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The Welfare Cost of Capital Taxation: An Asset Market Approach

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Abstract

We use an asset pricing perspective to provide a novel interpretation of the marginal welfare cost of capital income taxes. We show that the marginal welfare cost can be interpreted as the normalized present discounted value of consumption distortions from capital income taxes. Such an interpretation emphasizes the importance of the discount rate used to value future consumption distortions, especially in the presence of uncertainty. We find that the discount rate decreases as the capital income tax rate increases, thus increasing the welfare cost of taxes. The variations in the discount rate are caused by the amplified responses of consumption to exogenous shocks as a result of capital income taxation. We find that the welfare cost may be underestimated if variations in risky discount rates are ignored, especially when tax rates are high.

1 Introduction

This paper studies the determinants of the marginal welfare cost of capital income taxes in a stochastic production economy. Starting from a balanced-growth equilibrium consistent with an arbitrarily given capital income tax rate, we examine the welfare cost of a permanent shift to a marginally higher tax rate, under the assumption of a lump-sum full rebate of tax revenues.[1] We measure the welfare cost of taxes as the compensation required to make the representative household indifferent between consumption plans with and without the marginal shift in tax rates. We relate this measure of welfare cost to the market value of a security that is a claim to consumption distortions resulting from the marginal shift in tax rates. Such an interpretation brings to the forefront the importance of the discount rate used to value future consumption distortions. We find that the discount rate decreases as the capital income tax rate increases, thus increasing the welfare cost of taxes. The variations in the discount rate are caused by amplified responses of consumption to exogenous shocks as a result of capital income taxation. We find that the welfare cost may be underestimated if variations in risky discount rates are ignored, especially when tax rates are high.

Interpreting the marginal welfare cost as the market value of a security allows us to examine the welfare cost from an asset market perspective. Just like prices of any other risky securities, the marginal welfare cost of taxes is determined by three factors: the stream of consumption distortions caused by inefficient allocation of resources, the discount rate used to discount the stream of consumption distortions, and the covariance between consumption distortions and systematic risk.

We study the welfare cost of taxes in a general equilibrium production economy with varying degrees of uncertainty. We show that the marginal welfare cost of capital income taxes increases with the tax rate in both deterministic and stochastic environments. However, the marginal welfare cost curve is higher and steeper in the stochastic case than in the deterministic case. In the deterministic case, the upward slope of the marginal welfare cost curve is mostly driven by increasing consumption distortions as the tax rate increases. In the stochastic case, however, variations in the discount rate and in the covariance between consumption distortions and systematic risk

[1] We assume lump-sum tax rebates to distinguish between the inefficiencies resulting from distorted investment decisions (substitution effects) and the inefficiencies resulting from the use of tax revenues (income effects).

also play important roles. We denote consumption distortions, variations in the discount rate and variations in the covariance term as the distortion, discounting and insurance effects of capital income taxes, respectively.

The discounting and insurance effects that capture the variations in the discount rate and in the covariance term are unique for a stochastic environment. We find that as the tax rate increases, the discount rate used to value future consumption distortions tends to decrease, thus raising the welfare cost. Santoro and Wei (2011) show that capital income taxes can lead to amplified responses of the marginal utility of consumption to exogenous shocks. Those amplified responses are the main reason behind the decrease in the discount rate as the tax rate increases. At the same time, those amplified responses alter the magnitude of the covariance term. The covariance term becomes increasingly negative as the capital income tax rate rises, thus mitigating the marginal welfare cost. We find that the discounting effect increasingly dominates the insurance effect as the capital income tax rate rises.

Since the discounting and insurance effects can only affect the shape of the marginal welfare cost in the stochastic environment, the increasingly dominant impact of the declining discounting effect leads to a marginal welfare cost curve not only steeper than but also above that in a deterministic environment. We find that as the degree of uncertainty increases, the gap between the marginal welfare cost curves in deterministic and stochastic economies widens, reflecting the strength of the discounting effect as the degree of uncertainty increases.

In addition to the degree of aggregate uncertainty, the marginal welfare cost of capital income taxes depends on the preference and production specifications. We need a production economy, in which both consumption and investment are endogenously determined, to study the distortionary effect of capital income taxes. Since we use an asset market approach to price consumption distortions, it seems important to have a production-based model that is not only able to mimic some basic asset pricing features but also tractable enough to make transparent the mechanisms introduced by capital income taxes. Jermann (1998) and Boldrin, Christiano and Fisher (2001) feature such kind of models. They show that the key ingredients for such a model are habit formation in preferences and adjustment costs in production technology. Based on this consideration, we assume moderately high habit persistence and capital adjustment costs in the benchmark calibration.

We also conduct a sensitivity analysis in the same stochastic environment but with neither habit persistence nor capital adjustment costs[2]. Although the marginal rates of substitution are less volatile, we find that both the discounting and the insurance effects are still present in such a setting, albeit at a lower magnitude. The discounting effect dominates the insurance effect as the tax rate increases, just as in the benchmark case, resulting in a higher welfare cost in the stochastic environment.

Our quantitative findings can be related to those of Chamley (1981) and Lucas (1990), which use a deterministic dynamic general equilibrium model to evaluate the welfare gain obtained by abolishing the capital income tax. According to Lucas (2003), "the overall welfare gains amount to perhaps 2 to 4 percent of annual consumption, in perpetuity." Since tax reforms typically involve discrete changes in tax rates rather than abolition of a tax, we focus specifically on the welfare cost of a marginal shift in the capital income tax rate, and integrate the marginal welfare cost over the given range of tax rates to compute the total gain from discrete changes in tax rates. Our calculation yields an overall welfare gain from abolishing capital income taxes that is at the high end of Lucas's estimate. The impact of capital income taxes on the discount rate, which is absent in a deterministic setting, contributes to the higher welfare cost of capital income taxes in the presence of aggregate uncertainty.

Judd (1987) examines the marginal efficiency cost of various factor taxes in a deterministic model. He states that "any biases of the deterministic approach relative to a more realistic model with uncertainty must arise from decreasing returns in capital intensity and third-order properties of utility functions." Decreasing returns to capital and third-order properties of utility functions are important for capital income taxes to have a strong effect on the discount rate. It is the omission of the possible effect of capital income taxes on the discount rate that biases the estimate of the deterministic approach.

Our results advance the insights gained from Gordon and Wilson (1989), which examines the marginal welfare loss of capital taxation in a stochastic production economy similar to ours.[3] They argue that past measures that

[2]The risk-free interest rate can be overly volatile in models such as Jermann (1998) and Boldrin, Christiano and Fisher (2001). One purpose of the sensitivity analysis is to examine the robustness of our mechanism when the marginal rates of substitution, and also the risk-free rates, are less volatile.

[3]Bulow and Summers (1984) and Gordon (1985) also study the welfare cost of taxing risky capital income. An important limitation of their work is that they both employ a two-period framework, which alters the risk characteristics of any long-lived securities.

ignore the negative covariance between consumption distortions and the stochastic discount factor "likely overstate the efficiency costs of a rise in the tax rate, perhaps dramatically." The negative covariance stressed by Gordon and Wilson (1989) is also present in our framework. However, since Gordon and Wilson (1989) only examine the marginal welfare loss at a single tax rate, they do not study the declines in the discount rate accompanied by increases in the capital income tax rates. We find that the declines in the discount rate are significant enough to dominate the increasingly negative covariance as the tax rate increases.

We also relate our findings to the literature that uses an asset pricing approach to study welfare and budgetary issues. Alvarez and Jermann (2004) use an asset market measure to study the welfare cost of consumption fluctuations. Geanakoplos and Zeldes (2010) highlight the importance of proper discounting in deriving the market value of Social Security claims.

We organize the paper as follows. In Section 2, we derive our measure of the marginal welfare cost of capital income taxes and describe its asset pricing interpretation. In Section 3, we use a stylized model as a laboratory to examine the properties of the marginal welfare cost curves. In Section 4, we decompose the marginal welfare cost to analyze the distortion, discounting and insurance effects separately. Section 5 describes the sensitivity analysis. Section 6 concludes.

2 Asset Pricing Interpretation of the Marginal Welfare Cost

In this section, we define the marginal cost of capital income taxes in a stochastic economy and relate that cost to the normalized market value of consumption distortions.

2.1 Measures of Welfare Cost

Let τ be an arbitrarily given capital income tax rate. Assume that in each period t, the history of events up to and including period t is denoted by $s^t = (s_0, s_1, ...s_t)$. Here s_t is a vector that includes both exogenous and endogenous variables which may depend upon τ.

We consider an economy characterized by the given tax rate τ and an initial state s_0. Let $\left\{C_t^{\tau,0}\right\}_{t=0}^{\infty}$ represent the consumption stream for a representative household in an economy with the tax rate τ at each period t. The first superscript, τ, stresses the dependence of consumption on the given tax rate, and the second superscript, 0, indicates that the tax rate remains unchanged at τ. $C_t^{\tau,0}$ is a function of the history of events, s^t. Now we consider a permanent marginal change of ε in the tax rate at period 0. After that, the economy moves on a dynamic path from the initial state (s_0) consistent with a tax rate τ to a long-run state compatible with the higher marginal tax rate, $\tau+\varepsilon$. We use $\{C_t^{\tau,\varepsilon}\}_{t=0}^{\infty}$ to represent the alternative consumption stream along this transition path resulting from the marginal tax rate increase. Here the first superscript, τ, stresses that the initial state of the economy is consistent with the tax rate τ, and the second superscript, ε, indicates that the tax rate shifts up from τ by ε.

We measure the welfare cost of this marginal tax change in terms of the compensation that renders the representative household indifferent between the consumption streams with or without the permanent marginal increase in the tax rate. This compensation is measured as a fraction of consumption for each period in the economy with the marginally higher tax rate.

Let θ be the fraction that will serve as a compensating consumption supplement, and define the indirect utility function V for the representative household by:

$$V(\theta,\varepsilon|\tau) = E_0 U\left(\{(1+\theta)\, C_t^{\tau,\varepsilon}\}_{t=0}^{\infty}\right).$$

where $U(\bullet)$ represents lifetime utility from any consumption stream.

Here $V(\theta,\varepsilon|\tau)$ is interpreted as the utility the consumer would enjoy from the alternative consumption stream $\{C_t^{\tau,\varepsilon}\}_{t=0}^{\infty}$ and a consumption supplement $\theta C_t^{\tau,\varepsilon}$ for each period.

We define the unique, positive value of θ that satisfies the condition:

$$V(\theta,\varepsilon|\tau) = V(0,0|\tau), \tag{1}$$

as the welfare cost of raising the capital income tax rate from τ to $\tau+\varepsilon$. This measure reflects the welfare cost of the marginal increase in the tax rate in the long run, as well as during the transition period. Here θ is a function of both ε and τ.

Using the implicit function theorem, we can derive the derivative of $\theta(\varepsilon,\tau)$ with respect to ε :

$$\theta_\varepsilon(\varepsilon,\tau)\,|_{\varepsilon=0} = -\frac{V_\varepsilon(0,0|\tau)}{V_\theta(0,0|\tau)}. \tag{2}$$

The welfare cost of a permanent discrete change from τ to $\tau + \varepsilon$ can be approximated locally as[4]

$$\theta(\varepsilon, \tau) \simeq \varepsilon \theta_{\varepsilon}(\varepsilon, \tau)|_{\varepsilon=0}. \tag{3}$$

Here $\theta_{\varepsilon}(\varepsilon, \tau)$ is the marginal welfare cost of a marginal tax rate increase of ε in an economy characterized by an initial capital income tax rate of τ. We show in the next section that this traditional measure can be interpreted as a normalized net present value.

2.2 Asset Pricing Interpretation

Assume that $U(\bullet)$ takes the following form of expected utility,

$$U\left(\{C_t\}_{t=0}^{\infty}\right) = E_0 \sum_{t=0}^{\infty} \left[\beta^t u\left(C_t, \{C_{t-j}\}_{j=1}^{N}\right)\right], \tag{4}$$

where $u\left(C_t, \{C_{t-j}\}_{j=1}^{N}\right)$ can accommodate both time-separable and non-time-separable preferences. We have the following representation of the marginal welfare cost of taxes, based on equation (2):

$$\theta_{\varepsilon}(\varepsilon, \tau) = \frac{E_0 \sum_{t=0}^{\infty} \left\{\beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}} \frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial \varepsilon}\right\}}{W_{\tau}}, \tag{5}$$

where $\Lambda_t^{\tau,0}$ is the derivative of the lifetime utility $U\left(\left\{C_t^{\tau,0}\right\}_{t=0}^{\infty}\right)$, as defined in equation (4) with respect to $C_t^{\tau,0}$, and

$$W_{\tau} = E_0 \sum_{t=0}^{\infty} \left\{\beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}} C_t^{\tau,0}\right\}.$$

The marginal welfare cost of capital income taxes is now the ratio of the values of two securities. The numerator represents the value of a security that is a claim to the differences in the consumption streams (consumption distortions) with the marginal increase in the capital income tax rate. The

[4] The complete approximation takes the form of $\theta(\epsilon, \tau) \simeq \theta(0, \tau) + \theta_{\epsilon}(\epsilon, \tau)\epsilon$. According to our definition, $\theta(0, \tau)$ is equal to zero.

denominator, instead, represents the value of a security that is a claim to the consumption stream in an economy without the marginal increase in the current tax rate τ.

The marginal welfare cost of capital income taxes is now effectively the normalized market value of consumption distortions, with the denominator serving as the normalizing factor.

2.3 Comparison with the Lucas Method

Lucas (1990) uses Bernheim's (1981) formula to measure the welfare cost of eliminating capital income taxes. That formula involves a weighted sum of the percentage changes in consumption in the initial period and in the new long-run state of the economy characterized by a higher marginal tax rate. The weights are mainly determined by the subjective time preference and the speed of convergence.

Our measure of the marginal cost of taxation can be written in a form similar to Bernheim's formula.

$$\begin{aligned} \theta_{\varepsilon}(\varepsilon,\tau) &= \frac{E_0 \sum_{t=0}^{\infty} \left\{ \beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}} \frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon} \right\}}{W_\tau} \qquad (6) \\ &= E_0 \sum_{t=0}^{\infty} \left\{ \omega_t \frac{\partial\left(-C_t^{\tau,\varepsilon}\right)/\partial\varepsilon}{C_t^{\tau,0}} \right\}, \text{ where} \\ \omega_t &= \beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}} \frac{C_t^{\tau,0}}{W_\tau}. \end{aligned}$$

In an environment without uncertainty, the variables $\Lambda_t^{\tau,0}$, $C_t^{\tau,0}$, and $\frac{C_t^{\tau,0}}{W_\tau}$ are constant for all t. As a result, the weight ω_t becomes $\frac{\beta^t}{\sum_{j=0}^{\infty}\beta^j}$. It is clear that additional mechanisms other than the speed of convergence come into play in the presence of uncertainty. The time-varying marginal utility of consumption and its covariance with consumption distortions now play important roles in valuing future consumption distortions, which may lead to a different welfare cost of capital income taxes than previously accounted for in a deterministic environment.

Since consumption distortions resulting from capital income taxes extend into the infinite horizon, the rate at which those distortions are discounted is important for the measure of welfare cost. As a result, the possible impact of taxes on the marginal utility of consumption, and hence on the stochastic discount rate, may play an important role in determining the welfare cost. In a model without uncertainty, capital income taxes have no impact on the discount rate, which is a constant depending only upon β.

3 Valuing Consumption Distortions

In this section, we first describe a model that features a stochastic production economy. We then use it as a laboratory to compute the market value of consumption distortions as a way of measuring the marginal welfare cost of capital income taxes.

3.1 A Stochastic Production Economy

In this section, we sketch the major elements of a stylized production economy. The details are contained in Appendix A.

There is a continuum of infinitely-lived identical households that own a representative firm. The government levies taxes on capital income and rebates them back in a lump sum to the households. The economy grows at a constant rate g.

The representative household maximizes its lifetime utility subject to a standard budget constraint. The specification of the instantaneous utility function, $u(C_t, C_{t-1})$, allows for time-separable and non-time-separable preferences.

The firm produces output, Y_t, using the Cobb-Douglas production technology:

$$Y_t = Z_t K_t^{\alpha} \left[(1+g)^t L_t\right]^{(1-\alpha)}, \tag{7}$$

where L_t represents labor, K_t is the capital stock, and the logarithm of the stochastic productivity level, Z_t, follows

$$z_t = \rho z_{t-1} + \sigma \xi_t, \qquad \xi_t \sim N(0,1). \tag{8}$$

Here ρ is the persistence parameter and σ indexes the degree of aggregate uncertainty. The capital accumulation process is given by:

$$K_{t+1} = (1-\delta)\, K_t + \Psi(K_t, I_t), \tag{9}$$

where I_t represents investment. We specify the functional form of $\Psi(K_t, I_t)$ as

$$\Psi(K_t, I_t) = \left[\frac{(g+\delta)^\eta}{1-\eta}\left(\frac{I_t}{K_t}\right)^{1-\eta} + \frac{\eta(g+\delta)}{\eta-1}\right] K_t, \tag{10}$$

where the capital supply is inelastic when η approaches infinity.[5] The concavity of this function captures convex costs of adjustment.

In equilibrium, output is equal to the sum of consumption and investment:

$$Y_t = C_t + I_t. \tag{11}$$

All the asset markets clear. Given that leisure does not enter the utility function, households will allocate their entire time endowment to productive work. Labor supply is constant and normalized to 1. It is important to note that since labor supply is inelastic in our stylized model, capital income taxes are the only source of distortion.

The first-order condition for investment is:

$$\frac{1}{\Psi_I(K_t, I_t)} = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \left[\alpha(1-\tau)\frac{Y_{t+1}}{K_{t+1}} + \frac{(1-\delta) + \Psi_K(K_{t+1}, I_{t+1})}{\Psi_I(K_{t+1}, I_{t+1})} \right] \right\}, \tag{12}$$

where $\beta \frac{\Lambda_{t+1}}{\Lambda_t}$ represents the stochastic discount factor and Λ_t is given by

$$\Lambda_t = u'(C_t - bC_{t-1}) - b\beta E_t u'(C_{t+1} - bC_t).$$

Here b is the parameter that indexes the degree of habit persistence in the non-time-separable preferences. $\Psi_I(\bullet)$ and $\Psi_K(\bullet)$ are the derivatives of capital adjustment costs with respect to investment and capital stock. Capital income taxes distort investment decisions by reducing the marginal benefit of investment.

Proposition 1 *The deterministic steady state of the economy, where $Z_t = 1, \forall t$, can be characterized as follows,*
(a) The equilibrium capital stock, K_{ss}^τ, is given by

$$K_{ss}^\tau = \left[\frac{\alpha\beta^*(1-\tau)}{1-\beta^*(1-\delta)}\right]^{\frac{1}{1-\alpha}},$$

[5] Capital adjustment costs have been studied by Eisner and Strotz (1963), Lucas (1967) and others. The particular specification we adopt is similar to that of Jermann (1998).

where β^*, which is equal to $\beta(1+g)^{-\gamma}$, is the trend-adjusted subjective time preference.
(b) The equilibrium consumption, C_{ss}^{τ}, is given by

$$C_{ss}^{\tau} = (K_{ss}^{\tau})^{\alpha} - (g+\delta) K_{ss}^{\tau}.$$

(c) The equilibrium investment-output ratio, given by $(g+\delta)(K_{ss}^{\tau})^{1-\alpha}$, is monotonically decreasing in τ. The opposite is true for the equilibrium consumption-output ratio.

Proof. Equilibrium allocations are derived from the first-order condition for investment (12) and the aggregate resource constraint (11). ■

3.2 The Marginal Welfare Cost

In this section, we use the stylized model described above as a laboratory to compute the marginal welfare cost. We describe the benchmark parameterization in Table 1. Since our focus is on valuing stochastic consumption distortions, it is important to have a model that can reproduce key asset market statistics. Jermann (1998) shows that a combination of high habit persistence and capital adjustment costs help to generate reasonable asset return statistics in a model like ours. Based on this consideration, we introduce moderately high habit persistence and capital adjustment costs by setting b equal to 0.6 and η equal to 4.24.[6]

For any arbitrarily given τ, we consider two scenarios starting from the initial state s_0 characterized by $\{K_{ss}^{\tau}, C_{ss}^{\tau}, Z_{ss}\}$, which represent, respectively, the steady-state values of capital stock and consumption given τ and the steady-state level of the aggregate productivity. In the first scenario, the marginal tax rate remains unchanged at τ, and under the alternative scenario, there is a permanent marginal shift in the capital income tax rate from τ to $\tau+\varepsilon$ starting in the initial period. We assume the same aggregate technology process, $\{Z_t\}_{t=0}^{\infty}$, with or without the shift in the marginal tax rate. In the deterministic environment, Z_t is equal to 1 for all t.

We apply equation (5) to compute the marginal welfare cost for all of the tax rates. The details of the computation are contained in Appendix

[6]The value of b lies within the range widely used by the literature (see Cochrane and Hansen, 1992). The value of η is similar to that used by Jermann (1998).

B. Figure 1 plots the marginal welfare cost, $\theta_{\varepsilon}(\varepsilon, \tau)$, for a range of initial tax rates τ in both a deterministic ($\sigma = 0$) and a stochastic ($\sigma = 0.016$) environment.[7,8] These two environments are otherwise the same except for the differences in σ. Here we approximate ε with a value of 0.01.

Figure 1 displays three salient features about the marginal welfare cost curves:

1. The marginal welfare cost curves are convex and upward sloping in the tax rate in both the deterministic and stochastic cases.

2. The marginal welfare cost curve is higher and steeper in the stochastic case than in the deterministic case.

3. The gap between the marginal welfare cost curves in the stochastic and deterministic cases widens as the tax rate increases.

Since the total welfare cost of a discrete change in the tax rate is the integral of the marginal welfare cost for the given range of tax rates, steeper slopes imply higher total welfare costs. The gap between the two marginal welfare cost curves can thus be translated into the difference in the total welfare gain of eliminating capital income taxes by integration.

We find that the total welfare gain of eliminating capital income taxes with $\tau = 0.36$ is equivalent to 3.47 percent of lifetime consumption in the deterministic case, but is equal to 4.51 percent in the stochastic case. The part of total welfare gain attributed to uncertainty amounts to almost one third of the total welfare gain in the deterministic case. Ignoring uncertainty may lead to an underestimation of the welfare cost of capital income taxes.

In a deterministic environment, the upward slope of the marginal welfare cost curve is mostly driven by increasing consumption distortions as the tax rate increases. In a stochastic environment, however, variations in both the stochastic discount factor and the covariances between the stochastic discount factor and consumption distortions also play important roles.

[7] McGrattan and Prescott (2005) estimate the corporate tax rate to be 0.43 for periods of high tax rates. For illustration, we pick this as one of the extreme values that the capital income tax rate can take. We use an intermediate tax rate of 0.36 to relate our measure of welfare cost to Lucas (1990).

[8] We set the standard deviation of the aggegate productivity shock at 0.016 in accord with Bloom, Floetotto and Jaimovich (2010).

In the next section, we decompose the marginal welfare cost and distinguish the impact of variations in the discount rates from those in the covariance terms.

4 Decomposition of the Marginal Welfare Cost

In this section, we decompose the numerator of the marginal welfare cost to shed light on its determinants in a stochastic environment. The marginal welfare cost of taxes is decomposed as follows:

$$\theta_{\varepsilon}(\varepsilon,\tau)=\frac{\sum_{t=0}^{\infty}\left\{E_0\left(\beta^t\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}\right)E_0\left[\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right]\right\}+\sum_{t=0}^{\infty}cov_0\left[\beta^t\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}},\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right]}{W_{\tau}} \tag{13}$$

In the decomposition, the numerator of equation (5), $E_0\sum_{t=0}^{\infty}\left\{\beta^t\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right\}$, is decomposed into two elements. In the first element, the product of $E_0\left[\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right]$ and $E_0\left(\beta^t\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}\right)$ represents expected consumption distortions at period t valued at the price of the t-th period discount bond. The second element captures the covariance between the stochastic discount factor and consumption distortions. For future reference, we define

$$\Phi_{\tau}=\frac{\sum_{t=0}^{\infty}\left\{E_0\left(\beta^t\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}\right)E_0\left[\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right]\right\}}{W_{\tau}},\ \Omega_{\tau}=\frac{\sum_{t=0}^{\infty}cov_0\left[\beta^t\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}},\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right]}{W_{\tau}}. \tag{14}$$

The decomposition reveals multiple forces determining the marginal welfare cost in the presence of uncertainty. In Φ_{τ}, $E_0\left[\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right]$ represents the expected consumption distortion that varies with the tax rate. We label this the *distortion effect* of capital income taxes. A corresponding term is also present in a deterministic environment.

Also in Φ_{τ}, $E_0\left(\beta^t\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}\right)$ is the price of a $t-th$ period discount bond, which determines the term structure used to value future consumption distortions. It is important to note that in a stochastic environment the discount rate not only varies with time, but also depends upon the capital income tax rate, τ. We call that the *discounting effect* of capital income taxes.

The presence of uncertainty brings in another effect of capital income taxes on the marginal welfare cost, which is reflected in $cov_0\left[\beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}, \frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right]$. This term denotes a covariance between the stochastic discount factor and consumption distortions conditional on the information at time 0. A negative covariance between the two implies that consumption distortions are large in states when such distortions are valued less. Such desirable coincidence reduces the welfare cost of taxes and acts as insurance in the presence of uncertainty (see, for example, Gordon and Wilson, 1989). We call this effect the *insurance effect* of capital income taxes.

In all, we identify three effects of capital income taxes on the marginal welfare cost: the distortion, discounting and insurance effects. The distortion effect is present in both stochastic and deterministic environments, while the second two effects, which are closely related to second-order moments, are present only in stochastic environments.

Since the latter two effects appear only in stochastic environments, the difference between the marginal welfare cost curves in the stochastic and deterministic environments reflects the strength of the discounting and insurance effects. In the next sections, we examine each of the three effects in detail.

4.1 Distortion

To understand the distortion effect, it is useful to examine the path of consumption distortions for a particular tax rate. In Figure 2, we plot the percentage differences in the two consumption paths $\left\{C_t^{\tau,0}\right\}_{t=0}^{\infty}$ and $\{C_t^{\tau,\varepsilon}\}_{t=0}^{\infty}$ as a fraction of $\left\{C_t^{\tau,0}\right\}_{t=0}^{\infty}$ for $\tau = 0.36$. As mentioned above, $\left\{C_t^{\tau,0}\right\}_{t=0}^{\infty}$ represents the consumption path under the first scenario, where the marginal tax rate remains unchanged at τ, while $\{C_t^{\tau,\varepsilon}\}_{t=0}^{\infty}$ represents the corresponding path under the second scenario, where the tax rate shifts permanently to $\tau+\varepsilon$ at period 0.

Although those two scenarios start at the same initial steady state $\{K_{ss}^{0.36}, C_{ss}^{0.36}, Z_{ss}\}$ and are subject to the same aggregate technology process, their consumption paths diverge as soon as the marginal tax change is implemented. The resulting path of consumption distortions demonstrates the short-run and long-run trade-offs following a marginal shift in the tax rate, a point also made by Bernheim (1981) and Lucas (1990).

Under the second scenario, consumption temporarily increases above what it would be under the first scenario, resulting in negative consumption distortions. Increases in consumption in the initial periods accommodate the convergence of the capital stock to a lower level consistent with a higher tax rate in the long run. However, as time passes, distorted investment decisions also lead to both lower capital stock and lower output. Eventually consumption declines and converges to a lower long-run equilibrium level relative to that under the first scenario, where the tax rate remains unchanged at τ. Consequently, the amount of consumption distortions fluctuates around a positive value in the long run.

In Figure 2, we also plot a corresponding relative measure of consumption distortions in a deterministic economy. The same short-run and long-run trade-off applies, and the amount of consumption distortions eventually converges to a positive value.

4.2 Discounting

The rate that is used to discount consumption distortions is a function of the stochastic discount factor, $\beta\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}$, which in turn depends upon the given capital income tax rate τ. The discount rate plays a crucial role in determining the welfare cost of taxes. In this section, we examine how the discount rate varies across different capital income tax rates and determines the welfare cost of taxes together with other forces.

In order to isolate the effect of the stochastic discount factor on the marginal welfare cost, we compare two methods of valuing consumption distortions by discounting the same path of consumption distortions differently. In the first method, we use β, the discount factor in the deterministic case, while in the second method, we use the stochastic discount factor $\beta\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}$, which reflects consumption fluctuations across different states contingent upon the tax rate τ.

The upper left panel of Figure 3 plots the present discounted value of strips of deterministic consumption distortions, $\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}|_{\sigma=0}$, at $\tau = 0.36$ using the above two methods. The horizontal axis denotes the time period for the strip, and the vertical axis denotes the market value of the corresponding strip at period 0. The market value of the n-th strip computed with the first method, $\beta^t \frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}|_{\sigma=0}$, stays below the value computed with the second method, $E_0\left(\beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}} \frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}|_{\sigma=0}\right)$ for almost all the time periods. This indicates that the values of discount bonds in the stochastic environment are higher than those in the deterministic case. We plot the differences between $\beta^t \frac{\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}|_{\sigma=0}$ and $E_0\left[\beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}} \frac{\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}|_{\sigma=0}\right]$ in the upper right panel of Figure 3. This difference measures the strength of the discounting effect resulting from the presence of uncertainty. In the left panel of Figure 4, we plot the differences between those two terms for three different tax rates: $0.001, 0.13, 0.36$. As the tax rate increases, the discounting effect becomes stronger.[9]

The intuition for the increasing discounting effect is as follows. As the capital income tax rate increases, investment is further discouraged. As a result, investment constitutes an increasingly smaller share of aggregate output in the steady state. A lower investment-output ratio implies that investment needs to respond more to exogenous shocks in order to smooth consumption, which now constitutes a larger fraction of output. Santoro and Wei (2011) show that such a mechanism can be responsible for amplifying the responses of consumption, and consequently of the marginal utility of consumption, Λ, to aggregate technology shocks in an economy with higher tax rates. The amplified responses of the marginal utility of consumption typically lead to precautionary motives, which tend to reduce the discount rate used to value future consumption distortions.

4.3 Insurance

If the amount of consumption distortions is procyclical, the negative correlation between distortions and the stochastic discount factor will likely reduce the present discounted value of consumption distortions. This desirable correlation may alleviate the marginal welfare cost of capital income taxes.

The lower left panel of Figure 3 plots $E_0\left[\beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}\right] E_0\left[\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right]$ and $E_0\left(\beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}} \frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right)$ for all periods when τ is equal to 0.36. As shown in equation (13), the difference between the two variables represents $cov_0\left(\beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}, \frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right)$, the covari-

[9] The humped shape of the curves is explained by both the rising and then stabilizing of consumption distortions and the decaying value of the discount factor as t increases.

ance between the stochastic discount factor and consumption distortions for all time periods. The lower right panel of Figure 3 plots this difference. The difference remains largely negative. Such a negative covariance implies that consumption distortions are higher in states when they are valued less, reflecting the *insurance effect* of capital income taxes. This desirable property would reduce the welfare cost of taxes in the presence of uncertainty.

In the right panel of Figure 4, we plot $cov_0\left(\beta^t\frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}, \frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}\right)$ for all time periods with a tax rate of 0.001, 0.13 or 0.36. As the tax rate increases, the covariance becomes more negative, reflecting a stronger insurance effect. The increasingly negative covariance can be explained by both amplified responses of the marginal utility of consumption and higher consumption distortions as the tax rate increases.

4.4 Distortion, Discounting and Insurance Across Tax Rates

In this section, we use the decomposition analysis above to examine the gap between the marginal welfare cost curves in stochastic and deterministic environments shown in Figure 1. We follow the decomposition equation (13) and plot the two decomposed elements, Φ_τ and Ω_τ. The sum of the two elements represents the marginal welfare cost for each tax rate.

The left panel of Figure 5 plots Φ_τ for both the deterministic ($\sigma = 0$) and stochastic ($\sigma = 0.016$) cases for all tax rates. The curve Φ_τ corresponding to the stochastic case stays above that of the deterministic case. The fact that Φ_τ is increasing even without uncertainty shows that the distortion from capital income taxes increases with the tax rate. Since the distortion effect on consumption is present in both the deterministic and stochastic cases, and the discounting effect is present only in the presence of uncertainty, the gap between the curves across different degrees of uncertainty mainly reflects the importance of the discounting effect.[10] The widening gap between the two curves reflects stronger discounting effects as the tax rate increases.

The right panel of Figure 5 plots Ω_τ for both the deterministic ($\sigma = 0$) and stochastic ($\sigma = 0.016$) cases for all tax rates. The curve Ω_τ corresponding to

[10] Our simulation shows that the difference between the paths of the deterministic consumption distortions and the expected path of the stochastic consumption distortions is fairly small.

the stochastic case is increasingly negative as the tax rate increases, whereas it is equal to zero in the deterministic case.

To further demonstrate the effect of capital income taxes on the discount rate as the tax rate increases, we plot $E_0 \sum_{t=0}^{\infty} \left[\beta^t \frac{\Lambda_t^{\tau,0}}{\Lambda_0^{\tau,0}}\right]$, which is effectively the market value of a perpetual bond (the sum of the values of all t-th period discount bonds) and its reciprocal, which is equal to the long-term risk-free interest rate. As shown in Figure 6, the price of the perpetual bond increases, and the long-term risk-free discount rate declines as the tax rate increases.

As the capital income tax rate increases, the decline in the discount rate used to value consumption distortions raises the marginal welfare cost of taxes, while the increasingly negative covariance between consumption distortions and the stochastic discount factor reduces it. Two opposite forces are at work. We find that the discounting effect from declines in the discount rate increasingly dominates the insurance effect as the tax rate increases. This dominance explains the widening gap between the marginal welfare cost curves in the stochastic and deterministic environments, as displayed in Figure 1.

5 Sensitivity Analysis

In this section, we conduct a sensitivity analysis to examine the impact that preference and production specifications have on the marginal welfare cost.

We use the same economic environment but assume that there is neither habit persistence nor capital adjustment costs by setting both b and η to 0. Figure 7 plots the marginal welfare cost curves for both the stochastic and deterministic environments under this specification. The marginal welfare cost curve in the stochastic environment stays above that in the deterministic environment for medium-high and high tax rates and has a steeper slope throughout. A decomposition of the marginal welfare curves into Φ_τ and Ω_τ, as shown in Figure 8, indicates that both the discounting and the insurance effects are present, albeit at a smaller magnitude. The discounting effect is weaker relative to the insurance effect for relatively low tax rates. However, as the tax rate increases, the discounting effect dominates the insurance effect, just as in the benchmark case.

We compute the total welfare gain from eliminating capital income taxes at the rate of $\tau = 0.36$ just as in the benchmark framework. The total welfare gain is now 4.62 percent of perpetual consumption in the stochastic environment, as opposed to 4.51 percent in the deterministic environment. The part of the total welfare gain attributed to uncertainty amounts to only 2.5 percent of the total welfare gain in the deterministic case.

As shown in Santoro and Wei (2011), as long as investment plays a major role in smoothing consumption and the investment-output ratio declines as tax rate increase, there will be discounting and insurance effects at work.[11]

The strong discounting effect in the benchmark framework is due to the presence of both habit formation and capital adjustment costs. This is consistent with Santoro and Wei (2011), which finds that the impact of capital income taxes on asset prices is the strongest for this combination of preference and production specifications.[12]

6 Conclusion

In this paper, we employ an asset market approach to analyze the determinants of the marginal welfare cost of capital income taxes. We show that the marginal welfare cost can be thought of as the normalized market value of consumption distortions. Such a perspective brings to the forefront the importance of the discount factor and its covariance with consumption distortions in valuing future consumption distortions.

We find that the marginal welfare cost curve is convex in and upward sloping with the tax rate in both stochastic and deterministic environments. Everything held equal, the marginal welfare cost curve in the stochastic environment lies above and is steeper than that in the deterministic case. A decomposition of the marginal welfare cost shows that such features can be explained by three effects of capital income taxes: the distortion, discounting and insurance effects. The upward slope of the marginal welfare cost curve results from the increasingly inefficient allocation of resources as the tax rate

[11] We also investigate the discounting and insurance effects in the case of elastic labor supply. If labor elasticity is not very high in response to technology shocks (as indicated by the empirical literature), the self-insurance through adjustment of labor supply would be limited, and investment would remain the main vehicle to smooth consumption. As a result, the discounting effect still dominates, especially when tax rates are high.

[12] We also conduct a sensitivity analysis using different values of σ. We find that the discounting effect strengthens as the degree of aggregate uncertainty increases. Detailed results are available from the authors upon request.

increases, which represents the distortion effect common to deterministic and stochastic environments. The widening gap between the marginal welfare cost curves between the stochastic and deterministic cases is attributed to the dominance of the discounting effect over the insurance effect. The magnitudes of these two effects are determined, respectively, by the decrease in the discount rate and the increasingly negative covariance between the stochastic discount factor and consumption distortions as the tax rate increases. We find that the welfare cost of capital income taxes can be under-estimated if variations in risky discount rates are ignored, especially when tax rates are high.

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Appendix

A. Detailed Specifications of the Stylized Model

The representative household maximizes an expected life-time utility function subject to a standard budget constraint.

$$\max_{B_{t+1},C_t} E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - bC_{t-1})^{1-\gamma}}{1-\gamma}$$

subject to:

$$C_t + I_t + B_{t+1}V_t^B = (1-\tau) R_t K_t + W_t L_t + B_t + \chi_t.$$

Here β is the subjective time preference and C_t is real consumption at time t. The coefficient γ measures the curvature of the representative household's utility function with respect to its argument $C_t - bC_{t-1}$. When $b > 0$, the utility function allows for habit persistence based on the household's own consumption in the previous period.

In the budget constraint, I_t represents investment. B_t represents private risk-free bonds held from period $t-1$ to t. V_t^B is the price of the risk-free bond, and τ is the proportional capital income tax rate on capital income $R_t K_t$. The variable $W_t L_t$ represents labor compensation. The variable χ_t is a lump-sum transfer of all the tax revenues from the government.

We assume that there exists a representative firm owned by households. The representative firm chooses optimal capital and labor inputs given the wage and rental rate at each period. In equilibrium, the real wage and the gross rental rate are given, respectively, by

$$\begin{aligned} W_t &= (1-\alpha) \frac{Y_t}{L_t}. \\ R_t &= \alpha \frac{Y_t}{K_t}. \end{aligned}$$

B. Computing the Marginal Welfare Cost at τ

We assume that the economy starts from the deterministic steady state consistent with a given tax rate τ. Based on our stylized model, the initial state can be characterized by $\{K_{ss}^{\tau}, C_{ss}^{\tau}, Z_{ss}\}$, which represent, respectively, the steady-state values of capital stock and consumption given τ and the steady-state level of aggregate productivity.

Now we consider two scenarios starting from the initial state. In the first scenario, the marginal tax rate remains unchanged at τ. We denote the optimal decisions under such a scenario as

$$K_{t+1} = g^{\tau,0}\left(K_t, C_{t-1}, Z_t\right), C_t = h^{\tau,0}\left(K_t, C_{t-1}, Z_t\right)$$

where $g^{\tau,0}(\bullet)$ and $h^{\tau,0}(\bullet)$ represent respectively the optimal decision rules for next period capital stock and current consumption. The first superscript, τ, stresses the dependence of those decision rules on the given tax rate, and the second superscript, 0, indicates that the tax rate remains unchanged at τ. A dynamic path under this scenario is characterized by the initial state, $\{K_{ss}^{\tau}, C_{ss}^{\tau}, Z_{ss}\}$, and the optimal decision rules described above.

Now consider the alternative scenario, under which there is a marginal shift in the capital income tax rate from τ to $\tau + \varepsilon$ in the initial period. Under this scenario, the transition path to the new long-run equilibrium is characterized by the initial state $\{K_{ss}^{\tau}, C_{ss}^{\tau}, Z_{ss}\}$ and the optimal decision rules dictated by $g^{\tau,\varepsilon}(K_t, C_{t-1}, Z_t)$ and $h^{\tau,\varepsilon}(K_t, C_{t-1}, Z_t)$ for all t. Here the first superscript τ stresses that the initial state of the economy is consistent with the tax rate τ, and the second subscript stresses that the optimal decision rules are derived under the marginally higher tax rate, $\tau + \varepsilon$.

We assume the same aggregate technology process $\{Z_t\}_{t=0}^{\infty}$ with or without the marginal shift in the tax rate. In computing the marginal welfare cost, we approximate ε with a small positive value of 0.01. Consequently, the consumption distortion at period t, $\frac{\partial\left(-C_t^{\tau,\varepsilon}\right)}{\partial\varepsilon}$, is approximated by $\frac{C_t^{\tau,0} - C_t^{\tau,0.01}}{0.01}$ for all τ. Given the approximated values of consumption distortions, it is straightforward to use Monte-Carlo methods to compute the marginal welfare cost using equation (5).

Table 1: Benchmark Parameterization

Category	*Parameter Definition*	*Notation*	*Value*
Production	trend growth rate	g	0.005
	depreciation rate	δ	0.03
	share of capital income	α	0.36
Preferences	subjective time preference	$\beta\left(1+g\right)^{1-\gamma}$	0.992
	curvature of utility function	γ	5
Technology Process	persistence	ρ	0.995
	standard deviation	σ	0.016

Figure 1: The Marginal Welfare Cost Curves

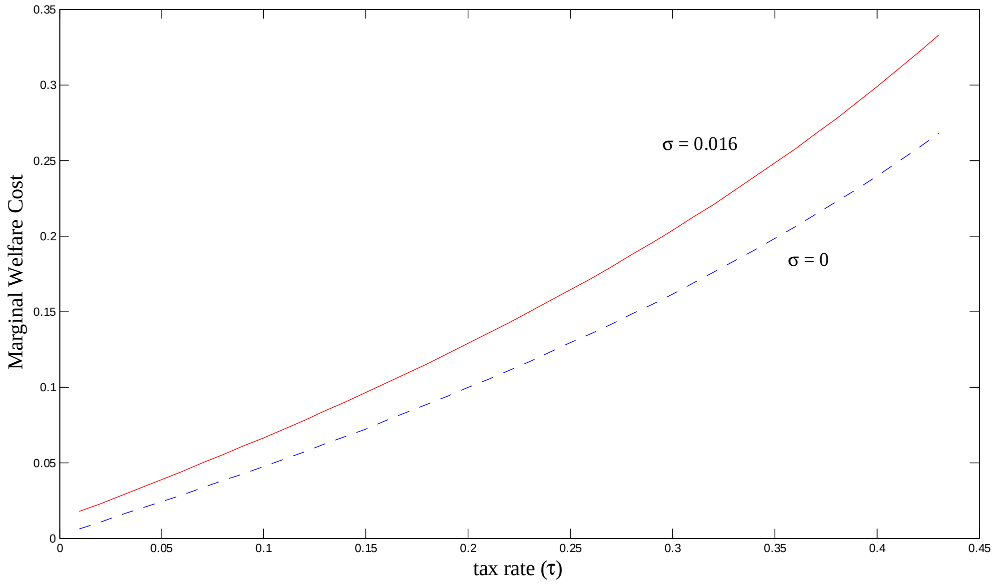

Figure 2: Paths of Consumption Distortions ($\tau = 0.36$)

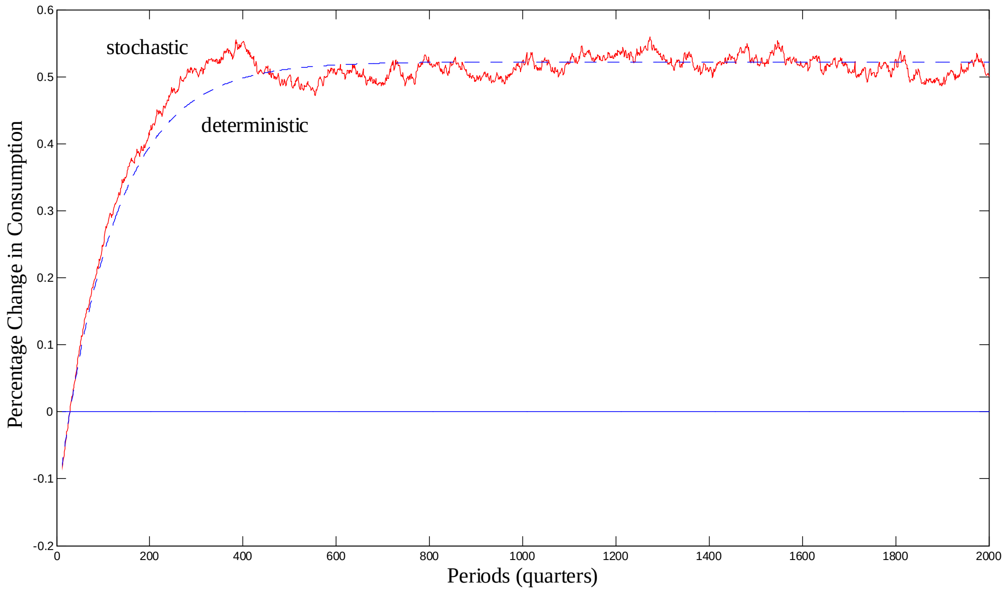

Figure 3: Amplification and Insurance Effects ($\tau = 0.36$)

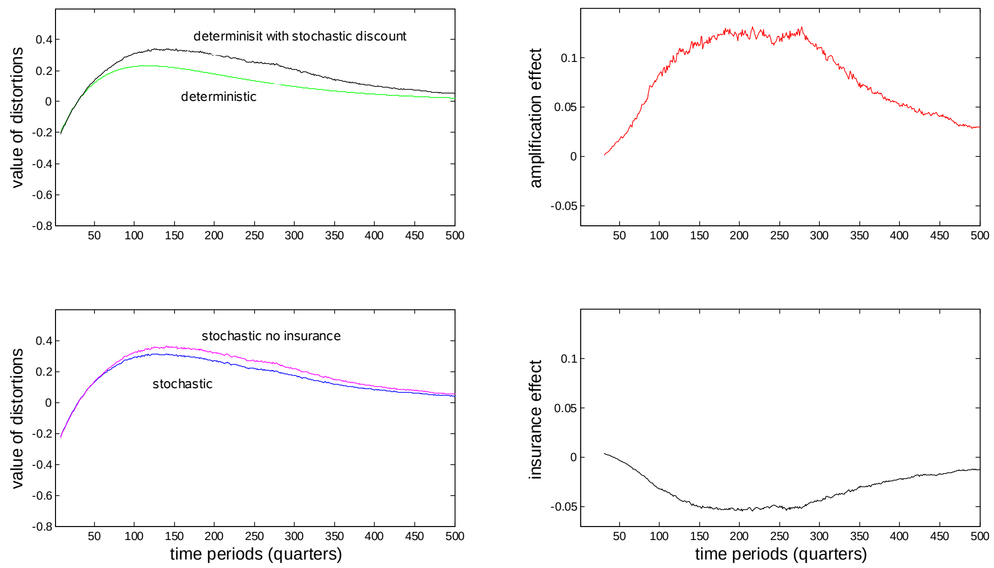

Figure 4: Discounting and Insurance Effects for Different Tax Rates

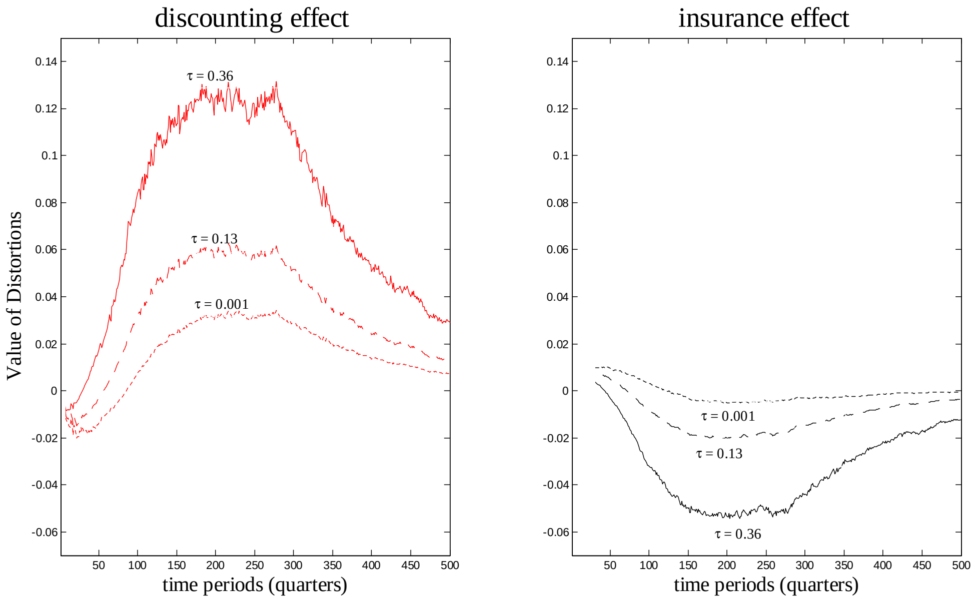

Figure 5: The Decomposition of the Marginal Welfare Cost Curves

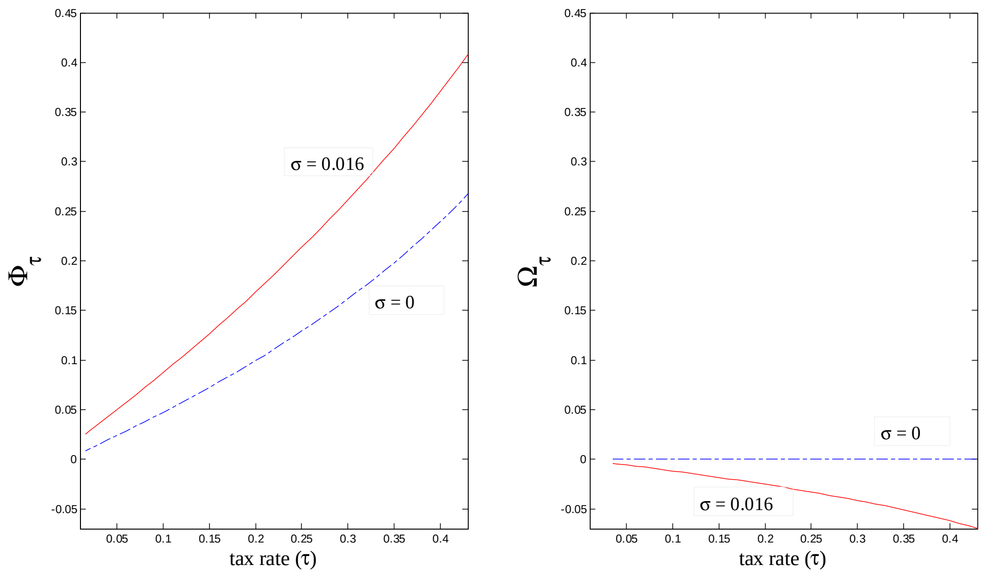

Figure 6: Prices of Perpetual Bonds and Discount Rates for Different Tax Rates

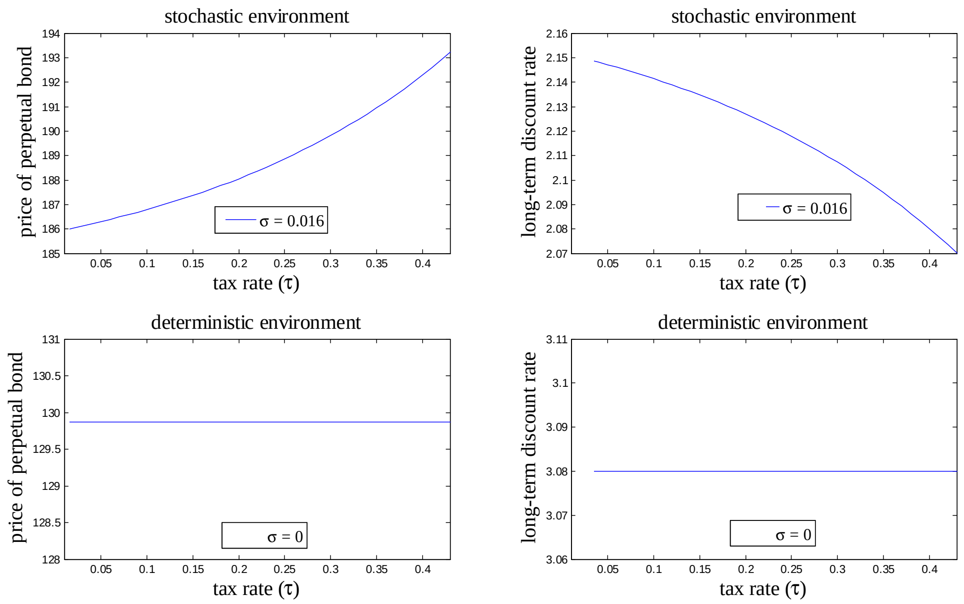

Figure 7: The Marginal Welfare Cost Curves ($b = 0$, $\eta = 0$)

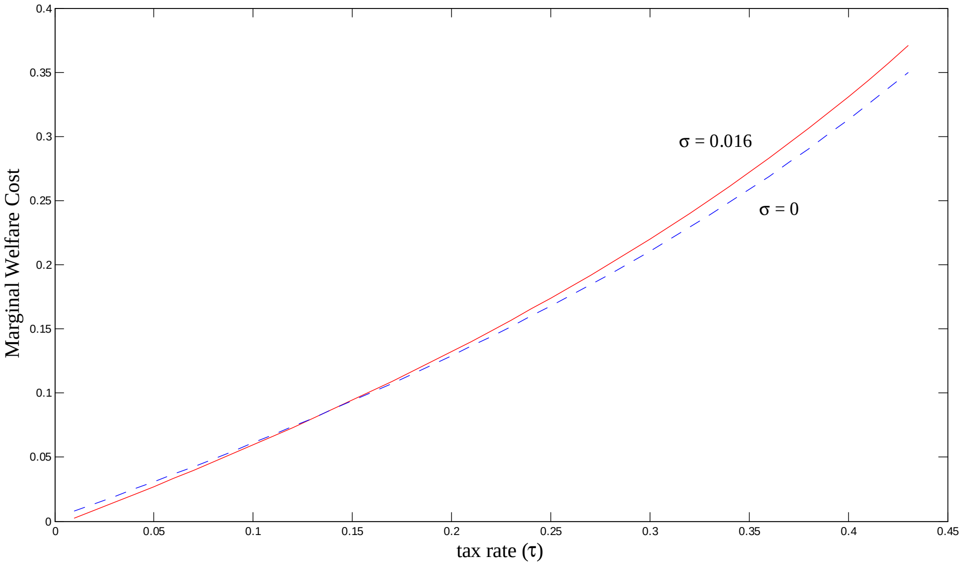

Figure 8: The Decomposition of the Marginal Welfare Cost Curves ($b = 0$, $\eta = 0$)