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ALTERNATIVE METHODS FOR PROJECTING EQUITY RETURNS: IMPLICATIONS FOR EVALUATING SOCIAL SECURITY REFORM PROPOSALS

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Abstract

The effect upon future Social Security benefits resulting from the introduction of individual accounts depends on both the potential risks and returns of private equities, yet the historical evidence about determinants of stock market risks and returns is mixed. In particular, correlations between equity returns and market fundamentals (such as the dividend price ratio) are weak at annual frequencies, which has led some to conclude that a random returns (fixed mean and variance) model is the preferred specification for simulating the future path of equity returns. Although choosing between random returns model and models based on market fundamentals do equally well for explaining variation of equity returns in the short run, the distinction is important when projecting equity returns over longer periods, as shown here in the context of a Monte Carlo simulation of Social Security reform. If equity returns are even weakly correlated with market fundamentals then (1) the expected future average return may be a function of the starting values for market fundamentals, and (2) the overall range of cumulative outcomes is more narrow than the random returns model suggests.

1. Introduction[1]

How would introducing individual accounts affect outcomes for future Social Security beneficiaries? Individual accounts with investment in corporate equities could raise expected benefits, but the welfare cost of increased risk could offset those gains (Congressional Budget Office (CBO), 2003; Feldstein and Rangulova, 2001; MaCurdy and Shoven, 2001). When attempting to quantify the trade-off between risk and return in a policy simulation context, one key modeling decision is how the probability distribution of future equity returns should be tied to underlying economic fundamentals. In particular, a mean-reverting process which ties equity returns to the underlying growth of capital income can lead to policy conclusions which differ significantly from a textbook random-returns specification in which the probability distribution of annual returns is fixed.

The choice between random returns and mean reversion in equity prices matters for two reasons. First, the starting values of financial valuation ratios will affect *average* equity returns if market fundamentals play a role, because starting with an overvalued stock market implies below average future returns while the market is in transition to its long run equilibrium (Campbell and Shiller, 1998; Diamond, 1999). Second, the specification matters for the *range* of cumulative stock returns, because the extent of mean reversion (holding annual variation in stock returns constant) determines the variance of longer-run cumulative outcomes. Thus, if one interprets the historical data as supporting mean reversion, the implication is that while the stock market undergoes a correction expected returns will be lower than they were historically, but the range of

[1]The authors would like to thank Amy Harris, Josh O'Harra, Steve Lieberman, Kevin Perese, and Michael Simpson of CBO's Long-Term Modeling Group within CBO's Health and Human Resources Division for their support and helpful comments.

the long-run cumulative equity returns will be more narrow than that suggested by a random returns model.

This paper focuses on projecting equity returns in a model developed for analyzing Social Security policy in a Monte Carlo setting. The underlying macro model generates the stable-economy of the Social Security Administration (SSA) baseline in which overall growth and the growth of capital income are stationary. Stationary growth is important for modeling equity returns because, in a mean reversion framework, the value of equities can be directly related to market fundamentals (Campbell, Lo, and MacKinlay, 1997). For example, in the case of simple autocorrelated equity returns, this principle reduces to a linear relationship in which future equity returns are a linear function of the log dividend-price ratio.

Historical data yield mixed results about the annual correlation between equity returns and dividend-price ratios. Indeed, depending on the time period, the textbook random returns process for annual equity returns fits the data as well as specifications involving the log dividend-price ratio (Goyal and Welch, 2002). The expected relationship between equity returns and the log dividend-price ratio holds when the equation is estimated over the period 1926 to 1990, and the coefficient on the log dividend-price ratio is significant. However, after adding data through the end of the 1990s when equity returns soared and dividend-price ratios plummeted, the statistical relationship disappears. Although it is not clear how the current stock market reversal will end and how the relationship will change with more data, adding the years 2000 to 2002 weakly reestablishes the statistical significance of the coefficient.

The historical data provide at best mixed guidance about the correlation between stock returns and underlying economic fundamentals; therefore the choice of the equity model must be based upon the credibility of the implications of each model. A random returns model with a fixed expected equity return produces results that are independent of economic outcomes and ignores the fact that the current indicators of stock market returns predict that the stock market is still overvalued. Because of this independence, the random returns model may produce values for the size of the stock market and the size of the economy that are not plausible. For instance, Diamond (1999) shows that using an equity return rate assumption of 7 percent and a 1.5 percent rate of real GDP growth the value of the stock market after 75 years will grow to be 20 times the size of the economy. In terms of simulating the effect of equity investments in a policy setting this choice also matters since the random returns model implies a much larger variation of equity returns over a 20 to 35 year time horizon. If mean reversion is the appropriate specification, conclusions about future risks and returns from a proposed reform may be biased.[2]

The strategy here is to quantify the extent to which the choice between random returns and mean reversion actually matters. The simulation model is a simple growth-accounting framework solved using Monte Carlo draws for crucial economic and demographic inputs. Policy experiments are conducted by solving the model 1,000 times and computing the outcomes for hypothetical example workers. Several interesting intermediate outputs are shown, such as the probability distributions for cumulative equity returns from the start of the simulation, but the focus is on the distribution of benefit outcomes for the individual workers before and after reform.

Preliminary results from a budget neutral policy simulation where 2 percent of payroll

[2]For example, in a recent analysis of Social Security individual accounts, Feldstein and Rangulova (2001) use the random returns model to project the probability distribution of future equity returns.

taxes are invested in the stock market starting in 2004 suggest that the specification for equity returns does in fact have a significant impact on the probability distribution of outcomes. If equities follow a mean reverting process then a below average expected gain (indeed, an expected loss in the near term) would result because, based on historical averages, the market is currently highly valued at the end of 2002. However, the variability of the gains and the probability of obtaining a loss under mean reversion are smaller, which suggests that potential losses under an equity-based investment plan would not be as large as the random returns model suggests.

2. Economic Fundamentals and Stock Returns

The focus of this section is on developing an empirical specification for the relationship between economic fundamentals (particularly the growth of capital income) and equity returns in a policy simulation model with Monte Carlo draws for productivity and other growth determinants. Starting with a present value relationship between the dividend-price ratio and expected future values for dividend growth and equity returns, a linear relationship between current equity returns and the lagged dividend-price ratio is derived. Given parameters for the equity returns equation, the predicted returns are tied to dividend growth in the underlying economic model to generate internally consistent predictions for cumulative returns in a stochastic environment. Implementing this approach involves specifying three dynamic relationships; the lagged dividend-price ratio is used to predict equity returns in any given year, and the actual equity returns and model-generated dividend growth are used to solve for dividend-yields and dividend-price ratios forward through time.

The following notation is used throughout this section to derive the relationships between equity returns, dividend-yields, and dividend-price ratios:

P_t	Real equity price
D_t	Real dividends
R_t	Gross equity return = $(P_t + D_t)/P_{t-1}$
p_t	$\log(P_t)$
d_t	$\log(D_t)$
r_t	$\log(R_t)$
G	the constant rate of dividend growth

A useful starting point is to note that the specification of equity returns in the random returns case is given by:

$$r_t = \mu + \quad_t,$$

where μ is set to the historical average of equity returns and   is normally distributed with mean zero and variance set to the historical variance of annual equity returns.

Campbell, Lo, and MacKinlay (1997) formally propose a model of the relationship between the dividend-price ratio and future equity returns using a present value model. If expected dividend growth is constant then it can be shown that the dividend-price ratio is equal to the expected present value of future equity returns less future dividend growth and can be represented as:

$$d_t \quad p_t = E_t \quad {}_{s=1} \quad {}^{s} (r_{t+i} - \quad d_{t+i}).$$

Thus, a low dividend-price ratio indicates to an investor that dividends must grow to justify the relative high value of the stock market or that future returns must decrease over the future.

Assuming that expected stock returns follow a first-order autoregressive process, $E_t\, [r_{t+1}]$ $= r + x_t$, where $x_t = \quad x_{t-1} + \quad_t$, the log dividend-price ratio can be used to predict the expected stock return, $E_t\, [r_{t+1}]$, since it can be expressed as:

$$d_t - p_t = r + x_t/(1- \quad)$$

The relationship between the log dividend-price ratio and the stock return is then:

$$r_{t+1} = \quad + \quad d_t \quad p_t \quad \eta_t$$

where η_t is a random variable normally distributed with mean zero.

This calculation is only the first step in modeling equity returns, because projecting forward through time involves incrementing the dividend-price ratio, and that in turn depends on the starting point and the subsequent model outcomes for equity returns and dividend growth. The values for the log dividend-price ratio are evolved forward through time using dividend growth generated by the underlying economic model and a simple decomposition of the relationship between the dividend-price ratio, dividend yield, and equity return.[3]

The time-path of the dividend-price ratio is constructed indirectly by first deriving a difference equation for the log dividend yield in terms of current capital income growth and previously obtained values of real equity returns and the log dividend yield. The log dividend yield $(d_t - p_{t-1})$ evolves over time according to the identity:

$$(d_t - p_{t-1}) = (d_{t-1} - p_{t-2}) + \quad d_t - \quad p_{t-1}$$

where the changes in log stock prices (p_{t-1}) can then be expressed as a function of real equity returns and last year's log dividend yield.[4] This substitution leads to the following equation:

$$(d_t - p_{t-1}) = (d_{t-1} - p_{t-2}) + \quad d_t - \log(\exp(r_{t-1}) - \exp(d_{t-1} - p_{t-2}))$$

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[3] The treatment here largely follows Holmer (2002). As described in the next section, the economic model actually generates values for total capital income. Dividends are assumed to grow proportionally with real capital income.

[4] This is derived from the definition of the log equity return,

$$p_t = \ln((P_t/P_{t-1} + D_t/P_{t-1}) - (D_t/P_{t-1})) = \ln(\exp(r_t) - \exp(d_t - p_{t-1}))$$

The log dividend-price ratio can be derived in a similar manner. The difference equation can be expressed as:

$$(d_t - p_t) = (d_{t-1} - p_{t-1}) + \quad d_t - \log(\exp(r_t) - \exp(d_t - p_{t-1}))$$

The next period's equity return can then be calculated using the linear relationship between the equity return and the log dividend-price ratio.

There is an additional modification made to the dividend ratio equations (i.e. the equations describing the time-path of the dividend yield and dividend-price ratio) to ensure that the dynamic model produces an expected long-run equity return that is more consistent with the historical average. To understand how the modifications are derived it is important to understand first how the modifications would be used in a static setting (a Gordon growth model) and then how those changes are used in this dynamic version.

In equilibrium, the difference equations for the dividend ratios are equal to the "Gordon Growth" equation;

$$R = G + \text{initial Dividend Yield (or } D_t/P_{t-1})$$

and therefore the projected long-run equity return will be, as Diamond (1999) shows, low relative to the historical average, due to the low value of the initial dividend yield. For example, without an adjustment to the dividend yield or to the value of the stock market the Gordon growth equation predicts that the future equilibrium rate of return over the next 75 years is between 3 to 4.5 percent.[5] Diamond proposes that the most credible projection would involve increasing the initial dividend yield in 1998 and immediately reducing the value of the stock market by 46

[5]Based on the assumption of future real GDP growth, G, equal to 1.5 percent and the initial adjusted dividend yield equal to 1.5 to 3.0 percent. In 1998, the (initial) dividend yield was 1.3 percent.

percent; thereafter, equity returns can return to the OCACT assumed value of 7 percent. Ten years after a 46 percent reduction of the value of the stock market, a long-run equilibrium stock return of 7 percent with a long-run real GDP growth of 1.5 percent is justified since the initial dividend yield (and constant in all years according to the Gordon growth formula) is equal to 5.5 percent.

Without a similar adjustment of all future dividend yields, the dynamic version of this equity model will result in long-run equity returns that range from 4.5 to 5.5 percent. As a result, an adjustment to dividend yields is made, such that the projected equity returns follow a more plausible time path. Equity returns are immediately low until the stock market is correctly valued and thereafter the long-run equity returns will be consistent with the historical average given the level of economic growth.

The adjustment is made directly in the dividend ratio equations. A dividend ratio adjustment factor equal to 0.006 (for the high mean reversion model) and 0.01 (for the low mean reversion model) is included. The new equations are expressed as follows:

$$(d_t - p_{t-1}) = (d_{t-1} - p_{t-2}) + \quad d_t + dividend\_ratio\_adjustment - \log(\exp(r_{t-1}) - \exp(d_{t-1} - p_{t-2})),$$

$$(d_t - p_t) = (d_{t-1} - p_{t-1}) + \quad d_t + dividend\_ratio\_adjustment - \log(\exp(r_t) - \exp(d_t - p_{t-1}))$$

These values were selected so that when the dynamic version of the Gordon Growth model uses the initial (1926) values of the equity return, dividend yield and dividend price ratio, and the historical values for dividend growth, the average equity returns projected over the historical sample equal the historical average.

The equation for predicting equity returns derived above is intuitive, and captures the straightforward prediction that if dividend growth is a stationary process, a stock market that is relatively overvalued (has a relatively low dividend-price ratio) is likely to produce lower than average returns in the future. This general prediction has passed empirical scrutiny (Campbell and Shiller, 1998) but the exact specification for the relationship is less obvious. Indeed, the next section shows that small changes in the time period used to estimate the relationship between equity returns and the dividend-price ratio–especially with respect to data after 1990–has a big effect.

Before describing the empirical results, it is worth contemplating in advance what the estimates for the coefficients of the equity return equation imply for the distribution of expected returns in a simulation exercise. By construction, the equation will produce a value for the mean equity return (in the sample period) when the lagged dividend-price ratio is set equal to its historical mean. Different values for the slope coefficient effectively determine the extent of mean reversion in a simulation, and thus the range of cumulative returns at various frequencies. A larger slope implies a larger response to deviations of market fundamentals from equilibrium values. Smaller values for the slope coefficient can cause problems with the simulations in the present context, because if there is too little correlation between dividend growth and equity returns, the implied equity price (shown by rearranging the equation for the dividend-price ratio and solving for p_t-p_{t-1}) can actually become negative in simulations where the dividend yield rises, but equity returns, and therefore the change in stock prices, do not increase enough to limit the growth of the dividend yield. Since the slope coefficient is zero in the random returns model, it is certain that this model would violate the condition that the stock market retains a positive value.

3. Annual Correlations Between Equity Returns and the Dividend-Price Ratio

The theoretical relationship between equity returns and market fundamentals suggests an empirical form which is straightforward to implement in practice. This section describes estimates of the parameters when the equation is fitted over various time periods, with a focus on data from the late 1990s and early 2000s. During the unprecedented run-up in stock prices prior to 2001, many researchers observed that stock prices were high relative to a number of indicators of stock market value, such as price-earnings ratio, dividend-price ratios, and gross domestic product (GDP). Thus, the episode of the late 1990s is at odds with the theoretical relationship, because equity returns were high (stock prices continued to grow) even though economic fundamentals suggested the opposite. The last few years have reversed some of that inconsistency, and point estimates of the correlation between equity returns and lagged dividend-price ratios are very sensitive to exactly which years are included.

A useful starting point for presenting the effect of time period on the parameter estimates is considering underlying variation in stock prices. Table 1 shows the estimates of means for equity returns and dividend-price ratios along with the standard deviations for the two series over several time periods. There are two sets of starting periods, 1926 and 1954. The various ending points are 1990 (which is before the beginning of the 1990s stock market boom) then 2000 (which was the end of the boom) and finally 2002 to capture the period over which prices were falling.

The starting year 1926 is based on when stock market data in a consistent time-series are readily available (Ibbotson Associates, 2003). Table 1 shows that the mean return for the entire period jumps dramatically when the 1990s data are added, from a value of 6.52 percent for 1926-

1990 to a value of 7.45 percent for 1926-2000. The standard deviation of returns is nearly three times the average return and not much affected by the time period chosen. The log dividend price ratio falls significantly when the 1990s data are added, and the standard deviation rises dramatically. This result is consistent with the 1990s experience, i.e. rapidly falling dividend yields and soaring stock returns.

Extending the data through 2002 reverses some of the growth in the average equity return and brings the long-run average down to 6.71 percent. Although the long run average for equity returns for the period 1926 to 2002 is still above the mean for 1926 to 1990, it is not clear that the current market downturn is concluded.

The last three rows of Table 1 report equity returns and variability for the period 1954 forward, because it can be argued that the U.S. economy has been on a fundamentally different trajectory since the middle part of the 20th century. In particular, the probability of significant events like the Great Depression is arguably much lower now than it was in the past, and since those sorts of events are associated with an implosion of stock prices, the probability of large negative equity returns has fallen as well. Again, the actual mean returns over the post 1954 period depend significantly on where the end point is set, but in both cases, the variability of returns is diminished by about one-fourth.

Prior estimates of the relationship between equity returns and dividend-price ratios have suggested the expected relationship (Campbell, Lo, and MacKinlay, 1997), though the magnitude and statistical significance of the relationship depends on the frequency at which returns are measured. Also, more recent work has shown that statistical relationships that existed prior to the 1990s market boom have disappeared after 1991 as stock returns averaged 15 percent and dividend yields fell (Goyal and Welch, 2002). Those results are generally confirmed here, though the interpretation is somewhat open.

Table 2 shows estimates of the linear specification derived in the last section using annual data for various time periods. The estimates for the pre-1990s boom show a statistically significant correlation between equity returns and the lagged log dividend-price ratio. The point estimate is 0.173, which suggests that a one percent change in the ratio of dividends to prices implies an expected increase in returns of 0.173 percent. The t-statistic is significant (2.04) but by no means overwhelmingly convincing. Also, the overall fit of the equation (R-squared of 0.06) is very low, and the standard error (0.1992) is not much different than the underlying variability in equity returns over the period (0.2027, Table 1).

The fact that the relationship between returns and market fundamentals is weak becomes even more clear when one extends the period used for estimation to include the 1990s boom and subsequent declines through 2002. The middle row shows that the boom itself completely eliminated any trace of correlation between the lagged dividend-price ratio and current returns, and although the market reversal has restored some of the statistical significance in the coefficient (the last row) the point estimate remains much lower. As noted in the last section, the value of that coefficient determines the extent of mean reversion in equity prices, and plays a significant role when making inferences about the likely range of cumulative returns in the future.[6]

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[6]The discussion here is narrowly focused on one class of models in the behavioral finance literature. For a more general discussion of the sorts of models used to study aggregate stock market yields see the interesting symposium between Malkiel (2003) and Shiller (2003). One alternative to the dividend-price ratio is developed in papers by Lettau and Ludvigson (2001 and 2002), who demonstrate that by using a present value relationship between equity returns and consumption-wealth ratios equity returns are predictable and that consumption-wealth ratios contain separate "business-cycle" variation in addition to the low frequency variation that dividend-price ratios contain. They find that consumption-wealth ratios outperform dividend-price ratios at forecasting equity returns. Their results are also suggestive of the type of activity that was occurring in the stock

4. Projecting Equity Returns in a Monte Carlo Policy Simulation Setting

The goal of analyzing the theoretical and empirical relationship between equity returns and market fundamentals is to project future returns in a policy simulation model. The model used here is designed to project Social Security finances and individual outcomes under current law and various reform scenarios, including policy alternatives that involve investment in private equities. The underlying economic model generates aggregate production and income forward through time, and dividend growth is proxied by the growth rate of capital income.[7] Given starting values for the dividend-price ratio and dividend yields along with stochastically generated values for the growth of capital income, the model solves for equity returns using the equations introduced above.[8]

The underlying economic model is simple and largely based on Solow growth accounting principles. National output is determined by a Cobb-Douglas production function where the annual technology shock to output is set equal to the accumulation of annual total factor productivity growth rates. Labor force is (basically) exogenous relative to population, though the overall ratio of labor force to population depends on trends and the age-sex mix. Labor income is solved for using labor force and standard first-order conditions for wage rates, and capital

market during the 1990's. Log dividend-price ratios were predicting a large reduction in long-term returns and still do. Consumption-wealth ratios predict a far less severe correction.

[7] The model is an extension of the framework used by the Congressional Budget Office to produce ten year forecasts, as described in CBO (2001-B).

[8] Other stochastic models have included non-random projections of equity returns but focused on different economic variables. Anderson, Tuljapurkar, and Lee (1999) project annual equity returns and real annual effective interest rates using a vector auto-regressive model. Holmer (2002) estimates annual equity returns as a unction of annual twenty year interest yields, inflation, unemployment and the dividend yield. It is worth noting that the Technical Panel on Assumptions and Methods (1999) recommended modeling the time-varying expected value of equity returns as a function of the log dividend-price ratio over methods such as these.

income is a residual. One distinctive behavioral feature of the model is private saving; effectively, aggregate saving in the model adjusts to target a long run capital-output ratio. That assumption (along with stationary total factor productivity growth) generates stable output growth and interest rates and thus mimics Social Security Administration baseline economic projections.[9]

The simulation strategy here involves solving the model using Monte Carlo draws for the major demographic and economic input assumptions. In a basic simulation there are three key stochastic demographic inputs (rate of mortality improvement, fertility, and immigration) and six key stochastic economic inputs (inflation, unemployment, the gap between the marginal product of capital and the risk free interest rate, disability incidence and termination, and rate of growth in total factor productivity). The time-series equations range from single equations for inputs like immigration to a VAR for the first three economic inputs and single equations with correlated errors for mortality improvement across 40 age-sex groups.[10]

In simulations involving equity investments the model uses the various specifications for equity returns introduced above. The simplest version is a pure random returns model, with fixed mean and variance. The random returns simulations here are based on data from 1926 to 1990, so the mean and the standard deviation over this period are equal to 6.5 percent and 20.3 percent respectively. These values should generate a conservative and realistic distribution of

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[9]The focus on replicating the Social Security Administration stable baseline is consistent with taking the historical patterns of interest rates and equity returns as given, and investigating how different interpretations of the same historical data affect one's conclusions about the risk and return of future equity investments. In that sense the goal here is very different from papers like Abel (2001), which seek to first explain the existence of the equity premium and then use that explanation to predict what will happen to average equity returns and interest rates when Social Security equity investments are introduced.

[10]For a detailed description of the time series equations used in the Monte Carlo simulations, see CBO (2001-A). The only equation not described in CBO (2001-A) is for total factor productivity growth, which, based on historical data, is estimated to be a simple white noise process with a standard deviation of 2.02 percent.

cumulative equity returns and will also be directly comparable to the high mean reversion model that uses the same sample to estimate the coefficient on the dividend-price ratio.[11]

Implementing a mean reversion model in the Monte Carlo simulations requires choosing values for the coefficients and using the 2002 values of the dividend yield and dividend-price ratio, which are equal to 1.18 percent and 1.36 percent, respectively, based on data from Ibbotson (2003). As noted above, the problem with reading the historical tea leaves is that it is not obvious which slice of history will most resemble the future, so the solution is to test the various combinations of coefficients in a sensitivity analysis exercise.

The last component of the economic model is the calculation of tax and benefit outcomes for example workers. All of the results shown here are for a hypothetical average earner, with an age-earnings profile derived from historical Social Security earnings data.[12] The model calculates standard benefits and the effects of reform on benefits under any combination of the stochastic draws for economic and demographic inputs. The various equity return models are applied to example worker outcomes, yielding estimates of median returns and the variability of those returns over time.

5. Probability Distributions for Cumulative Equity Returns

The sensitivity analysis strategy that emerges from the foregoing considerations suggests

[11]One other approach that was implemented but ultimately not reported involved adding a constant term to the random returns model to capture systematic changes in the projected marginal product of capital. Since the Social Security Administration baseline adopted for this exercise does show real interest rates that are slightly higher than the historical average, the expected equity return is shifted up slightly as the marginal product of capital rises. In practice, however, because of the combination of the stable baseline and large variance for annual equity returns, this approach gave results that are very similar to the random returns model.

[12]For a description of the hypothetical worker earnings profile, see SSA (2001).

an examination of three basic models: first, no mean reversion (a random returns model); second, a "low" mean reversion model (where the coefficient, is set to 0.096, the estimate from the 1926-2002 sample); and, third, a "high" mean reversion model (where the coefficient, is set to 0.173, the estimate from the 1926-1990 sample).[13]

Figure 1 confirms the first implication of choosing between random returns and mean reversion models. When starting with actual dividend-price ratios (which, at the end of 2002, are still below historical averages) the median expected equity return outcomes across the 1,000 simulations are predictably lower during the first twelve years of the projection when the stock market is transitioning to its long run equilibrium; after these first twelve years equity returns gradually adjust to its long-run value.[14] The rate of transition is predictably faster for the high mean reversion model than the low mean reversion model, but at the same time, the high mean reversion median expected returns in the first few years are lower than the low mean reversion returns. In addition, Figure 1 also shows that the mean reversion models project nearly the same long-run expected annual return slightly less than 6.5 percent in year 2077. This latter result is due to a combination of assumptions–the upward adjustment of the dividend ratios and the

[13]As mentioned before, as the slope coefficient decreases the equity model becomes more like the random returns model; therefore the likelihood that the value of the stock market becomes negative increases. As a result, the low mean reversion model is prone to this problem and a related problem due to weak responses to changes in the dividend yield. When the model generates a large number of negative random equity shocks, η_t, the dividend ratio equations will produce dividend yields and dividend price ratios that increase since the change in price term, p_t, will be below its expected value. In this case, future equity returns will tend to be consecutively high while the dividend ratios remain large. This problem is eliminated by limiting the size of the dividend yield to 13.5 percent–roughly 3 percent higher than the historical maximum of the dividend price ratio. This value was chosen to reduce the number of times this correction is used, yet at the same time it ensures that low mean reversion model does not produce equity returns that are consistently large. When the dividend yield recovers, any differences that were taken out of the dividend yield when it was capped at 13.5 percent are added back to subsequent dividend yields.

[14]In Figure 1, the median expected equity return is equal to + median$(d_t - p_t)$

slowdown in capital income growth inferred from the Social Security Administration baseline.[15]

The median annual expected equity returns shown in Figures 1 tell half the story, and the other half is shown by comparing the entire probability distributions for cumulative equity returns in Figures 2 through 4. The three figures show various percentiles (between the 1st and 99th) of the cumulative equity returns distribution for each set of simulations mentioned above. The differences in cumulative variance come through clearly in the figures: the overall range of cumulative outcomes is remarkably lower in the mean reversion models than it is in the random returns model, and the range of the high mean reversion model is more narrow than low mean reversion.

The slices of these cumulative return distributions for 2003 to 2022 and from 2003 to 2077, reported in Table 3, also confirm the differences in cumulative variability, and reinforce the differences in long run median outcomes (Figures 1) as well. Indeed, the median outcomes for the mean reversion models are well below the 6.5 percent for the random returns model for the first twenty years (this shows up as a slight "U" shape in the cumulative distributions in Figures 3 and 4) but much closer over the entire 75 year period. The range of returns is more narrow in both of the mean reversion models, suggesting lower probabilities of extreme outcomes than one would get from a random returns model.

The underlying economy in these simulations is stable, and the Monte Carlo framework generates variability in productivity, wage growth, and other aggregates which match history. The natural question is 'How well do the various models reflect the historical variation in equity

[15]There is no comparable NIPA series back to 1926, but data from 1952 through 2002 show slightly higher growth of capital income than what the model derives based on the Social Security Administration stable baseline assumptions.

yields at various holding-period frequencies?' Figures 5 and 6 answer this question in two parts. Figure 5 shows that all three models generate similar variability in cumulative returns for holding periods of one to five years. The variances from the models should match by construction at an annual frequency, and the fact that the variances are similar through five years confirms that mean reversion models simulate the variation for annual returns and for other short-run holding periods just as well as the random returns model.

However, Figure 6 shows that when looking at holding periods of five to thirty-five years, the gap between the variances of cumulative returns grows and becomes quite noticeable, which is just another way of stating that the range for cumulative returns is more narrow in the mean reversion models.[16] Of course, there are not many 35 year periods in history to use for computing an empirical estimate of the variance, so at this stage it is only possible to recount out how the models differ mechanically in the long run. The observations about variances simply reflect the fact that large, fundamental differences between capital income growth and equity returns lead to a reversal in the expected returns in subsequent periods under the mean reversion models. Thus, one can describe the mean reversion models as forcing long run consistency between capital income and equity returns, while at the same time generating short run variability that is consistent with observed annual and short-run variation.

6. Implications for Social Security Individual Accounts

The differences in expected returns and the range of cumulative returns reflect the

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[16]The difference between figures 5 and 6 may also explain why MaCurdy and Shoven (2001) found that varying the holding period did not matter for their bootstrap approach to measuring variability in equity returns. They sampled historical outcomes through holding periods up to five years, and the mean reversion models estimated here suggest the real differences show up for longer periods.

expected outcomes given the nature of the differences between random returns and mean reversion. The last question to address in this sensitivity analysis is whether or not the differences in equity returns make any real differences when the various models are applied to a specific policy proposal. This section shows that the estimated risk and return characteristics of a simple individual account experiment are significantly affected by the choice between random returns and mean reversion.

Tables 4 and 5 show the effects of choosing between the various models of equity returns when evaluating a prototypical Social Security individual account proposal. Table 4 shows the payment value of the estimated annuities from the individual account, and Table 5 shows the net change in benefits, for example workers turning 65 in 2012, 2032, 2052, and 2072, under each of the three models presented in the last section. In each simulation, an individual account with a 2 percent carve-out, 100 percent participation rate, and the SSA (2002) default portfolio is implemented. The proposal is basically cost neutral to the government, because participants implicitly "borrow" the 2 percent at the trust fund interest rate in order to fund the accounts, paying back the loan through a reduction in benefits.[17]

Table 4 shows the value of the individual account annuity for the 10th, 50th , and 90th percentile annuities for four cohorts of retirees under the 2 percent carve out. Annuities are in real dollars, but they rise over time because of real wage growth and because future cohorts are exposed to the plan for a longer period. The table repeats the theme of the last section–the random returns model shows a slightly larger median annuity value (between 1 to 2 percent

[17] A description of the various details (carve out, participation, benefit offset, and default portfolio) for the individual account experiments can be found in the President's Commission to Strengthen Social Security (CSSS) 2001 report. The SSA (2002) default portfolio is composed of 50 percent equities, 30 percent corporate bonds, and 20 percent treasury bonds.

greater) but contains a larger amount of variation. For example, in 2072, the random returns model has a range between the 10^{th} and 90^{th} percentile annuity outcomes that is 17 percent greater compared to low mean reversion model and 31 percent greater than the high mean reversion model.

Table 5 shows what is probably more pertinent from the perspective of participants: the change in the median benefits and the probability of attaining a loss due to the proposed reform. As would be expected the probability of attaining a loss is high in the first years under the mean reversion models. For example, in 2012 the probability of loss is between 63 to 69 percent according to the mean reversion models but only 36 according to the random returns model. Since stock returns make a more rapid recovery in the high mean reversion model, by 2032, the probability of attaining a loss is always less than the random returns model. By 2072, the difference in the probabilities of loss is quite large (8 percent for the high mean reversion model versus 20 percent for the random returns model). The results for the low mean reversion model are similar in nature but less dramatic. The probability of a loss is not less than the returns model until 2052, and the only major difference only occurs in 2072 where the probability of a loss is only 13 percent compared to the 20 percent predicted probability under the random returns model.

In all cases, the median gains from Individual Accounts are well below the annuities themselves, because participants have to give up some basic benefits in order to get the annuity. While the median gain for the mean reversion models are slightly less than the random returns model–in 2072 the median gains for the random returns and the low and high mean reversion models are $2,475, $2,226, and $2,370 respectively--the difference in variability is more pronounced. Why? Because these measures capture the difference in overall equity return variability seen in Table 4, but also reflect the correlation between standard benefit determinants and equity returns. In particular, a high productivity economy is good for standard benefits (through the current law formula) and good for the stock market under a mean reversion model.

7. Conclusion

In this study we develop a stochastic and dynamic equity model that is analogous to the static Gordon growth model used by Diamond (1999). The mean reversion models are favored as the model of choice since they allow the value of the stock market to be consistent with economic fundamentals in the long-run; in the random returns model, economic and stock market outcomes are independent which can lead to conclusions about the economy that are not plausible. In addition, this stochastic and dynamic version of the Gordon growth model also shows that the amount of variation of equity returns over longer holding periods is significantly less compared to the random returns model. For example, the mean reversion models produces a variation of average equity returns over a 35 year period that is half as much as the random returns model.

This difference has a dramatic impact on the evaluation of equity investment in a policy simulation. Using a simple budget neutral policy simulation where 2 percent of payroll taxes are invested in the stock market starting in 2004, it is shown that even with the stock market correction the median gains to investments in Individual Accounts are similar in the long-run for each equity model, but when projecting equity returns that are mean-reverting the investments in individual accounts have a much higher probability of attaining a gain compared to the random returns model. In summary, this sensitivity analysis shows that when long-run economic and equity outcomes are linked, then the perception of equity return variation is much lower compared to the textbook economic point of view.

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Table 1
Historical Equity Returns and Log Dividend Price Ratios
Means and Standard Deviations

	Equity Returns		Log Dividend Price Ratio	
Time Period	**Mean**	**Standard Deviation**	**Mean**	**Standard Deviation**
1926 to 1990	6.52	20.27	-3.14	0.30
1926 to 2000	7.45	19.59	-3.24	0.41
1926 to 2002	6.71	19.88	-3.26	0.43
1954 to 1990	6.54	17.39	-3.30	0.20
1954 to 2000	8.01	16.79	-3.43	0.36
1954 to 2002	6.84	17.47	-3.46	0.38

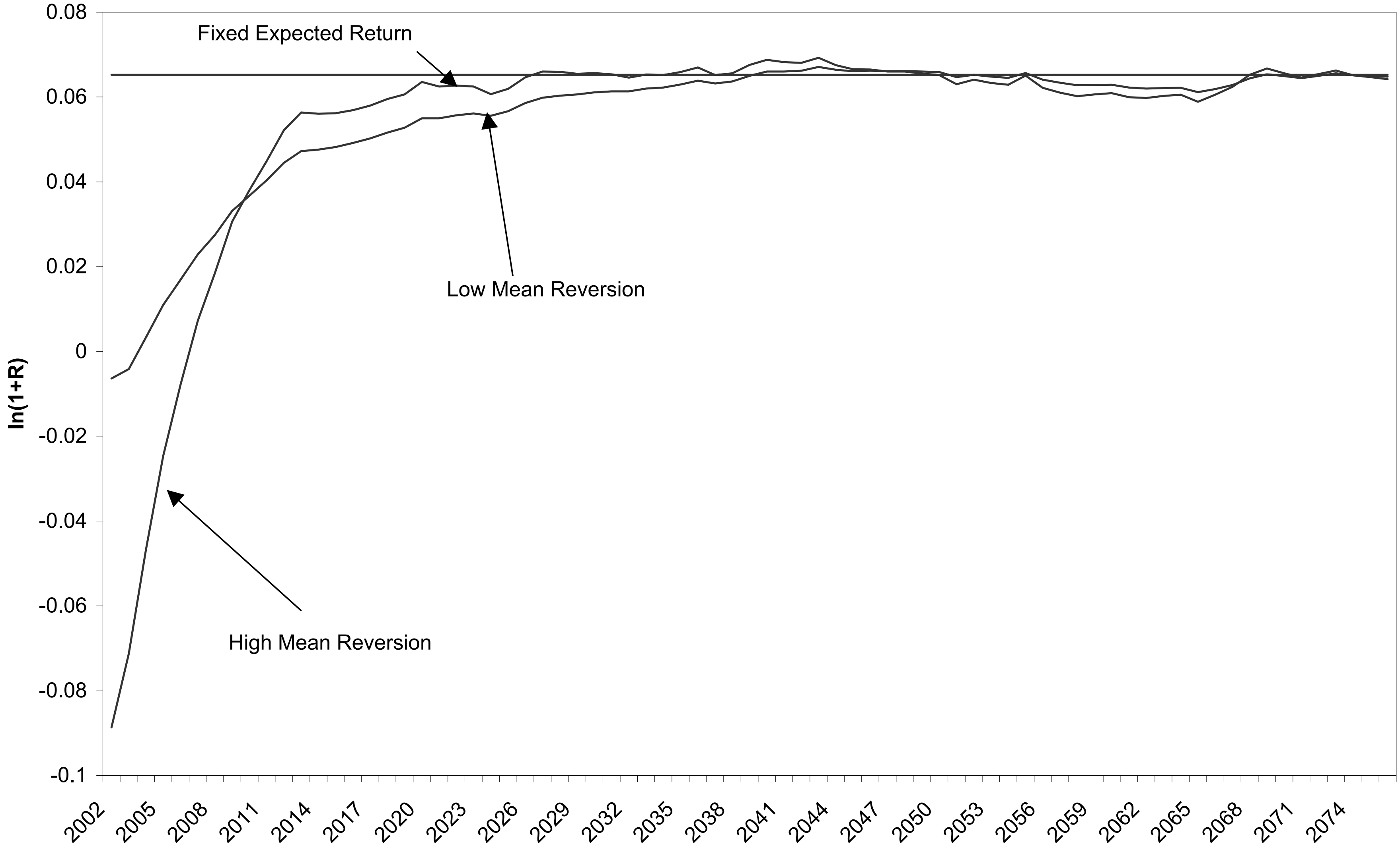
Source: Ibbotson Associates (2003)

Table 2
Regressions of Equity Returns on the Lagged Log Dividend-Price Ratio

Time Period	Regression Intercept	Regression Slope	Standard Error	R^2
1926 to 1990	0.605	0.173	0.1992	0.0628
	(2.274)	(2.035)		
1926 to 2000	0.275	0.062	0.1971	0.0149
	(1.418)	(1.033)		
1926 to 2001	0.317	0.076	0.1963	0.0244
	(1.732)	(1.357)		
1926 to 2002	0.38	0.096	0.1972	0.0414
	(2.147)	(1.778)		

Note: Estimates based on data from Ibbotson Associates (2003). T-statistics are in parenthesis.

Figure 1. Expected Log Equity Returns



Note: The results presented in Figures 1 through 6 are based upon the 75-year projections of the CBOLT model.

Figure 2: Cumulative Equity Returns, Random Returns

Expected Annual Return = 6.5%

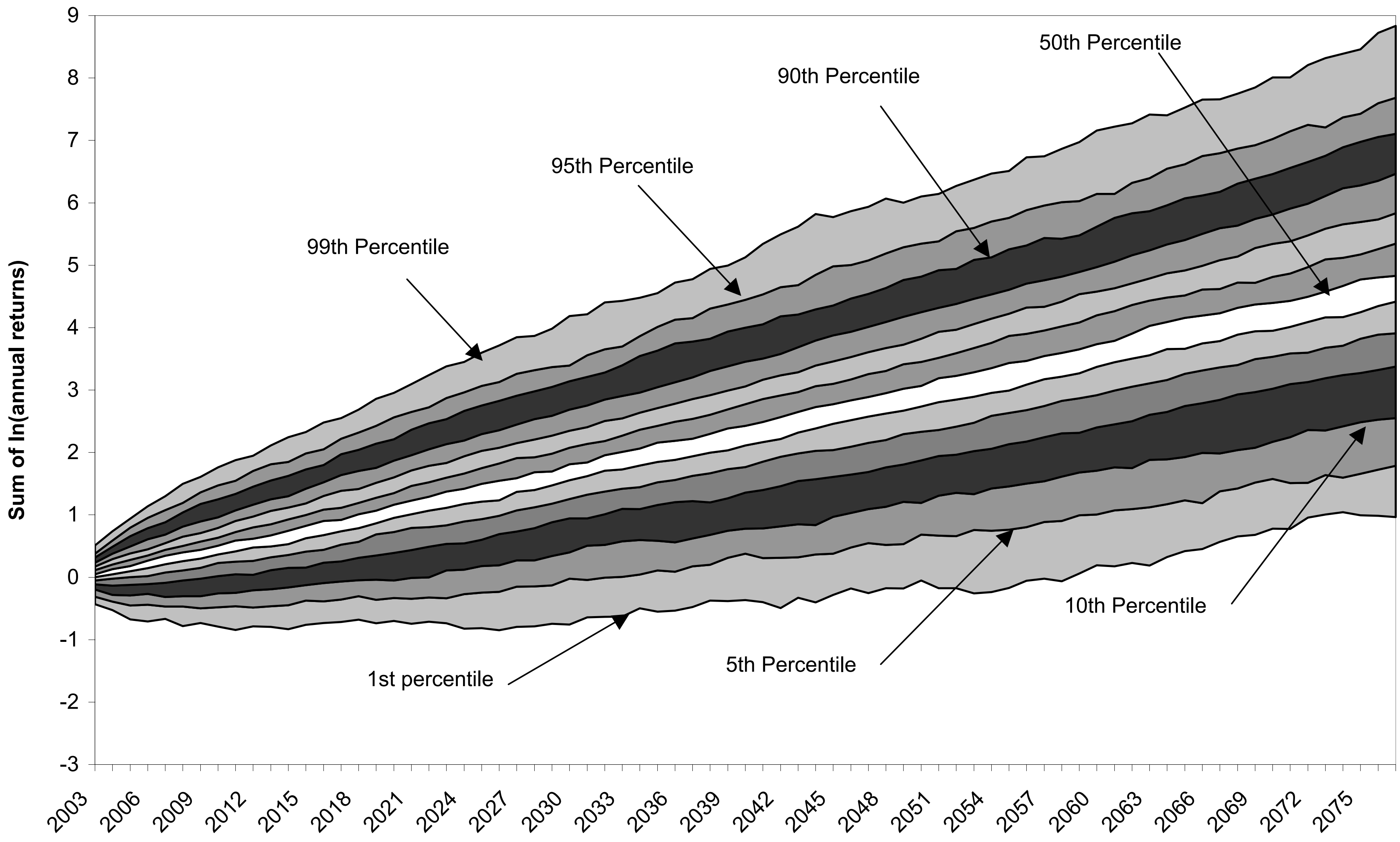


Figure 3: Cumulative Equity Returns, Low Mean Reversion Model
(beta = 0.096)

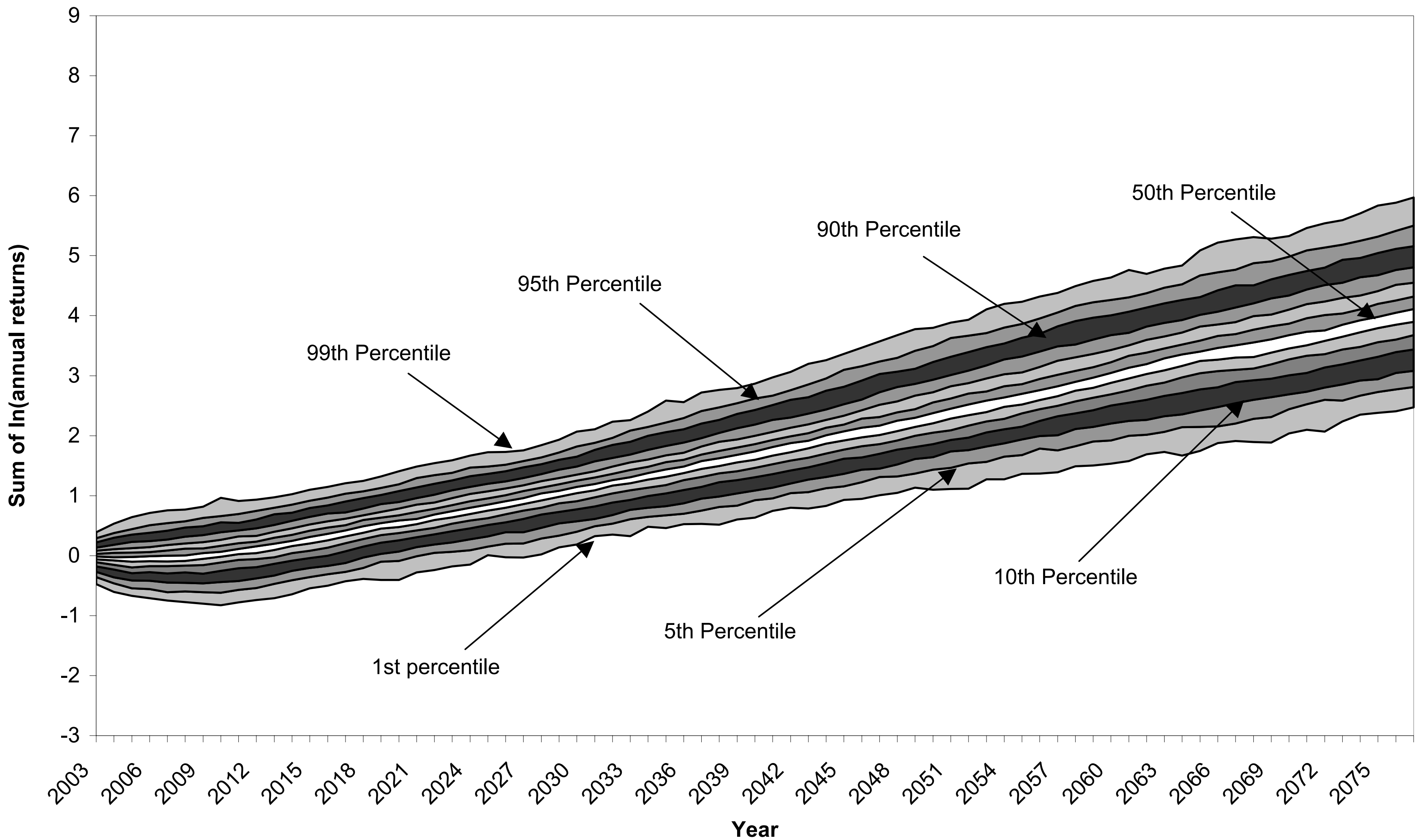


Figure 4: Cumulative Equity Returns, High Mean Reversion Model
(beta = 0.173)

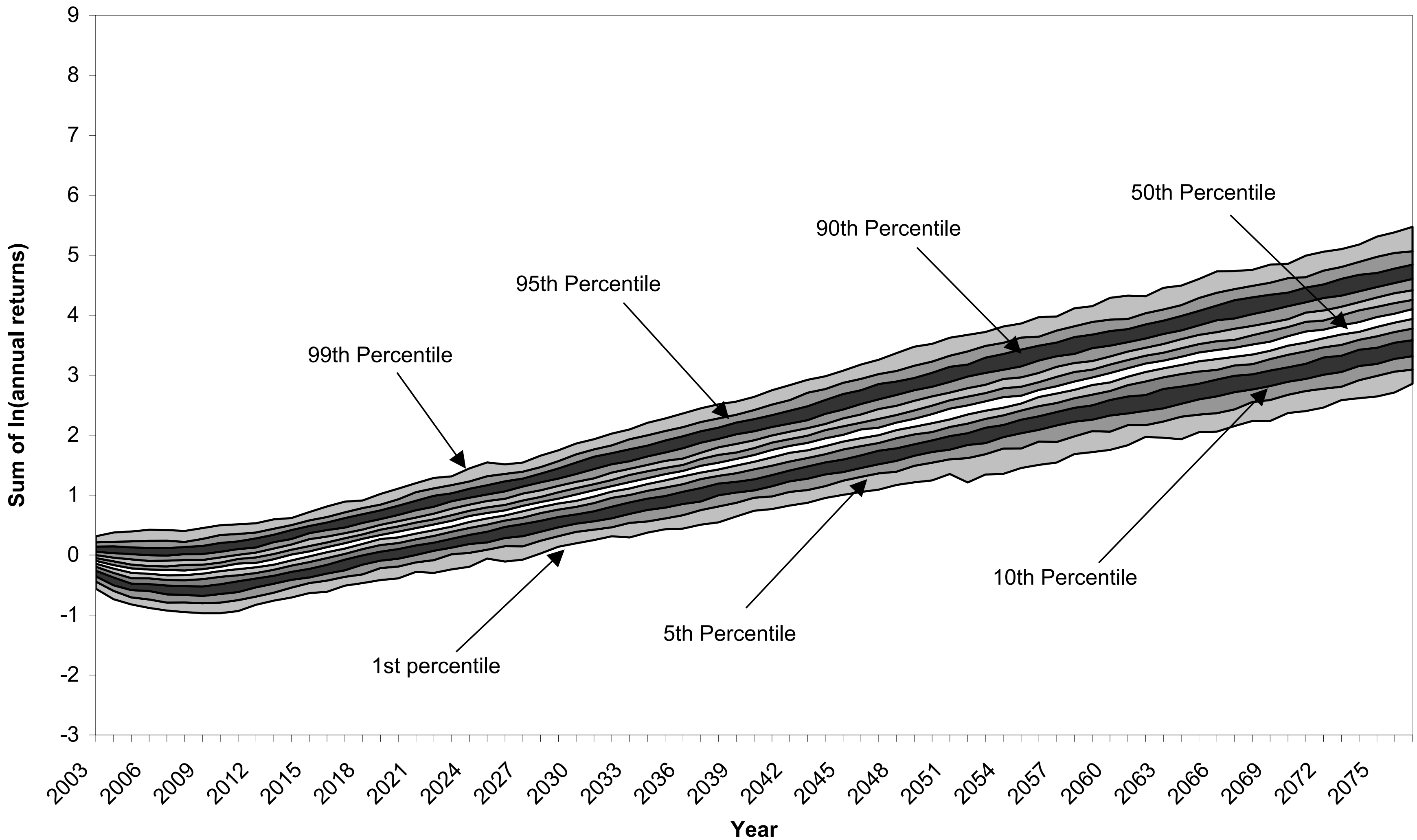


Table 3
20 Year Average Equity Returns from years 2003 to 2022

	Percentile of 20 Year Average Return(%)				
	1st	**10th**	**50th**	**90th**	**99th**
White Noise	-3.55	-0.01	6.46	12.34	16.21
Mean Reversion Models					
Low Mean Reversion	-1.20	0.94	3.42	5.99	7.71
High Mean Reversion	-1.48	0.36	2.55	4.94	6.43

75 Year Average Equity Returns from years 2003 to 2077

	Percentile of 75 Year Average Return(%)				
	1st	**10th**	**50th**	**90th**	**99th**
White Noise	1.29	3.40	6.44	9.47	11.78
Mean Reversion Models					
Low Mean Reversion	3.30	4.11	5.48	6.87	7.96
High Mean Reversion	3.81	4.42	5.47	6.46	7.30

Note: The results presented in Tables 3 through 5 are based upon the 75-year projections of the CBOLT model.

Figure 5. Variance of Mean Equity Returns, Years 1 through 5

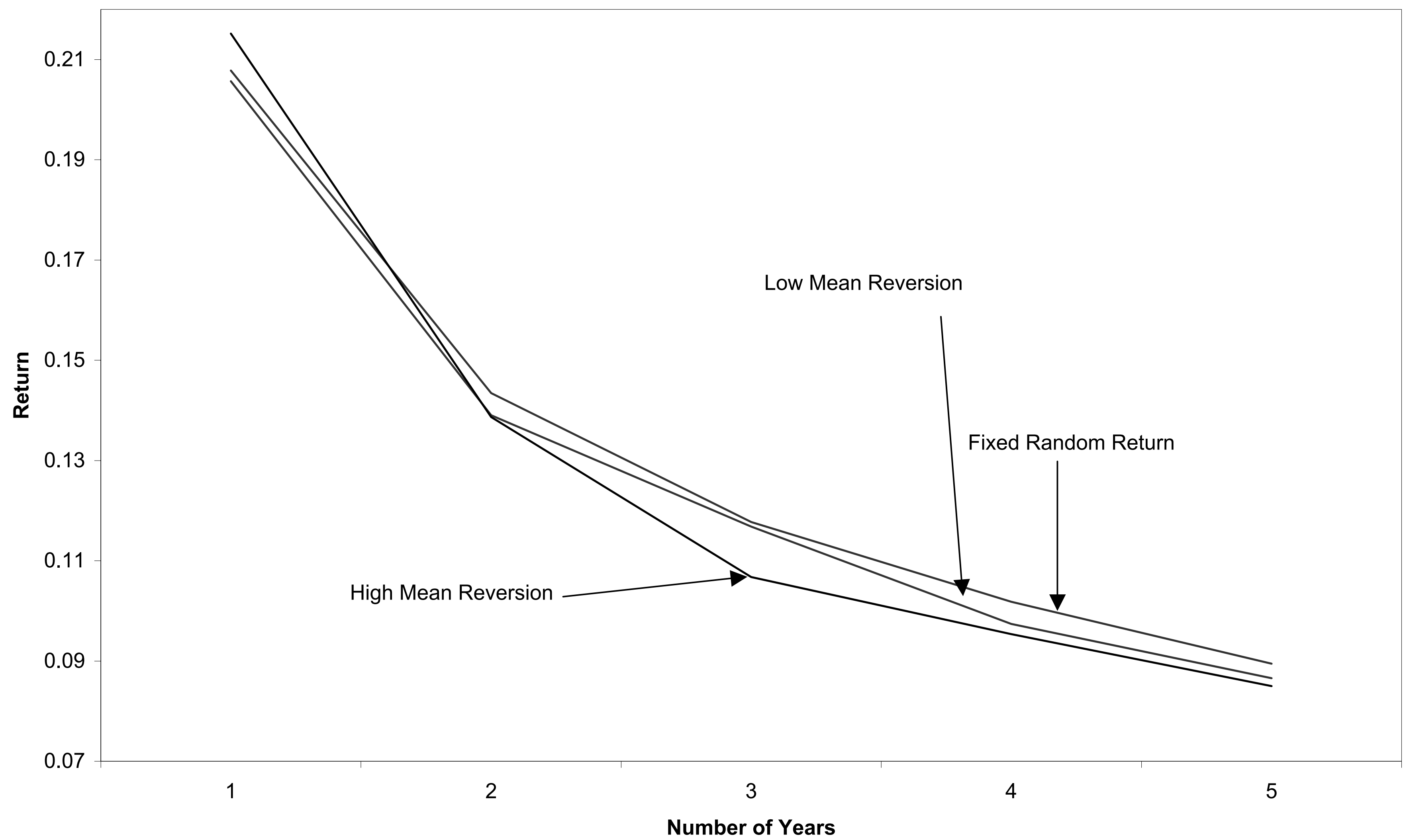


Figure 6. Variance of Mean Equity Returns, Years 5 through 35

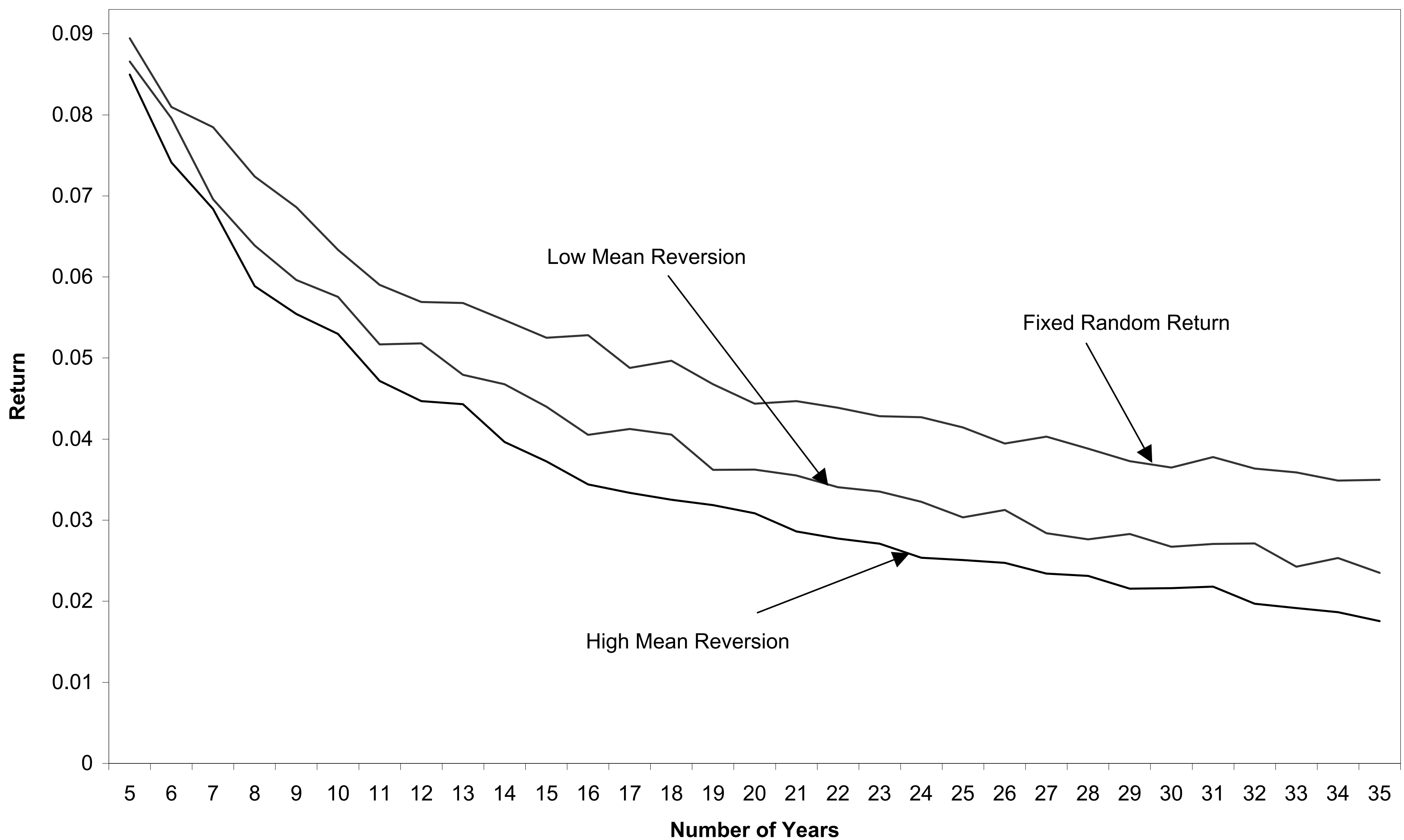


Table 4
Distribution of Example Worker IA annuities at age 65

	Year			
	2012	**2032**	**2052**	**2072**
White Noise				
Percentile Value of Individual Accounts Annuity				
10th	239	1,520	3,310	3,511
50th	377	2,852	6,510	7,833
90th	555	5,108	12,501	15,637
90th - 10th	316	3,588	9,191	12,126
Mean Reversion Models				
Low Mean Reversion				
Percentile Value of Individual Accounts Annuity				
10th	221	1,564	3,284	3,950
50th	343	2,690	6,437	7,657
90th	522	4,337	11,457	14,299
90th - 10th	301	2,773	8,173	10,349
range(white noise)/range(low mean reversion)	4.98%	29.39%	12.46%	17.17%
High Mean Reversion				
Percentile Value of Individual Accounts Annuity				
10th	217	1,661	3,772	4,324
50th	335	2,855	6,744	7,683
90th	511	4,517	11,377	13,567
90th - 10th	294	2,856	7,605	9,243
range(white noise)/range(high mean reversion)	7.48%	25.63%	20.85%	31.19%

Policy Simulations are based upon a 2 percent contribution rate from payroll taxes, 100 percent participation rate, and the PCSSS default portfolio.

Note: All figures are listed in dollar amounts unless otherwise noted.

Table 5
Distribution of Example Worker Gains at age 65

	Year			
	2012	**2032**	**2052**	**2072**
White Noise				
Percentile Gain from Individual Accounts				
10th	-59	-481	-800	-935
50th	22	588	1,912	2,475
90th	113	2,353	6,862	8,766
90th-10th	172	2,834	7,662	9,701
Probability of Negative Gain	36	26	19	20
Mean Reversion Models				
Low Mean Reversion				
Percentile Gain from Individual Accounts				
10th	-82	-407	-386	-287
50th	-18	316	1,739	2,226
90th	59	1,444	5,481	6,975
90th-10th	141	1,851	5,867	7,262
Probability of Negative Gain	63	29	15	13
range(white noise)/range(low mean reversion)	21.99%	53.11%	30.59%	33.59%
High Mean Reversion				
Percentile Gain from Individual Accounts				
10th	-88	-271	187	204
50th	-26	426	2,134	2,370
90th	45	1,510	5,210	5,951
90th-10th	133	1,781	5,023	5,747
Probability of Negative Gain	69	20	7	8
range(white noise)/range(high mean reversion)	29.32%	59.12%	52.54%	68.80%

Policy Simulations are based upon a 2 percent contribution rate from payroll taxes, 100 percent participation rate, and the PCSSS default portfolio.

Note: All figures are listed in dollar amounts unless otherwise noted.