertemporal budget constraint is modified to

$$T_{P,t}(\tau_{O,t}) = \big(1+(1-\xi)r_t\big)D_{G,t}(d_{G,t}) - (1+\mu)(1+\nu)D_{G,t+1}(d_{G,t}) - T_{I,t}(\varphi_t) - T_{C,t}(\tau_{C,t})$$

$$+ C_{G,t}(c_{G,t}) + TR_{LS,t}(tr_{LS,t}) + TR_{SS,t}(\psi_{O,t}) \qquad \text{for } t = 1, \ldots, \infty.$$

Government debt per capita, $d_{G,t}$, is fixed at the 2013 baseline level—that is, $d_{G,t+1} = d_{G,t} = d_{G,0}$—and the OASI payroll tax rate, $\tau_{O,t}$, is endogenous throughout the transition path.

Figure 9 shows the results of this policy experiment. The Social Security system in the model economy is pay-as-you-go—that is, the government collects revenue from the payroll tax on the labor income of working-age households and transfers it to elderly households as Social Security OASI and HI benefits. Compared with the corresponding baseline economies, the labor supply will increase gradually and by 1.8–2.8 percent in the long run. This increase in the labor supply is caused mostly by the gradual reduction in the effective payroll tax rate. When future Social Security benefits are reduced, households will have to accumulate larger wealth for their retirements. Therefore, the private saving rate will go up by 0.4 percentage points in year 1 and by 1.4–2.0 percentage points in year 50. The change in the saving rate is gradual because the policy change is phased in. The capital stock will increase by 14.7–22.6 percent in the long run, and gross domestic product will increase by 6.5–10.0 percent in the long run.

Private consumption will be smaller than that in the baseline economies for the first 22–24 years, because the current young households and households entering the economy in the near future will have to save more to prepare for their retirements, but their payroll tax payments will not decrease as much as their future Social Security benefits decrease. In order to continue paying Social Security benefits to the current elderly households, the government will have to keep the payroll tax rate relatively high for many years. For example, in the stationary-population economy, the OASI payroll tax rate will be only 12.2 percent lower in year 20 than the rate in the baseline level, although households that reach age 40 in that year will receive about 33 percent less in OASI benefits when those households reach age 66.

OASI benefits will decrease by 29.9–30.9 percent in the long run. The long-run decreasing rates are smaller than 33 percent because labor income per capita will increase by 6.6–10.0 percent due to the increases in the labor supply and the wage rate. In the long run, the government can reduce the OASI payroll tax rate by 59.6 percent in the stationary-population economy. The long-run decreasing rate is almost twice as large as the decreasing rate of OASI benefits. This is because other tax revenues will increase as GDP goes up, and the government's debt-service cost will decrease as the interest rate falls. The payroll tax rate will decrease by 86.9–95.3 percent in economies with the aging population. The decreasing rates are even larger because, in the aging-population baseline economies, the Social Security budget is partially financed

Figure 9: A 40-Year Phased-In Reduction of OASI Benefits With a Cut in Payroll Taxes

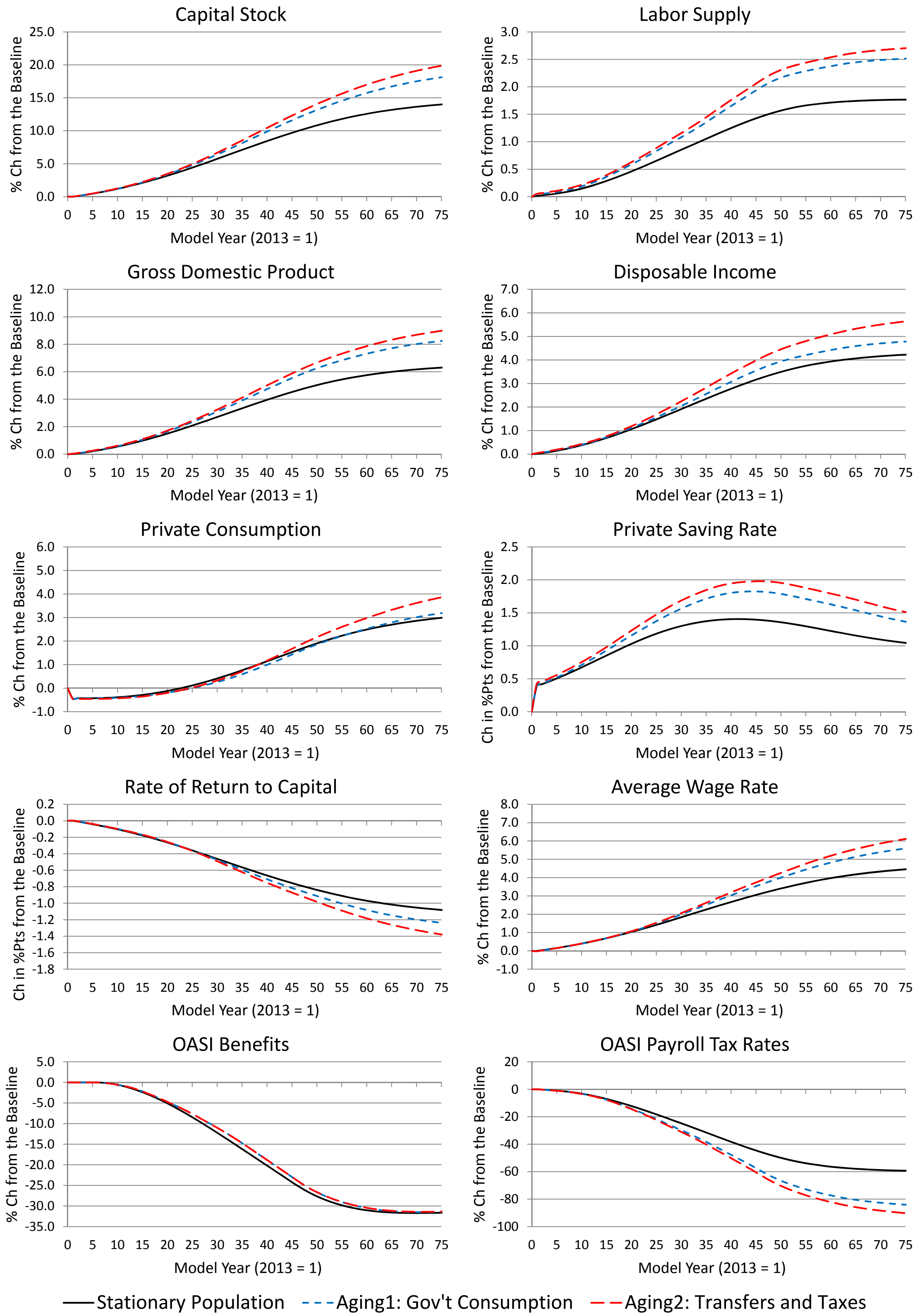
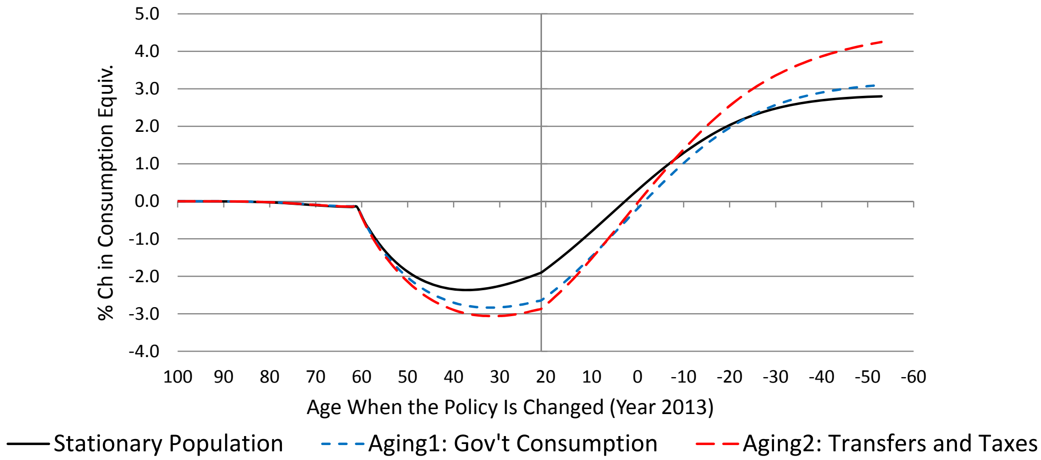


Figure 10: A 40-Year Phased-In Reduction of OASI Benefits With a Cut in Payroll Taxes: Average Welfare Change



by the rest of the government budget, and the OASI payroll tax revenue is only 57.0–57.5 percent of the OASI benefit expenditure in the long run.

Figure 10 shows the average welfare change by age cohort. In an economy with a pay-as-you-go Social Security system, the government has an unfunded liability to the elderly households. When Social Security benefits are cut by 33 percent, the unfunded liability will also be reduced by 33 percent, other things being equal. This benefit for the government is equal to the cost shared by the current and future households. It is often referred to as the transition cost of Social Security privatization. In this policy experiment, the welfare levels of current households ages 61 and older will be roughly unchanged; current households and households in the near future will be, on average, worse off in the long run; but households later in the future will be, on average, better off in the long run.

Again, the effects of the policy change look similar for the first 20 years in the three baseline economies, although the difference in the private saving rate is larger in this model. However, the difference in effects between the stationary-population economy and the two aging-population economies gets larger in this model after 20 years. For example, the percentage changes in the long-run capital stock will differ by almost 8 percentage points. This is mainly because the initial size of the Social Security system is significantly different between the stationary-population economy and the aging-population economies. Therefore, the impact of 33 percent cuts in the future benefits produces different effects quantitatively, although the qualitative policy implications are similar.

6 Conclusion

We extend a heterogeneous-agent overlapping-generations model with idiosyncratic uncertainties about future wages and life spans by incorporating the projected aging of the population in the United States. The paper has two goals: first, to understand the effect of the aging population on the government budget, the aggregate economy, and social welfare; and second, to learn when and to what extent considering the aging population in the model economy is important in evaluating possible changes in fiscal policy.

Three findings emerge regarding the first goal of this paper: First, the effect of the aging population on the U.S. economy is significant even without considering additional effects through the government budget. Compared with the results for the model without the aging population, GDP per capita is smaller by 6.1 percent in 2040 and by 7.8 percent in the long term. Second, the direct budgetary cost of the aging population is significant. The government tax revenue is smaller by 1.2 percent of GDP in 2040 and by 1.5 percent in the long term if tax rates remain unchanged. In addition, government transfer expenditures are larger by 2.3 percent of GDP in 2040 and by 3.5 percent in the long term. Third, the overall (general-equilibrium) effect of the aging population is even larger, especially in the long term. If the government finances the budgetary cost of the aging population by cutting non-Social Security transfers and increasing marginal income tax rates, roughly on an equal basis, GDP per capita would be smaller by 6.8 percent in 2040 and by 11.4 percent in the long term.

Regarding the second goal, it is less important to consider the aging population when analyzing the effects of changes in tax policy for the next 10 or 20 years. The aging population becomes more important if fiscal policy involves some intergenerational transfers—for example, between working-age households and elderly households. Not surprisingly, using the model with the aging population is most important when analyzing the effect of age-dependent government transfer programs, such as Social Security, over a time horizon of more than 20 years.

A Appendix: Computational Algorithm

We solve the household's optimization problem recursively from age $i = I$ to age $i = 21$ by discretizing the asset space, $A = [0, a_{\max}]$, into J nodes, $\hat{A} = \{a_1, a_2, \ldots, a_J\}$, the average historical earning space, $B = [0, b_{\max}]$, into K nodes, $\hat{B} = \{b_1, b_2, \ldots, b_K\}$, and the working ability space, $E = [0, e_{\max}]$, into L

nodes for each age, $\hat{E}^i = \{e_1^i, e_2^i, \ldots, e_L^i\}$.[68] When the working ability of very old households, $i > 75$, is assumed to be $e_1^i = 0$, the total number of nodes for which we solve the household's optimization problem in each period (year) t is $55JKL + (I - 75)JK$.[69]

Let $\mathbf{\Omega}_t$ be a time series of vectors of factor prices and government policy variables that describes a transition path of the aggregate economy,

$$\mathbf{\Omega}_t = \left\{r_s, w_s, c_{G,s}, tr_{LS,s}, \varphi_s, \tau_{O,s}, \psi_s, \tau_{C,s}, d_{G,s}\right\}_{s=t}^{\infty}.$$

The household's value function is shown as $v(\mathbf{s}, \mathbf{S}_t; \mathbf{\Psi}_t, \mathbf{\Phi}_t)$, and the factor prices and endogenous government policy variables are shown as $r_s(\mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s)$, $w_s(\mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s)$, $\varphi_s(\mathbf{S}_s; \mathbf{\Psi}_s, \mathbf{\Phi}_s)$, and so on, for $s \geq t$. It is impossible to solve the model of this form, because the dimension of $\mathbf{S}_t = (x(\mathbf{s}), d_{G,t})$ is infinite. However, households in the model economy use the information of $(\mathbf{S}_t, \mathbf{\Psi}_t, \mathbf{\Phi}_t)$ only for predicting $\mathbf{\Omega}_t$. In the absence of aggregate productivity shocks, policy uncertainty, or population shocks, we can avoid the curse of dimensionality by replacing $(\mathbf{S}_t, \mathbf{\Psi}_t, \mathbf{\Phi}_t)$ with $\mathbf{\Omega}_t$. Because the time series $\mathbf{\Omega}_t$ is deterministic and perfectly foreseeable, it will suffice to find the fixed point of $\mathbf{\Omega}_t$ to solve the model economy for an equilibrium-transition path.[70]

This section first explains the algorithm to solve the household's optimization problem for each individual state node,

$$\mathbf{s} = (i, a, b, e) \in \hat{A} \times \hat{B} \times \hat{E}_i \times \{21, 22, \ldots, I\},$$

taking $\mathbf{\Omega}_t$ as given. Then, the section explains how to solve the model for a stationary (steady-state) equilibrium (balanced-growth path) and an equilibrium-transition path.

[68]This appendix comes, with some notational changes, from Section 4 in Nishiyama and Smetters (2013).

[69]We set $I = 100$, $J = 70$, $K = 16$, and $L = 7$, and thus the total number of nodes per period is 459,200. To find the model economy's equilibrium-transition path of 240 periods, which usually takes 8 to 10 iterations, we need to solve the household's two-period optimization problem 110,208,000 times for each iteration.

[70]If there are any aggregate shocks in the model economy, $\mathbf{\Omega}_t$ is a stochastic process of vectors of factor prices and government policy variables, and thus the state space of the model economy is infinitely dimensional. It is infeasible to solve the household's problem for all sample paths.

A.1 Solving the Household's Problem

We solve the household's optimization problem backward, from $i = I$ to $i = 21$, by assuming the terminal value to be zero,

$$v(\mathbf{s}_{I+1}; \mathbf{\Omega}_{t+1}) = 0 \quad \Longrightarrow \quad v_a(\mathbf{s}_{I+1}; \mathbf{\Omega}_{t+1}) = v_b(\mathbf{s}_{I+1}; \mathbf{\Omega}_{t+1}) = 0,$$

where $\mathbf{s}_i$ is the individual state vector of a household of age i. The following computational algorithm is a modified version of that in Nishiyama (2010, 2011).[71]

The Household's Problem. Let $l = h_{\max} - h$ be leisure hours.[72] The optimization problem of a household at age i in period t, equations (1) to (5), is modified to

$$v(\mathbf{s}; \mathbf{\Omega}_t) = \max_{c,l} \Big\{ u(c, l) + \tilde{\beta}\phi_{i,t} E\big[\, v(\mathbf{s}'; \mathbf{\Omega}_{t+1}) \,\big|\, \mathbf{s} \,\big] \Big\}$$

subject to the constraints for the decision variables,

$$\begin{aligned}
0 < c \le \frac{1}{1+\tau_{C,t}} \big[\, & (1+\tilde{r}_t)a + w_t eh + tr_{SS,t}(i,b) + tr_{LS,t} + q_t(i) - \tau_{I,t}(w_t eh, \tilde{r}_t a) - \tau_{P,t}(w_t eh) \\
& - (1+\mu) a_{\min}(i+1) \,\big] = c_{\max}, \\
0 < l = h_{\max} - h \le h_{\max},
\end{aligned}$$

and the law of motion of the state variables,

$$\begin{aligned}
\mathbf{s}' &= (i+1, a', b', e'), \\
a' &= \frac{1}{1+\mu} \big[\, (1+\tau_{C,t})(c_{\max} - c) \,\big] + a_{\min}(i+1), \\
b' &= \mathbf{1}_{\{i<I_R\}} \frac{1}{i-20} \Big[\, (i-21)\, b\, \frac{w_t}{w_{t-1}} + \min(\eta w_t eh, \vartheta_{\max}) \Big] + \mathbf{1}_{\{i \ge I_R\}} b.
\end{aligned}$$

[71] Auerbach and Kotlikoff (1987) solve the household's optimization problem cohort by cohort. In the presence of idiosyncratic wage shocks, however, it is easier to solve the problem period by period, because the algorithm to solve the model for an equilibrium-transition path of many periods will be similar to the algorithm to solve the model for a stationary (steady-state) equilibrium.

[72] As we will see below, the first-order conditions for the household's problem and the corresponding complementarity problem look simpler if we use leisure hours, l, instead of working hours, h.

The Complementarity Problem. Let the objective function be

$$f(c,l;\mathbf{s},\mathbf{\Omega}_t) = u(c,l) + \tilde{\beta}\phi_{i,t} E\big[\, v(\mathbf{s}';\mathbf{\Omega}_{t+1}) \,\big|\, \mathbf{s} \,\big].$$

Then, the first-order conditions for an interior solution are

$$\text{(24)} \quad f_c(c,l;\mathbf{s},\mathbf{\Omega}_t) = u_c(c,l) - \frac{\tilde{\beta}\phi_{i,t}}{1+\mu}\, E\big[\, v_a(\mathbf{s}';\mathbf{\Omega}_{t+1}) \,\big|\, \mathbf{s} \,\big] = 0,$$

$$\begin{aligned}\text{(25)} \quad f_l(c,l;\mathbf{s},\mathbf{\Omega}_t) = {} & u_l(c,l) - w_t e \left[\, 1 - \tau_{I,1,t}(w_t e h, \tilde{r}_t a) - \tau'_{P,t}(w_t e h) \,\right] u_c(c,l) \\ & - \mathbf{1}_{\{i<I_R,\, w_t e h < \vartheta_{\max}\}} \frac{w_t e}{i-20}\, \tilde{\beta}\phi_{i,t} E\big[\, v_b(\mathbf{s}';\mathbf{\Omega}_{t+1}) \,\big|\, \mathbf{s} \,\big] = 0,\end{aligned}$$

where $\tau_{I,1,t}(w_t e h, \tilde{r}_t a)$ is the marginal income tax rate on labor income, and $\tau'_{P,t}(w_t e h)$ is the marginal payroll tax rate. Equation (24) is the Euler equation, and equation (25) is the condition for the marginal rate of substitution of consumption for leisure.

With the inequality constraints for the decision variables, the Kuhn–Tucker conditions of the household's problem are expressed as the following nonlinear complementarity problem,

$$\begin{aligned} f_c(c,l;\mathbf{s},\mathbf{\Omega}_t) &= 0 \quad \text{if} \quad 0 < c < c_{\max}, && > 0 \quad \text{if} \quad c = c_{\max}, \\ f_l(c,l;\mathbf{s},\mathbf{\Omega}_t) &= 0 \quad \text{if} \quad 0 < l < l_{\max}, && > 0 \quad \text{if} \quad l = l_{\max}. \end{aligned}$$

As described in Billups (2000, 2002) and Miranda and Fackler (2002), the above complementarity problem is expressed more compactly as the nonlinear system of equations,

$$\text{(26)} \quad CP(c,l) = \min\left\{ \max\left[\begin{pmatrix} f_c(c,l;\mathbf{s},\mathbf{\Omega}_t)/u_c(c,l) \\ f_l(c,l;\mathbf{s},\mathbf{\Omega}_t)/u_l(c,l) \end{pmatrix}, \begin{pmatrix} \varepsilon - c \\ \varepsilon - l \end{pmatrix} \right], \begin{pmatrix} c_{\max} - c \\ l_{\max} - l \end{pmatrix} \right\} = \mathbf{0},$$

where ε is a small positive number—for example, $\varepsilon = 10^{-3}$. In a heterogeneous-agent economy, the marginal utilities and marginal values in equations (24) and (25) can be very large or very small, depending on the state of the household. To minimize the numerical rounding error, we often need to normalize $f_c(c,l;\mathbf{s},\mathbf{\Omega}_t)$ and $f_l(c,l;\mathbf{s},\mathbf{\Omega}_t)$ by dividing these by $u_c(c,l)$ and $u_l(c,l)$, respectively.

We use a Newton-type iteration method to solve the above system of nonlinear equations. Because Newton-type equation solvers use a Jacobian matrix to update the solution, it is not desirable for the equations to have kinks generated by the maximum and minimum operators. Following Billups and following Miranda and Fackler, therefore, we also replace the $\min(a, b)$ and $\max(a, b)$ operators with the Fischer–Burmeister function and its variation,

$$\phi^-(a, b) \equiv a + b - \sqrt{a^2 + b^2}, \qquad \phi^+(a, b) \equiv a + b + \sqrt{a^2 + b^2},$$

respectively, to make the above system of equations differentiable without altering the solutions.

We solve equation (26) for $c(\mathbf{s}; \mathbf{\Omega}_t)$ and $l(\mathbf{s}; \mathbf{\Omega}_t)$ by using a Newton-type nonlinear equation solver (the subroutine NEQNF of the IMSL Fortran Numerical Library). The library function uses a modified Powell hybrid algorithm and a finite-difference approximation to the Jacobian. We evaluate the marginal values, $v_a(\mathbf{s}'; \mathbf{\Omega}_{t+1})$ and $v_b(\mathbf{s}'; \mathbf{\Omega}_{t+1})$, between nodes in equations (24) and (25) by using either bilinear interpolation or two-dimensional quadratic interpolation (the subroutine QD2VL) of corresponding marginal value functions, equations (28) and (29), explained below.

Value and Marginal Value Functions. After the optimal decision for each state is obtained, the value of the household with state $\mathbf{s}$ in period t is obtained as

$$\text{(27)} \quad v(\mathbf{s}; \mathbf{\Omega}_t) = u\left(c(\mathbf{s}; \mathbf{\Omega}_t), l(\mathbf{s}; \mathbf{\Omega}_t)\right) + \tilde{\beta}\phi_{i,t} E\left[\, v(\mathbf{s}'; \mathbf{\Omega}_{t+1}) \,|\, \mathbf{s} \,\right],$$

and the corresponding marginal values are obtained as

$$\text{(28)} \quad v_a(\mathbf{s}; \mathbf{\Omega}_t) = \left\{\, 1 + r_t\left[\, 1 - \tau_{I,2,t}\big(w_t e h(\mathbf{s}; \mathbf{\Omega}_t), \tilde{r}_t a\big) \,\right] \right\} u_c\big(c(\mathbf{s}; \mathbf{\Omega}_t), l(\mathbf{s}; \mathbf{\Omega}_t)\big),$$

$$\text{(29)} \quad v_b(\mathbf{s}; \mathbf{\Omega}_t) = tr'_{SS,t}(i, b)\, u_c\big(c(\mathbf{s}; \mathbf{\Omega}_t), l(\mathbf{s}; \mathbf{\Omega}_t)\big) + \left(\mathbf{1}_{\{i<I_R\}} \frac{i - 21}{i - 20} \frac{w_t}{w_{t-1}} + \mathbf{1}_{\{i \geq I_R\}} \right) \tilde{\beta}\phi_{i,t} E\left[\, v_b(\mathbf{s}'; \mathbf{\Omega}_{t+1}) \,|\, \mathbf{s} \,\right],$$

where $\tau_{I,2,t}\left(w_t e h, \tilde{r}_t a\right)$ is the marginal income tax rate on capital, and $tr'_{SS,t}(i, b)$ is the marginal Old-Age and Survivors Insurance (OASI) benefits with respect to b,

$$tr'_{SS,t}(i, b) = \mathbf{1}_{\{i \geq I_{\tilde{R}}\}} \psi_{O,t} (1 + \mu)^{60-i} \left\{ \mathbf{1}_{\{b<\vartheta_1\}} 0.90 + \mathbf{1}_{\{\vartheta_1 \leq b < \vartheta_2\}} 0.32 + \mathbf{1}_{\{\vartheta_2 \leq b < \vartheta_{\max}\}} 0.15 \right\}.$$

A.2 Finding the Distribution of Households

Let $x_t(\mathbf{s}) = x_t(i, a, b, e)$ be the discrete population distribution function of households in period t. Because households at age 21 are assumed to have no wealth and working history,

$$\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} x_t(21, 0, 0, e_l) = \sum_{l=1}^{L} x_t(21, 0, 0, e_l) = p_{21,t}.$$

Then the law of motion of growth-adjusted population distribution is, for $i = 21, \ldots, I-1$,

$$x_{t+1}(i+1, a', b', e_{l'}) = \frac{\phi_{i,t}}{1+\nu}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L} \mathbf{1}_{\{a'=a'(\mathbf{s};\mathbf{\Omega}_t),\, b'=b'(\mathbf{s};\mathbf{\Omega}_t)\}}\pi_i(e_{l'} \mid e_l) x_t(i, a_j, b_k, e_l).$$

Because $a'(\mathbf{s};\mathbf{\Omega}_t)$ and $b'(\mathbf{s};\mathbf{\Omega}_t)$ are in general not on a node in $\hat{A} \times \hat{B}$, the population in the next period is distributed linearly into 4 adjacent nodes, (a_j, b_k), (a_j, b_{k+1}), (a_{j+1}, b_k), and (a_{j+1}, b_{k+1}), such that $a_j \le a' < a_{j+1}$ and $b_k \le b' < b_{k+1}$.

The algorithm to calculate $x_{t+1}(\mathbf{s}')$ from $x_t(\mathbf{s})$ is as follows: First, set $x_{t+1}(\mathbf{s}') = 0$ for all $\mathbf{s}'$ and set $x_{t+1}(21, 0, 0, e_l) = \pi_{21}(e_l)$ for $l = 1, \ldots, L$, where $\pi_{21}(e_l)$ is the working ability distribution of age $i = 1$ households. Then, for $i = 21, \ldots, I$; $j = 1, \ldots, J$; $k = 1, \ldots, K$; and $l = 1, \ldots, L$, do the following:

1. Find the indices j' and k' that satisfy

$$a_{j'} \le a'(i, a_j, b_k, e_l; \mathbf{\Omega}_t) < a_{j'+1}, \qquad b_{k'} \le b'(i, a_j, b_k, e_l; \mathbf{\Omega}_t) < b_{k'+1}.$$

   If $a'(i, a_j, b_k, e_l \ge a_J$, set the index as $j' = J - 1$.

2. Calculate the interpolation weights,

$$\omega_a = \frac{a'(i, a_j, b_k, e_l; \mathbf{\Omega}_t) - a_{j'}}{a_{j'+1} - a_{j'}}, \qquad \omega_b = \frac{b'(i, a_j, b_k, e_l; \mathbf{\Omega}_t) - b_{k'}}{b_{k'+1} - b_{k'}}.$$

3. For $l' = 1, \ldots, L$, update the next period distribution as

$$x_{t+1}(i+1, a_{j'}\!:\!a_{j'+1}, b_{k'}\!:\!b_{k'+1}, e_{l'}) := x_{t+1}(i+1, a_{j'}\!:\!a_{j'+1}, b_{k'}\!:\!b_{k'+1}, e_{l'})$$
$$+ \frac{\phi_{i,t}}{1+\nu}\begin{bmatrix} (1-\omega_a)(1-\omega_b) & (1-\omega_a)\,\omega_b \\ \omega_a\,(1-\omega_b) & \omega_a\,\omega_b \end{bmatrix}\pi(e_{l'}|e_l)\, x_t(i, a_j, b_k, e_l),$$

where $x_{t+1}(i+1, a_{j'} : a_{j'+1}, b_{k'} : b_{k'+1}, e_{l'})$ is a 2×2 matrix.

The steady-state distribution of households can be found by replacing $x_{t+1}(\,\cdot\,)$ with $x_t(\,\cdot\,)$ in Step 3 and calculating $x_t(i, \,\cdot\,)$ recursively from age $i = 21$ to I.[73]

A.3 Solving the Model for an Equilibrium

The following computational algorithm is a modified version of that in Nishiyama and Smetters (2005, 2007). The procedure to find an equilibrium-transition path is also shown in Conesa and Krueger (1999) and Domeij and Heathcote (2004).

A Stationary (Steady-State) Equilibrium. The stationary (steady-state) equilibrium with a time-invariant government policy $\mathbf{\Psi} = \{c_G, tr_{LS}, \tau_I(\cdot), \tau_P(\cdot), tr_{SS}(\cdot), \tau_C, d_G\}$ is obtained as follows:

1. Set the initial values of factor prices and the government's policy variables, $\mathbf{\Omega}^0 = \{r^0, w^0, c_G^0, tr_{LS}^0, \varphi^0, \tau_O^0, \psi_O^0, \tau_C^0, d_G^0\}$.

2. Given $\mathbf{\Omega}^0$, find the decision rules, $\mathbf{d}(\mathbf{s}_i; \mathbf{\Omega}^0)$, value function, $v(\mathbf{s}_i; \mathbf{\Omega}^0)$, and marginal value functions, $v_a(\mathbf{s}_i; \mathbf{\Omega}^0)$ and $v_b(\mathbf{s}_i; \mathbf{\Omega}^0)$, of a household recursively from age $i = I$ to age 21, starting with $v(\mathbf{s}_{I+1}; \mathbf{\Omega}^0) = v_a(\mathbf{s}_{I+1}; \mathbf{\Omega}^0) = v_b(\mathbf{s}_{I+1}; \mathbf{\Omega}^0) = 0$.[74]

3. Find the stationary (steady-state) population distribution of households, $x(\mathbf{s}_i)$, recursively from age $i = 1$ to age I by using the obtained decision rules, $a'(\mathbf{s}_i; \mathbf{\Omega}^0)$ and $b'(\mathbf{s}_i; \mathbf{\Omega}^0)$, as well as the Markov transition matrix of the working ability shock.

4. Compute K, L, and new factor prices, (r^1, w^1), by using the decision rules and the population distribution function. Then compute new government policy variables, $(c_G^1, tr_{LS}^1, \varphi^1, \tau_O^1, \psi_O^1, \tau_C^1, d_G^1)$, that satisfy the government budget constraint.

5. If the difference between $\mathbf{\Omega}^1 = \{r^1, w^1, c_G^1, tr_{LS}^1, \varphi^1, \tau_O^1, \psi_O^1, \tau_C^1, d_G^1\}$ and $\mathbf{\Omega}^0$ is small enough, for example, $\max\{\left|\mathbf{\Omega}^1 - \mathbf{\Omega}^0\right| / \left(0.01 + \left|\mathbf{\Omega}^0\right|\right)\} < 10^{-4}$, then stop. Otherwise, update $\mathbf{\Omega}^0$ by using $\mathbf{\Omega}^1$ and return to Step 2.[75]

[73] To preserve the measure of households, the distribution of households in the next period must be calculated with linear or bilinear interpolation. However, that procedure is much more efficient than a procedure that finds the distribution by simulation.

[74] Within a given age i, a household's problem at any state can be solved independently of the other states, thereby creating a large opportunity for parallelizing the computations, which is especially useful if more state variables are added.

[75] A simple Gauss–Jacobi type iteration of factor prices and government policy variables, $\mathbf{\Omega}$, is more efficient than a Newton-type iteration, because the household decision rules are sensitive to the changes in $\mathbf{\Omega}$.

In many cases, only one or two government policy variables are endogenous, and the others are exogenous. In Step 5, it will suffice to find the convergence of $(K/L)^0$ instead of (r^0, w^0), but we usually need to dampen the iteration process of K/L as

$$(K/L)^1 := \eta(K/L)^1 + (1-\eta)(K/L)^0, \qquad \eta \in (0,1).$$

An Equilibrium-Transition Path. Assume that the economy is in the initial stationary (steady-state) equilibrium with a government policy schedule $\mathbf{\Psi}_0$ in period $t = 0$ and that the government introduces a new policy schedule $\mathbf{\Psi}_1$ at the beginning of period 1. The equilibrium-transition path from the initial steady state to a final steady state is computed as follows:

1. Choose a large number T such that the economy is said to reach the new stationary (steady-state) equilibrium within T periods. Then set initial values of factor prices and the government's policy variables, $\mathbf{\Omega}_1^0 = \{r_t^0, w_t^0, c_{G,t}^0, tr_{LS,t}^0, \varphi_t^0, \tau_{O,t}^0, \psi_{O,t}^0, \tau_{C,t}^0, d_{G,t}^0\}_{t=1}^T$, that are consistent with the new policy $\mathbf{\Psi}_1$.

2. Given $\mathbf{\Omega}_T^0$, compute the final stationary (steady-state) equilibrium in period T—that is, find the decision rules, $\mathbf{d}(\mathbf{s}_i; \mathbf{\Omega}_T^0)$, value function, $v(\mathbf{s}_i; \mathbf{\Omega}_T^0)$, and marginal value functions, $v_a(\mathbf{s}_i; \mathbf{\Omega}_T^0)$ and $v_b(\mathbf{s}_i; \mathbf{\Omega}_T^0)$, of a household from age $i = I$ to age 21.

3. Given $\mathbf{\Omega}_1^0$, find the decision rules, $\mathbf{d}(\mathbf{s}_i; \mathbf{\Omega}_t^0)$, value function, $v(\mathbf{s}_i; \mathbf{\Omega}_t^0)$, and marginal value functions, $v_a(\mathbf{s}_i; \mathbf{\Omega}_t^0)$ and $v_b(\mathbf{s}_i; \mathbf{\Omega}_t^0)$, of a household from period $t = T-1$ to period 1, using $v(\mathbf{s}_{i+1}; \mathbf{\Omega}_{t+1}^0)$, $v_a(\mathbf{s}_{i+1}; \mathbf{\Omega}_{t+1}^0)$, and $v_b(\mathbf{s}_{i+1}; \mathbf{\Omega}_{t+1}^0)$ recursively.[76]

4. Set $x_1(\mathbf{s}) = x_0(\mathbf{s})$ and $d_{G,1}^1 = d_{G,0}$, since the economy is still in the initial stationary (steady-state) equilibrium at the beginning of period $t = 1$. Compute aggregate variables, (K_t, L_t), factor prices (r_t^1, w_t^1), government policy variables, $(c_{G,t}^1, tr_{LS,t}^1, \varphi_t^1, \tau_{O,t}^1, \psi_{O,t}^1, \tau_{C,t}^1, d_{G,t}^1)$, and the distribution function of households, $x_{t+1}(\mathbf{s})$, recursively from period $t = 1$ to T.

5. If the difference between $\mathbf{\Omega}_1^1 = \{r_t^1, w_t^1, c_{G,t}^1, tr_{LS,t}^1, \varphi_t^1, \tau_{P,t}^1, \psi_{O,t}^1, \tau_{C,t}^1, d_{G,t}^1\}_{t=1}^T$ and $\mathbf{\Omega}_1^0$ is small enough, for example, $\max\{|\mathbf{\Omega}_1^1 - \mathbf{\Omega}_1^0| / (0.01 + |\mathbf{\Omega}_1^0|)\} < 10^{-4}$, then stop. Otherwise, update Ω_1^0 by using Ω_1^1 and return to Step 2. If there is no change in $d_{G,T}^0$, then return to Step 3.

[76] In Step 3, we obtain all of the decision rules and value functions in the transition path without updating a set of factor prices and government policy variables, $\mathbf{\Omega}_1^0$. Thus, the procedure adopted here is a Gauss–Jacobi iteration.

If the policy change is deficit financing for the first several years before the debt-to-GDP ratio is stabilized, we need to calculate the final steady state repeatedly until $d_{G,T}$ converges.

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