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INPUT-OUTPUT MODEL ANALYSIS: PRICING CARBON DIOXIDE EMISSIONS

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Abstract

The risk of significant climate change caused by greenhouse gas emissions is currently one of the largest environmental and economic issues facing policymakers in the United States and around the world. Carbon dioxide (CO_2) is one of the most prevalent greenhouse gases released into the atmosphere, so some policymakers have placed their focus on reducing these emissions. Economists generally agree that efficient regulation of CO_2 emissions involves placing a price on them. An input–output (IO) model of the U.S. economy provides a framework that can be used to estimate detailed commodity price effects in response to a placing price on the emission of CO_2 into the atmosphere.

This paper provides a general overview of IO models and a specific application of an IO model to estimate the effect of a $20 tax per metric ton of CO_2 emissions. In comparison with previous work by other analysts using an IO model for this type of analysis, the model presented here uses more recent (though less detailed) data, holds the price of most imported commodities fixed while subjecting imported petroleum, natural gas, and coal to the tax, and makes adjustments for the noncombusted uses of fossil fuels.

Results from the model, which can only be interpreted as the first-order effects of the policy, imply that in response to a $20 tax on CO_2 emissions, energy commodities such as natural gas, electricity, and gasoline will experience price increases of approximately 10%, but the vast majority of commodities will experience much smaller price increases of approximately 1%. The distribution of the policy effects across sectors of the economy are based on the relative price increases and the mix of commodities consumed in each sector. Based on the estimated price increases and the mix of commodity consumption observed in the 2006 input–output tables, consumers would bear approximately 70% of the aggregate policy effects; federal, state, and local governments would bear about 12% of the aggregate policy effects; and private fixed investment costs would be approximately 8% higher.

1. Introduction

The risk of significant climate change caused by greenhouse gas emissions is one of the largest environmental and economic issues currently facing policymakers in the United States and around the world. The release of carbon dioxide (CO_2) into the atmosphere as a by-product of burning fossil fuels for energy accounted for approximately 80% of all greenhouse gas emissions in 2006.[1] As scientific consensus solidifies the link between CO_2 emissions from the combustion of fossil fuels and the atmospheric changes these emissions produce, economists have simultaneously been investigating the most economically efficient approaches for reducing CO_2 emissions to avert or reduce the consequences of significant global climate change.[2] In response to these scientific developments and economic findings, legislators in the United States have been considering an array of policy options to reduce CO_2 emissions.[3]

Economists generally agree that efficient regulation of CO_2 emissions involves placing a price on them. Pricing CO_2 emissions can be done either directly through a tax on emissions or indirectly by creating a cap-and-trade program. A cap-and-trade program would create emission allowances as a new commodity that would give entities covered by the program the right to emit a metric ton of carbon into the atmosphere in exchange for an allowance they own or purchase.[4,5]

A carbon tax and a carbon cap-and-trade system are both market-based approaches to reducing carbon emissions. Each would produce two effects: The number of carbon emissions would be reduced, and the cost of emitting each metric ton of carbon would increase. Levying a tax on carbon emissions would directly increase the price of carbon emissions, which would result in market forces reducing carbon emissions. Conversely, a cap-and-trade policy would explicitly limit the number of emissions allowed, which would result in market forces driving up the price

[1] U.S. Department of Energy (2008).
[2] Congressional Budget Office (2009).
[3] The U.S. House of Representatives passed the American Clean Energy and Security Act of 2009 (H.R. 2454) in June 2009—a significant portion of which is devoted to the creation of a carbon cap-and-trade program for greenhouse gas emissions. The U.S. Senate's Environment and Public Works Committee passed the Clean Energy Jobs and American Power Act (S. 1733) in November 2009, which includes a related cap-and-trade program. Numerous other approaches have been introduced as legislation.
[4] A metric ton is equal to 1,000 kilograms or approximately 2,205 pounds.
[5] The terms "carbon," "carbon dioxide," and "CO_2"are used interchangeably throughout this paper, but the design of the policy modeled in this paper is based on CO_2 emissions. The relationship between carbon and carbon dioxide is based on the atomic weights of the components such that a policy that placed a price on carbon alone would be approximately 12/44ths the price on carbon dioxide emissions.

of carbon emissions. A carbon tax operates directly on the *price* of emitting carbon as the policy lever; under a cap-and-trade system, the *quantity* of emissions serves as the policy lever.

An input–output model (or IO model) of the U.S. economy provides a framework that can be used to estimate detailed commodity price effects in response to a carbon policy. An IO model is constructed from a large database of intermediate transactions in the production of all goods and services as well as the distribution of all the final goods produced in an economy. At the heart of the input–output model is the intermediate transaction matrix, which describes the mix of production inputs required for every commodity output in an economy. These data can be used to estimate how a price on carbon emissions (through either a direct tax or a cap-and-trade policy) would filter through to every good and service produced and sold in the economy. Such a model is capable of capturing not only the direct effects based on the carbon intensity of inputs used in production, but also the sum of all the indirect effects based on the carbon intensity of all the secondary, tertiary, and higher-order inputs to production (that is, the inputs to the inputs to the inputs, and so on).

The IO model described here is based largely on previous models used to analyze carbon policies (most notably Fullerton, 1996, and Metcalf, 1999). Similar to these models, the one presented here makes two important assumptions. First, the model assumes that labor and capital markets are perfectly competitive and that a price on carbon is passed on to consumers in the form of higher prices for carbon intensive energy sources and for commodities that rely heavily on these energy sources in their production process.[6] The second assumption, inherent to most IO models, is that production functions are fixed, which precludes any factor substitution in response to higher (or lower) input prices. Because of that assumption, the results from these models can only be interpreted as the short-run, first-order effects of a carbon pricing policy. Firms will, however, respond to the carbon policy and will seek lower-priced alternative inputs to their production processes. To the extent that production substitution is able to lower the initial cost of the policy, the estimated effects presented here are likely to be upper bounds beyond the short term.

[6] For modeling simplicity, the full cost of carbon dioxide emissions is assumed to be passed on to consumers, though the cost of the policy will likely be at least partially shared by primary factor inputs (labor and capital) in the form of lower wages and/or lower profits. The ultimate incidence of a given carbon policy is, however, beyond the scope of the model described here.

There are however, some important differences between the model presented here and those used by other researchers to analyze the impact of various carbon policies. Relative to previous models, the model described in this paper:

- Uses more recent but less detailed data;
- Is commodity-based rather than industry-based;
- Holds prices fixed for most imported commodities; and
- Makes adjustments for the non-combustive uses of carbon.

This paper provides both a general background on input–output modeling and a description of a specific application of input–output modeling to analyze the effects of a carbon policy on commodity prices. Section 2 of the paper provides an overview of input–output models, and Section 3 provides an overview of the Make–Use framework in which modern input–output data are collected. Section 4 provides details on the conversion of Make–Use tables to a square input–output matrix.[7] For those who are well versed in input–output modeling techniques and issues, Sections 5, 6, and 7 cover the specific application of an IO model to analyze a policy that places a price on carbon emissions, accounts for international imports, and provides some model results from the analysis.

2. Overview of Input–Output Analysis

The foundation of modern input–output analysis is based on work started in the 1930s by Wassily Leontief.[8] Economic theory abstractly describes the relationships between prices and quantities with respect to supply and demand in a market economy. The ways in which these relationships unfold in reality, however, are based on innumerable individual transactions involving a vast array of inputs, products, and services. By collecting, aggregating, and tabulating detailed industrial output data into a matrix, in which the output of every industry may serve as the input to a variety of other industries in an economy, Leontief created an analytic tool

[7] For a more comprehensive background on input–output models, see United Nations (1999) and Horowitz and Planting (2006). For more detailed information on the conversion of Make–Use tables to symmetric input–output tables, see Guo, Lawson, and Planting (2002).
[8] Leontief published the first input–output tables for the 1919 and 1929 U.S. economy in 1936. See Leontief (1986) for his collected essays on his life work, for which he was awarded a Nobel Prize in economics in 1973.

that bridges the gap between the abstraction of economic theory and the empirical detail found in economic data.

A stylized depiction of the basic IO framework is shown in Figure 1. The primary matrix, F, is an industry-by-industry transaction matrix (or "flow" matrix). This matrix captures the balance of supply and demand among industries, with the values representing intermediate industry inputs to the production of industry output. The columns represent the variety of industrial input requirements (demand) and the rows represent the distribution of industrial output (supply). In addition to the square industry-by-industry transaction matrix, the model includes a vector at the bottom for value added and a vector along the right-hand side of the matrix for final demand. The value-added vector comprises primary factor inputs to production such as capital and labor services. The final demand vector comprises the components that make up gross domestic product (GDP): consumption, investment, imports, exports, and government. Because of the basic accounting identity that all outputs in an economy must equal all inputs, the total output for a given industry can be calculated as either the column sum of intermediate inputs and value added or as the row sum of intermediate and final demand for its output. In addition, total value added (the row sum of the vector), which represents all the income in the economy, must equal total final demand (the column sum of the vector), which represents the output of the economy.

The values in the F matrix and the w vector represent dollar transaction values, each comprising a price component and a quantity component. The nominal transaction values in the F matrix and the w vector can be converted into coefficients by dividing each column value by the value of total industry output. The calculated coefficients represent the proportions of inputs required to produce a single unit of output, and each column sums to 1. This matrix of coefficient values, known as the A matrix or the direct requirements matrix, can be thought of as the production "recipes" for each industry. When viewed as a whole, the entire matrix provides a snapshot of the current technological state of an economy.

Figure 1.

Stylized Symmetrical Input–Output Matrix[9]

	Industries 1, 2, …, n	Final Demand	Total Output
Industries 1 2 . . n	$\boldsymbol{F}$	$\boldsymbol{y}$	$\boldsymbol{x}$
Value Added	$\boldsymbol{w}$		
Total Input	$\boldsymbol{x'}$		

$\boldsymbol{F}$: Transaction matrix, (or "flow" matrix), (i x i)
$\boldsymbol{y}$: Final demand, which comprises components of GDP such as consumption investment, imports, exports, and government, collapsed for simplicity (i x 1)
$\boldsymbol{x}$: Total industry output (i x 1)
$\boldsymbol{w}$: Value added, which comprises wages and salaries, net profits, and indirect taxes and subsidies, collapsed for simplicity (1 x i)
$\boldsymbol{x'}$: Total industry input, which is equal to the transpose of total industry output (1 x i)

A Leontief IO model is based on the assumption that an entire economy can be described by an interconnected system of fixed and linear production functions. This simplification greatly eases IO analysis but diverges from what might be expected in the real world. In a dynamic economy, production processes are more flexible and can shift in response to changes in supply, demand, and prices. Because of the fixed, linear production functions used in the IO model (and thus the absence of any factor substitution in response to price changes of inputs), estimates from IO models specified in this manner are limited to producing descriptions of only the short-term, first-order responses to changes in exogenous variables.

[9] Technically speaking, a "symmetric matrix" means that values on either side of the diagonal are mirror images, whereas a "square matrix" means that the matrix is comprised of an equal number of column and row vectors. In input–output parlance, however, a "symmetric matrix" is used to mean "square matrix."

By constructing a matrix of economic data and quantifying the production requirements of industries, it becomes possible to use the Leontief IO model to estimate the distribution of total output requirements (the sum of both the direct and indirect requirements) in response to exogenous changes in final demand. If demand for a given commodity increases, there would clearly be an increase in demand for the commodities that serve as inputs to production for that commodity (these inputs are known as the "direct requirements"). But, to the extent that the given commodity required inputs to production that are themselves intermediate goods, there would also be a ripple effect of increased demand for the inputs to those products. The model is capable of estimating the increase in demand for the primary inputs as well as secondary, tertiary, and all other higher-order input requirements through the entire supply chain (the "indirect requirements").

The stylization presented above can be more formally described by two sets of equations:

$$\begin{aligned} x_{11} + x_{12} + \cdots + x_{1j} + y_1 &= x_1 \\ x_{21} + x_{22} + \cdots + x_{2j} + y_2 &= x_2 \\ \vdots + \vdots + \cdots + \vdots + \vdots &= \vdots \\ x_{i1} + x_{i2} + \cdots + x_{ij} + y_i &= x_i \end{aligned} \tag{1}$$

where x_{ij} is the amount of output from industry i used as an input to production for industry j, y_i is the amount of final demand from industry i, and x_i is the total output from industry i. Note that, because the number of industries down the rows is equal to the number of industries across the columns there is a system of n equations, where $n = i = j$. These n equations state that total output for any given industry is equal to the sum of output from that industry used as inputs by other industries and final demand from that industry.

A second set of n equations identified in the IO model is closely related to the first set, but it defines the value of each industry's total output as a function of its intermediate inputs and its value added, rather than final demand:

$$\begin{aligned} x_{11} + x_{21} + \cdots + x_{j1} + w_1 &= x_1 \\ x_{12} + x_{22} + \cdots + x_{j1} + w_2 &= x_2 \\ \vdots + \vdots + \cdots + \vdots + \vdots &= \vdots \\ x_{1i} + x_{2i} + \cdots + x_{ji} + w_j &= x_j \end{aligned} \tag{2}$$

where w_j is the vector of value-added from each industry. Note the transpose of the subscripts in the second set of equations, which can essentially be viewed as the sum of the column inputs and value added in the Figure 1 totaling the industry output along the bottom row.

Equation (1) can be viewed as the input–output identity (intermediate uses plus final demand equal total output), which is constructed horizontally across the F matrix. That identity is commonly used in economic analyses to model output responses to exogenous changes in various components of final demand. Equation (2), however, is the second input–output identity (intermediate inputs plus value added equal total output), which is constructed vertically down the F matrix. It is this second set of equations that is used when estimating price effects in an input–output modeling framework, and it is thus the focus of this paper's analysis.

Equation (2) is expressed in dollar transaction values, which can be split into their separate price and quantity components:

$$\begin{matrix} x_{11}p_1 + x_{21}p_2 + \cdots + x_{j1}p_j + w_1 = x_1 p_1 \\ x_{12}p_1 + x_{22}p_2 + \cdots + x_{j1}p_j + w_2 = x_2 p_2 \\ \vdots \quad + \quad \vdots \quad + \cdots + \quad \vdots \quad + \ \vdots \ = \ \vdots \\ x_{1i}p_1 + x_{2i}\, p_2 + \cdots + x_{ji}\, p_j + w_j = x_j p_j \end{matrix} \qquad (3)$$

where p_j is price vector for the output from each industry. The quantity values in Equations (3) can then be converted into input coefficients by dividing both sides by x_j.

Define:

$$a_{ji} = \frac{x_{ji}}{x_j} \qquad (4a)$$

and

$$v_j = \frac{w_j}{x_j} \qquad (4b)$$

where a_{ji} is equal to the proportion of x_i used in the production of a single unit of output from industry j, and v_j is the proportion of value added in the production of a unit of output from industry j such that:

$$\sum_{i=1}^{n} a_{ji} + v_j = 1 \qquad (5)$$

Then Equation (3) can then be rewritten as:

$$
\begin{array}{l}
a_{11}p_1 + a_{21}p_2 \ + \cdots + a_{j1}p_j \ + v_1 = p_1 \\
a_{12}p_1 + a_{22}p_2 \ + \cdots + a_{j2}p_j \ + v_2 = p_2 \\
\quad \vdots \ + \quad \vdots \quad + \cdots + \quad \vdots \quad + \ \vdots \ = \vdots \\
a_{1j}p_1 + a_{2j}p_2 \ + \cdots + a_{ji}p_j \ + \ v_j = p_j
\end{array} \tag{6}
$$

From this system of equations, it follows that the price of a single unit of output is based on the prices of the inputs weighted by the proportions in which they are used in the production process plus the value added (or, in economic terms, the prices of the primary factors of production).

This system of equations in Equation (6) can then be more compactly written in matrix algebra notation as:

$$A'p + v = p \tag{7}$$

which can then be solved for p with some simple algebraic manipulations:

$$v = p - A'p$$

$$v = (I - A')p$$

$$p = (I - A')^{-1}v \tag{8}$$

where I is an identity matrix.[10] If the underlying data in the A matrix are physical quantities, the value added vector represents the monetary value of value added *per unit of physical output*. The values in the A matrix, as they are collected, however, are monetary values and the value added vector represents the value added *per dollar of output*. Solving the system of equations with the A matrix in monetary terms produces a price vector that is equal to a vector of ones in Equation (8). In essence, the price of every commodity in the economy is equal to 1 dollar, which is known as the unit price convention in tax incidence analyses.

[10] A similar IO identity, $X = (I - A)^{-1}Y$, can be derived based on the set of equations in Equation (1). $(I - A)^{-1}$ is known as the Leontief inverse or, as the Bureau of Economic Analysis (BEA) refers to it, the total requirements matrix. As the name implies, this matrix contains both the direct output factors and the indirect output factors that would equate any changes in total output necessary to respond to a change in final demand. Note that $(I - A')^{-1}$ is equal to the transpose of the Leontief inverse: $[(I - A)^{-1}]'$.

That unit price convention becomes useful in estimating exogenous shocks to the system in the form of technological changes imbedded in the A matrix or shifts in the composition of value added, which would include the imposition of a carbon tax or a cap-and-trade policy. Such exogenous changes to the production system would produce a column vector of *relative commodity price changes*. It is that vector of price changes in response to the carbon tax or cap-and-trade program that is the key output of interest from the IO model.

3. Make–Use Framework

The basic Leontief input–output framework requires a symmetrical matrix that describes the inter-industry production relationships for the entire economy. Originally, these data were collected and tabulated with the underlying assumption that each industry produces a single commodity and, conversely, each commodity is produced by a single industry. Because of this assumption, no distinction was made between industries and commodities. This homogeneous production assumption, which was more realistic in the 1930s than today, was integral to the development of an analytic tool to trace changes in final demand through the interdependencies of an entire economy. But as industries became more complex and the collection of inter-industry transaction data more difficult, the simplifying homogeneity assumption was no longer viable. In 1968, the United Nations proposed an updated System of National Accounts (SNA) to adapt to the increasing complexity of national economies and data collection. The Bureau of Economic Analysis (BEA), which collects and publishes macroeconomic data in the United States, converted to the 1968 recommendations of the U.N. System of National Accounts in 1972 and since then has been collecting, tabulating, and releasing input–output data as two separate tables—a Make table and a Use table.[11]

The Make table is an industry-by-commodity matrix. The row values in the matrix show the variety of commodities produced for any given industry, whereas the column values show the distribution of industries that produce any given commodity. The Use table contains several matrices and vectors. The primary matrix in the table is a commodity-by-industry matrix, which

[11] Most other countries release their IO data as a set of Supply–Use tables rather than the Make–Use table framework used in the United States. The Supply–Use framework differs from the Make–Use framework in the base prices used in the Supply vs. Make tables and where data on imports, taxes, and trade and transportation margins are presented.

captures all the intermediate input transactions in domestic production of goods and services. The column values contain the mix of commodity inputs required to produce the output of any given industry, whereas the row values show the distribution of where the commodity output is used across the industries in the economy. Other important components of the Use table include a matrix of value added in production, which includes separate rows for wages and salaries, net profits, and indirect taxes and subsidies; a matrix of final demand, which contains the components of Gross Domestic Product (GDP); and vectors for total commodity and total industry output (See the Stylized version of the Make and Use tables in Figures 2 and 3 below). See the Appendix for details on the specific data considerations and adjustments made in the analysis.

Figure 2.

Stylized Make Table

	Commodities 1, 2, …, c	Total Industry Output
Industries 1 2 . . i	$\boldsymbol{M}$	$\boldsymbol{g}$
Total Commodity Output	$\boldsymbol{q'}$	

$\boldsymbol{M}$: Make matrix (i x c)
$\boldsymbol{g}$: Total industry output (i x 1)
$\boldsymbol{q'}$: Total commodity output (c x 1)

Figure 3.

Stylized Use Table

	Industries 1, 2, …, i	Final Demand 1, 2, …, f	Total Commodity
Commodities 1 2 . . c	$\boldsymbol{U}$	$\boldsymbol{E}$	$\boldsymbol{q}$
Value Added	$\boldsymbol{w}$		
Total Industry Output	$\boldsymbol{g'}$		

$\boldsymbol{U}$: Intermediate use matrix (c x i)
$\boldsymbol{E}$: Final Demand (c x f)
$\boldsymbol{q}$: Total commodity output (c x 1)
$\boldsymbol{w}$: Value Added (1 x i)
$\boldsymbol{g'}$: Total industry output (i x 1)

4. Converting Make–Use Tables to a Square Input–Output Matrix

Although the Make–Use framework offers a more accurate depiction of an economy than the symmetrical input–output framework originated by Leontief, it lacks the necessary conditions to solve the system of equations provided by the Leontief framework. Because the Leontief IO framework provides such a useful analytic tool, analysts have sought methods to convert the Make–Use tables into symmetrical industry-by-industry or commodity-by-commodity tables. The IO modeling community, however, has yet to reach a consensus on a definitive method for converting the Make–Use tables to symmetrical input–output tables. There are generally two options for converting Make–Use tables to symmetrical input–output tables:

1. The industry-technology assumption (ITA), which assumes that all commodities produced by any given industry share the same input structure and production technology, and thus that the industry in which any commodity is produced should be the criterion used for classification of any secondary commodity output.

2. The commodity-technology assumption (CTA), which assumes that the input structure and technology used in production are unique for each commodity, and thus that the industry that produces a given commodity as its primary output should be the criterion used for classification of any secondary commodity output.

Both options hinge on assumptions, inherent in the Make–Use framework, about how to reallocate secondary output so that a symmetrical matrix is produced and the homogeneous production assumption is satisfied. Although the commodity-technology assumption is generally agreed to be the most intuitively accurate classification framework, the method often produces negative production coefficients in the direct requirements matrix (A), requiring a variety of ad hoc manual adjustments to create a non-negative A matrix.[12]

The BEA relies largely on the CTA approach when it redefines, reclassifies, and reallocates commodities in the production of its supplementary IO tables; the IO tables provide the starting point for this analysis. The BEA uses the ITA method to produce their direct and total requirements tables. This model follows the BEA's methodology by also using the ITA method to create a square coefficients matrix.

Notation and Calculations

First, define some standard notation:

U	The Use table: a commodity-by-industry matrix
M	The Make table: an industry-by-commodity matrix
q	Total commodity output: a commodity-by-1 vector
g	Total industry output: an industry-by-1 vector
I	An identity matrix
i	A summation vector containing only 1s
e	Total final demand: a commodity-by-1 vector
w	Total value added: an industry-by-1 vector
p	Commodity prices: a commodity-by-1 vector
^	A symbol indicating that a vector is expressed as a diagonal matrix (a square matrix with the vector elements on the main diagonal and zeros everywhere else)

[12] See Guo, Lawson, Planting (2002) for a more thorough discussion of the techniques available to convert Make–Use tables to symmetric IO tables. Also see Chapter 5 in Miller and Blair (2009) for a description of a more sophisticated "purification" technique developed by Clopper Almon from the Interindustry Forecasting Project at the University of Maryland (Inforum), to create a square commodity-by-commodity matrix using the commodity-technology assumption without producing negative coefficients.

The stylized versions of the Make and Use tables produced by the BEA and shown in Figures 2 and 3 can be written as a collection of matrices and vectors:

$$
\begin{array}{l}
\mathit{Make\ Table} \\
\begin{bmatrix} m_{11} & m_{21} & \cdots & m_{j1} \\ m_{12} & m_{22} & \cdots & m_{j2} \\ \vdots & \vdots & \vdots & \vdots \\ m_{1i} & m_{2i} & \cdots & m_{ji} \end{bmatrix} \quad \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_j \end{bmatrix} \\
\begin{bmatrix} q_1 & q_2 & \cdots & q_i \end{bmatrix}
\end{array}
\tag{9}
$$

$$
\begin{array}{l}
\mathit{Use\ Table} \\
\left\{ \begin{array}{l} \begin{bmatrix} u_{11} & u_{12} & \cdots & u_{1j} \\ u_{21} & u_{22} & \cdots & u_{2j} \\ \vdots & \vdots & \vdots & \vdots \\ u_{i1} & u_{i2} & \cdots & u_{ij} \end{bmatrix} \quad \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_i \end{bmatrix} \\ \begin{bmatrix} w_1 & w_2 & \cdots & w_j \end{bmatrix} \end{array} \right\} \quad \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_i \end{bmatrix} \\
\begin{bmatrix} g_1 & g_2 & \cdots & g_j \end{bmatrix}
\end{array}
\tag{10}
$$

The subscript i represents the commodity dimension and the subscript j represents the industry dimension in both the Make table and the Use table; however, unlike the Leontief model, $i \neq j$. Because the Make table is an industry-by-commodity matrix and the Use table is a commodity-by-industry matrix, the subscripts are transposed in the two matrices. The total commodity output vector (q) is presented as a row vector in the Make table released by BEA and the total industry output vector (g) and total value added vector (w) are presented as row vectors in the Use table released by BEA. In the calculations that follow, however, they are all column vectors.

Although there are several components of total final demand (e), in the Use table (such as personal consumption expenditures, investment, change in inventories, imports, exports, and government expenditure—i.e., all of the standard components of gross domestic product), for expositional purposes the values in this matrix are collapsed to a single column vector. Likewise, although total value-added vector (w) comprises primary factor inputs to production (such as wages and salaries, net business profits, and indirect taxes and subsidies), this matrix is collapsed to a row vector for computational simplicity.

From here on, U will refer only to the commodity-by-industry intermediate inputs portion of the table ($u_{11} \ldots u_{ij}$), e will refer to the final demand vector, and w will refer to the transpose of the value added vector (that is, w will be a column vector rather than a row vector). Similarly, M will refer only to the industry-by-commodity matrix ($m_{11} \dots m_{ji}$), and not the additional industry and commodity output vectors.

The vector of total commodity output (q) can be calculated from the Use table as the sum commodity output across all industries (Ui) plus the vector of final demand (e) or, alternatively, as the column sums of commodity inputs to production in the Make table ($M'i$):

$$q = Ui + e = M'i \tag{11}$$

Similarly, the vector of total industry output (g) can be calculated as the sum of all industry output across all commodities ($U'i$) plus the vector of value added (w) or, alternatively, as the sum of the industry output across the rows of the Make table (Mi):

$$g = U'i + w = Mi \tag{12}$$

In converting the Make–Use tables to a symmetrical matrix, it is possible to create either a commodity-by-commodity matrix or an industry-by-industry matrix. Because the model output of interest is the change in commodity prices, the Make–Use tables are converted into a square commodity-by-commodity matrix. The first two steps in applying the ITA to create a symmetrical commodity-by-commodity matrix is to scale both the Use table and the Make table by total industry output and total commodity output, respectively.

The scaled Use table is known as the commodity-by-industry direct requirements matrix (B) and is calculated as:

$$B = U\hat{g}^{-1} \tag{13}$$

This matrix contains the commodity proportions to produce a single unit of output for each industry. If the proportion of input attributable to value added is included, each column sums to 1.

The scaled Make table is an industry-by-commodity matrix and is known as the market-share matrix (D). The Market Share matrix is calculated as:

$$D = M\hat{q}^{-1} \tag{14}$$

Note that, if all production of secondary output were moved to the industry in which that output is primary, there would be no values in the off-diagonals in the Make table, and the market-share matrix would be equivalent to an identity matrix. Because the market-share matrix is not equivalent to an identity matrix—i.e., even after redefinitions, much secondary output remains in the Make table—it is necessary to multiply the commodity-by-industry direct requirements matrix (B) by the industry-by-commodity market share matrix (D) to construct a symmetrical commodity-by-commodity technical coefficients matrix (A):

$$A = BD \tag{15}$$

The technical coefficients matrix (A) is essentially a reweighting of the direct requirements matrix (B) by the distribution of commodities produced by each industry (D).

In addition to the intermediate portion of the Use table being converted into a symmetrical technical coefficients matrix, the nominal value added vector (w) is similarly transformed into a coefficient value added vector (v) by dividing the nominal vector by total industry output (g), and then multiplying by the market-share matrix (D):

$$v = w\hat{g}^{-1}D \tag{16}$$

With a symmetrical A matrix and a reweighted value-added coefficient vector, the standard Leontief IO calculations can be made, as shown in equations (6) to (8).

Special IO Commodities

In the Make–Use tables produced by the BEA, there are several IO commodities that require special attention when combining the two tables to create a symmetrical IO table. At the summary level of detail, there are three IO commodities that do not have corresponding industries in which they are produced: 1) scrap, used, and secondhand goods; 2) noncomparable imports; and 3) rest-of-world adjustments. Each of these shows up at the bottom of the Use table as a commodity row, but does not have a corresponding industry column. Noncomparable imports and rest-of-world adjustments do not show up at all in the Make table, but there is a commodity column for scrap, used, and secondhand goods (with, as in the Use table, no corresponding industry row).[13]

Noncomparable imports and rest-of-world adjustments are part of foreign transactions and are not produced by any industry. Noncomparable imports are distributed across intermediate production by industry as well as across final demand (specifically to personal consumption expenditures (PCE), imports, and government). These amounts are offset such that total output is equal to zero. This analysis removes noncomparable imports (nci) from the intermediate inputs matrix and combines them with the value-added matrix. As a result, the prices for these noncomparable imported commodities are held constant in the analysis.

Rest-of-world adjustments are offsetting adjustments made between PCE and gross exports and between government final demand and gross exports. These are made to align commodity treatment in IO table with the expenditure treatment in the national income and product account (NIPA) tables. These affect only final demand and are dropped from the Use table in our calculations.

To appropriately balance the equation such that p is a column vector of 1s, the special input commodity noncomparable imports (nci) needs to be included.[14] As stated above, these special input commodities are held constant in the model calculations by adding them to the value-added coefficient vector:[15]

$$p = (I - A')^{-1}(v + nci) \tag{18}$$

5. Applying Carbon Pricing in the Model

Pricing carbon emissions can be done either through a direct tax on those emissions or by creating a cap-and-trade program to reduce emissions. From a modeling perspective, the market

[13] At the detail level, the commodity scrap is separated from used and secondhand goods. Also, there are three industries without corresponding commodities at the detail level: federal government electric utilities, state and local government passenger transit, and state and local government electric utilities. See Fiedler (2009) for a more rigorous treatment of these details.

[14] The special commodity “rest-of-world adjustment” would need to be included if the calculations were performed using the 2002 benchmark data because those data have a positive value for rest-of -world adjustment in the intermediate input portion of the “General Federal nondefense government services” industry.

[15] The nci vector is also a coefficient vector calculated similarly to the way the value-added coefficient vector is calculated: $nci = nci\hat{g}^{-1}D$.

price of those allowances is analogous to a unit tax on each metric ton of CO_2 emission from covered entities that would produce similar emissions reductions. For expository purposes, the policy modeled in this analysis is a \$20 tax per metric ton of CO_2 emissions, which would result in a reduction of CO_2 emissions from 6 billion to 5 billion metric tons. This pricing policy is expected to collect \$100 billion in tax revenues.[16] This aggregate tax revenue amount is collected from the three primary fossil fuels in the most upstream point possible based on emissions data by fuel source released by the Energy Information Administration. Based on those data, the distribution of CO_2 emissions from energy consumption in 2006 was approximately 36%, 44%, and 20% for coal, petroleum, and natural gas, respectively.[17]

Fuel-specific tax rates are then calculated based on the amount of tax revenue from each fuel source divided by the total amount of fuel used in intermediate production.[18] Based on total coal output of \$30 billion in the 2006 Use table and an expected \$36 billion in tax revenue to be generated from coal emissions, any industry using coal as an input to its production process will face a 125% tax on coal inputs ($t_c = 1.25$). Unfortunately, the IO classification combines oil extraction and natural gas extraction into a single oil and gas extraction commodity class.[19] Because the majority of natural gas extraction is used as inputs to the natural gas distribution industry and the electricity industry, the tax rate for natural gas extraction is calculated as the expected revenue from natural gas divided by the total oil and gas extraction output going to the sum of the natural gas distribution industry and the electricity industry. The remainder of the oil and gas extraction output is used as the denominator for calculating the tax rate on oil extraction and is applied to all other industries in the Use table row for oil and gas extraction. This is a simplifying assumption, but there are insufficient data to estimate the proportion of oil and the proportion of natural gas from the oil and gas extraction commodity that goes to each industry.[20] Based on the amount of revenue to raise from each fuel source (the numerators) and the value of

[16] In 2006, there were approximately 6 billion metric tons of CO_2 emissions from energy use. The reduction to 5 billion metric tons in response to a \$20 tax is just a simplifying assumption for expository purposes. A more detailed analysis would require estimated carbon demand elasticities to arrive at a total revenue estimate.
[17] See Table 5 in U.S. Department of Energy (2008).
[18] The amount of raw coal, oil, or natural gas consumed as final goods is negligible and would likely be excluded from either the tax base or the requirement to hold emission allowances. Because of this, these final use amounts are excluded from the tax base in the rate calculations.
[19] This is true for even the benchmark tables at the detail level, which provides the finest disaggregation of data released by the BEA.
[20] This technique follows the one used in Metcalf (1999).

the oil and gas extraction output (the denominators), the tax rate on oil inputs is 11.3% ($t_o = 0.113$) and the tax rate on natural gas inputs is 20.7% ($t_{ng} = 0.207$).[21]

To apply these tax rates to the Use table, a tax matrix is constructed based on a commodity-by-industry null matrix (where all values in the matrix are equal to zero). The tax rate for coal, t_c, shows up in every column of the commodity row for coal. The tax rate for oil, t_o, is applied to almost every column of the commodity row for oil and gas extraction. The tax rate for natural gas, t_{ng}, shows up in the oil and gas extraction commodity row, but only for the natural gas distribution industry and electricity industry columns. A representation of the tax matrix T is below:

$$T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ t_c & t_c & t_c & t_c & t_c & t_c & t_c & \cdots & t_c \\ t_o & t_o & t_o & t_{ng} & t_{ng} & t_o & t_o & \cdots & t_o \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0_{ij} \end{bmatrix} \tag{19}$$

This matrix of tax rates on fossil fuel uses across the Use table is converted to a coefficient tax vector that is added as an indirect tax in the value added component of the model. The coefficient tax vector is calculated by first performing an element-by-element multiplication of the Use matrix and the Tax matrix. The resulting matrix, which contains nominal tax revenue amounts to be extracted by the industries that use coal, oil, or natural gas as inputs to their production based on the tax rate on the fossil fuels and the amount of these fuels used, is then converted into a coefficient vector by dividing by total industry output and multiplying by the market share matrix. Finally, this coefficient matrix is transposed and multiplied by a summation column vector of 1s. This calculation can be written as:

$$t = (\{U \otimes T\}\hat{g}^{-1}D\,)'i \tag{20}$$

[21] The calculated tax rates are *ad valorem* approximations of the specific tax rates based on the carbon intensity and average price for the fuel sources used in production. Because natural gas is a less carbon intensive energy source and a cheaper energy source relative to oil, it has a lower base price per metric ton of CO_2 emissions than oil. The conversion of the specific tax of \$20 per metric ton of CO_2 emissions, therefore, produces a higher *ad valorem* tax rate for natural gas than for oil.

where $\otimes$ represents element-by-element (or Hadamard) multiplication of two identically dimensioned matrices, where each element is equal to $u_{ij} \cdot t_{ij}$.

The calculated coefficient tax vector is added to the value added and noncomparable imports coefficient vectors so that the IO model to estimate relative commodity price changes then becomes:

$$p = (I - A')^{-1}(v + nci + t) \tag{21}$$

Without the tax coefficient vector, t, the price change vector is equal to a column of 1s. When the tax coefficient vector is introduced, the effect of the taxes filters through to the prices of all the commodities in the economy by means of the transpose of the Leontief inverse.

Adjustment for Noncombustive Uses

Not all uses of fossil fuels result in CO_2 emissions. Most notably, natural gas going to feed stocks and petroleum going to produce asphalt do not release the carbon in the fuels into the atmosphere. To model the likely exclusion of the non-emissive uses of oil, natural gas, and coal from a carbon tax or a cap-and-trade program, the incidence of the tax on industries is adjusted based primarily on data from the Manufacturing Energy Consumption Survey (MECS) conducted by the Energy Information Agency. That survey reports manufacturing purchases of energy as well as the amount of that energy that is combusted.[22]

In the IO tables, only the total value of transactions—not the CO_2 emissions—are reported. To account for the proportion of fossil fuels that do not get combusted (and for which rebates would be available), the tax matrix is altered by reducing the denominator in the tax rate calculations by the amount of non-combusted uses in manufacturing industries (equal to the amount of fossil fuel used in production times one minus the percent combusted, shown in Table 1).[23] This increases the tax rates on CO_2 emissions for all industries, but the industries with non-combustive use of fossil fuels in their production process are then given rebates for their non-combustive use of

[22] Data are available from the Energy Information Agency at http://www.eia.doe.gov/emeu/mecs/contents.html. For noncombustive emissions from the oil and gas extraction sector, estimates of oil refining emissions relative to the resulting petroleum products produced are estimated based on Phillips (2002).
[23] These proportions are for the IO commodity classes of "natural gas distribution" and "petroleum and coal products," which is one level down in the production stream from where the tax is levied (e.g., on the "coal mining" commodity and the "oil and gas extraction" commodity).

fossil fuels. The rebates exactly offset the higher rates such that, in aggregate, the total revenue raised remains constant, but the tax incidence across industries is shifted.

The IO model is then specified as:

$$p = (I - A')^{-1}(v + nci + t^{*} - r) \qquad (22)$$

where t^{*} is the adjusted tax coefficient vector, and r is a calculated coefficient rebate vector.

Table 1.

Percent of Natural Gas and Petroleum and Coal Products Combusted in Production, by Manufacturing Industry

	Natural Gas	Petroleum and Coal Products
Food	99.7%	100.0%
Textile Mills	100.0%	100.0%
Apparel	100.0%	100.0%
Wood Products	98.9%	88.9%
Paper	100.0%	100.0%
Printing and Related Support	100.0%	100.0%
Petroleum and Coal Products	100.0%	74.0%
Chemicals	79.6%	8.5%
Plastics and Rubber Products	99.2%	100.0%
Nonmetallic Mineral Products	99.8%	100.0%
Primary Metals	93.0%	62.3%
Fabricated Metal Products	100.0%	85.7%
Machinery	100.0%	100.0%
Computer and Electronic Products	100.0%	100.0%
Electrical Equipment, Appliances, and Components	97.6%	100.0%
Transportation Equipment	100.0%	76.0%
Furniture and Related Products	100.0%	100.0%
Miscellaneous Industries	100.0%	100.0%

Source: Manufacturing Energy Consumption Survey (EIA, 2009).

6. Imported Commodities

The basic IO model with an application of a carbon tax or a cap-and-trade program discussed so far has a limitation with respect to how imports are handled in the model. The Use tables released by the BEA include imported commodities in the intermediate transaction matrix.

These imported values are offset by a column of negative values in the final demand matrix. The negative value for a given commodity in the imports column offsets the total amount of imported commodities that were used as either intermediate inputs to production or as final personal consumption. The BEA, however, also releases a version of the Use table for the imported components, which is used in this analysis to decompose the combined intermediate-use cells into domestic and imported components. The imports matrix released by BEA is described in more detail in the Appendix.

Previous research that has used an input–output framework to analyze the impacts of a carbon tax on commodity prices has relied on the Armington assumption to address price effects on imported commodities.[24] The Armington assumption states that similar commodities are sufficiently differentiated by country of origin such that imported commodities are not perfect substitutes for domestically produced commodities.[25] Essentially, this implies that commodity prices are not set on a world market, but that domestic price changes can occur independent of imported price changes. This is a fairly broad assumption. Furthermore, unless the policy being modeled includes a comprehensive set of border tax adjustments so that all imports face price increases in accordance with their carbon content, the Armington assumption is insufficient to justify price changes for imported commodities in response to a domestic carbon policy.

To hold imported commodities prices fixed in response to a U.S. carbon policy in this analysis, virtually all imported intermediate inputs to production are isolated when calculating the A matrix. These imported commodities are then combined with the noncomparable imports commodity that is being held constant with the value-added component of production. The only exception in the model is for the three primary fossil fuels. The model presented here treats all imports of coal, unrefined petroleum, and natural gas as domestically produced commodities; thus they are subject to the tax and are modeled to face commodity price changes in tandem with domestically produced fossil fuels.[26]

[24] Fullerton (1996) and Metcalf (1999).
[25] Armington (1969).
[26] This is essentially the same assumption made by Ho, Morgenstern, and Shih (2008) in their medium- and long-run analysis of a \$10 tax per metric ton of CO_2 emission on U.S. manufacturing industries.

Model Extension to Handle Imports

Recall Equation (2), which shows that the total value of commodity output can be described as the sum of the intermediate inputs and value added. As the data are collected, however, each x_{ji} comprises both domestic and imported commodity inputs to production, which is more accurately expressed as:

$$\begin{aligned} x_{11}^c + x_{21}^c + \cdots + x_{j1}^c + w_1 &= x_1^d \\ x_{12}^c + x_{22}^c + \cdots + x_{j2}^c + w_2 &= x_2^d \\ \vdots + \vdots + \cdots + \vdots + \vdots &= \vdots \\ x_{1i}^c + x_{2i}^c + \cdots + x_{ji}^c + w_j &= x_j^d \end{aligned} \tag{23}$$

where x_j^d is the total domestic production for each commodity j, w_j is the value added to production, and x_{ij}^c is the composite (both domestic and imported) inputs to the production of each commodity j, each of which can be split in to its domestic (x_{ji}^d) and imported (x_{ji}^m) components:

$$x_{ji}^c = x_{ji}^d + x_{ji}^m \tag{24}$$

Equations in (23) can then be expanded to:

$$\begin{aligned} x_{11}^d + x_{11}^m + x_{21}^d + x_{21}^m + \cdots + x_{j1}^d + x_{j1}^m + w_1 &= x_1^d \\ x_{12}^d + x_{12}^m + x_{22}^d + x_{22}^m + \cdots + x_{j2}^d + x_{j2}^m + w_2 &= x_2^d \\ \vdots + \vdots + \vdots + \vdots + \cdots + \vdots + \vdots + \vdots &= \vdots \\ x_{1i}^d + x_{1i}^m + x_{2i}^d + x_{2i}^m + \cdots + x_{ji}^d + x_{ji}^m + w_j &= x_j^d \end{aligned} \tag{25}$$

Manipulations similar to those in Equations (2) through (8) can then be made. Separate domestic and imported inputs can be split into their quantity and price components, a unit price convention can be applied, and input coefficient values can be calculated by dividing both sides of the equation through by x_j^d, which results in the following set of equations:

$$\begin{aligned} a_{11}^d p_1^d + a_{11}^m p_1^d + a_{21}^d p_2^d + a_{21}^m p_2^m + \cdots + a_{j1}^d p_j^d + a_{j1}^m p_j^m + v_1 &= p_1^d \\ a_{12}^d p_1^d + a_{12}^m p_1^m + a_{22}^d p_2^d + a_{22}^m p_2^m + \cdots + a_{j2}^d p_j^d + a_{j2}^m p_j^m + v_2 &= p_2^d \\ \vdots + \vdots + \vdots + \vdots + \cdots + \vdots + \vdots + \vdots &= \vdots \\ a_{1i}^d p_1^d + a_{1i}^m p_1^m + a_{2i}^d p_2^d + a_{2i}^m p_2^m + \cdots + a_{ji}^d p_j^d + a_{ni}^m p_j^m + v_j &= p_j^d \end{aligned} \tag{26}$$

where $a_{ji}^d = x_{ji}^d / x_j^d$, $a_{ji}^m = x_{ji}^m / x_j^d$, and $v_j = w_j / x_j^d$

This system of equations can then be written in matrix algebra notation as:

$$A'_d p^d + A'_m p^m + v = p^d \tag{27}$$

which can then be solved for p^d in the following simple steps:

$$p^d - A'_d p^d = A'_m p^m + v$$

$$(I - A'_d) p^d = A'_m p^m + v$$

$$p^d = (I - A'_d)^{-1} (v + A'_m p^m) \tag{28}$$

where v is a column vector of value-added coefficients, I is an identity matrix, A'_d is the transpose of the domestic technical coefficients matrix, A'_m is the transpose of the imported technical coefficients matrix, p^d is a vector of price changes for domestic factors of production, and p^m is a vector of price changes for imported factors of production.

If p^m is held constant as a column vector of 1s, the equation will balance, and p^d will similarly be equal to a column vector of 1s, so long as the coefficient vector for noncomparable imports is also included.

$$p^d = (I - A'_d)^{-1} (v + nci + A'_m p^m) \tag{29}$$

Similar to the basic version of the model, a tax coefficient vector (minus a rebate vector for noncombustible uses of fossil fuels in production) can be added to the system as a fixed component of the value-added vector to produce a vector of domestic commodity price changes while holding imported commodity price changes constant.

$$p^d = (I - A'_d)^{-1} (v + nci + A'_m p^m + t^* - r) \tag{30}$$

When calculating the tax coefficient vector, the tax rates are calculated using only domestic output in the denominator, and the tax matrix containing these rates is applied only to the domestic portion of the Use matrix:

$$t = (\{U_d \otimes T\} \hat{g}^{-1} D)' i \tag{31}$$

In the policy simulated, however, imported coal, petroleum and natural gas are treated as domestically produced commodities, and because these are the only commodities that have a tax levied on them, there is no real distinction between $\{U_d \otimes T\}$ and $\{U \otimes T\}$.[27]

7. IO Model Results

Placing a price on the emission of CO_2 will increase direct costs for covered entities. In this model, a direct tax is placed as upstream as possible on producers of petroleum, natural gas, and coal. This model assumes that labor and capital markets are perfectly competitive and that this tax is passed on to consumers in the form of higher prices for carbon-intensive commodities.[28]

The IO model presented here captures the ripple effects of a $20 tax per metric ton of CO_2 emissions as it filters through the production structure of the U.S. economy as collected in the 2006 input–output data. It is important to remember that a fundamental assumption built into the model is that production functions are fixed, and thus the price changes estimated here represent only the short-term effects of a carbon policy. The mix of production inputs and technologies will, however, shift in response to the tax. Because firms will likely substitute to the lowest cost production process available, the estimated price changes presented in this analysis likely represent the upper bounds of the price effects.

Effects on Domestic Commodity Prices

The results presented below are based on the policy scenario laid out previously in the construction of the tax rates: Each metric ton of CO_2 is taxed at $20, which will result in 5 billion metric tons of CO_2 being emitted, and the resulting total revenue collected under the policy will be $100 billion. As would be expected under such a policy, commodity price changes would be largest for carbon-intensive commodities such as natural gas distribution (12.0%), coal mining (11.6%), electricity (9.3%), and petroleum and coal products (7.6%).

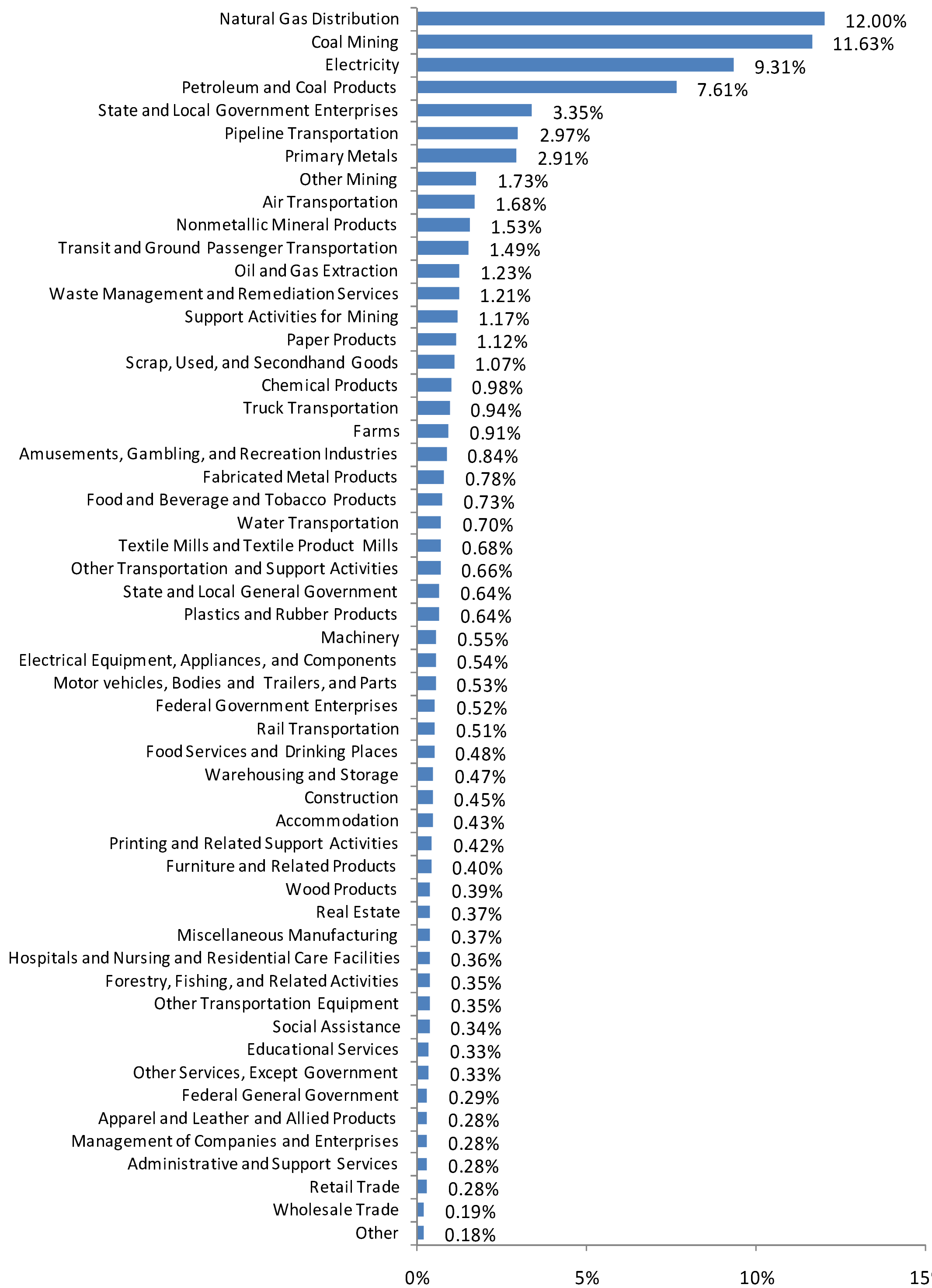
[27] Moving imported fossil fuels to be treated as domestically produced commodities, however, produces an imbalance in the commodity and industry output totals between the Make and Use tables. To address this, a RAS algorithm (described in the Appendix) is applied to the Make table to ensure the row and column output totals in the Use table exactly balance with the column and row totals in the Make table.
[28] Whether aggregate prices would increase is dependent on Federal Reserve actions and is not addressed explicitly in this model.

At first glance, the finding that natural gas distribution and coal mining have similar price increases seems incongruous with the fact that the combustion of coal releases much more CO_2 than natural gas. Recall, however, that the tax is levied in the most upstream location possible on the commodities coming out of the coal mining and oil and gas extraction industries. The tax rate on coal mining is 125%, but because the tax is passed forward and falls only on the *use* of the commodity, the price increase for coal mining is a function of the tax rate and the amount of intra-industry use of coal. Approximately 10% of the coal mining output is used by the coal mining industry, which is what determines the price increase for the commodity coming out of the coal mining industry (12%).

Natural gas distribution and electricity are each the next level down from where the tax is levied, and the difference in the price increases is determined by the mix of inputs to these two commodities and the underlying tax rates on the primary inputs. Approximately 54% of the inputs to natural gas distribution is from oil and gas distribution, the natural gas portion of which is taxed at 20.7% (54% of 20.7% = 11.2%, with the remaining 0.8% price increase coming from other direct and indirect inputs—e.g., intra-industry use of natural gas). The relatively lower price increase for electricity is attributable to the broader mix of inputs to producing electricity. Specifically, the two taxed inputs of natural gas extraction and coal mining make up 10% and 6% of the inputs to electricity production, respectively, which, when multiplied by the appropriate tax rates, approximately account for the 9.3% price increase in electricity (with negligible indirect price increase effects). The domestic commodity price increases for all other non-carbon intensive products are rather low in comparison, averaging less than 1% (see Figure 4).

Figure 4.

Domestic Commodity Price Changes in 2006 Producers' Prices



Although these results are not directly comparable with previous research using similar techniques, there are some comparisons to make with a recent Ho, Morgenstern, and Shih (2008) paper. That paper examines the impact of a carbon tax, particularly on the U.S. manufacturing industries, for four time horizons after enactment of the policy (the very short-run, the short-run, the medium-run, and the long-run). The latter two time frames employ a general equilibrium model for the analysis, and the former two time frames use an input–output modeling approach. In the very short-run time frame, prices cannot change, and the price increase associated with the policy results in reduced profits. This is analogous to the analysis presented here, except the incidence of the policy in this paper is assumed to be passed forward to consumers rather than passed backward to the primary input factors (profits and wages).

There are several methodological differences in the IO modeling approach as well as underlying data between this paper and the Ho, Morgenstern, and Shih paper. Their analysis uses 2002 benchmark data and relies heavily on industry-specific energy-intensity data from the 2002 Manufacturing Energy Consumption Survey (MECS), fossil fuel consumption for the electric utility industry from the Annual Energy Review (AER), and energy use for agriculture from the U.S. Department of Agriculture's Economic Research Service (ERS). They observe substantially different energy consumption amounts by industry between these outside energy sources and the quantities implied in the IO data, and they adjust the IO data to align with these aggregate energy data. Their analysis is based on a \$10 per metric ton of CO_2 emissions in 2005; the analysis in this paper is based on a \$20 per metric ton of CO_2 emissions policy in 2006. To be more directly comparable, the very short-run results that Ho, Morgenstern, and Shih present should be approximately doubled. Furthermore, they drop from their analysis all intra-industry transactions in the IO data.

The level of detail Ho, Morgenstern, and Shih provide for the manufacturing industries is much higher than the level of detail provided in Figure 4. With the energy adjustment they make, they find significantly higher production cost increases (Ho, Morgenstern, and Shih, 2008, Table 3) than for any of the manufacturing industries in the analysis presented here. They do, however, estimate a 4.2% increase for the petrochemical manufacturing industry. When that estimate is doubled to align to the policy simulation conducted here (ignoring the one-year price inflation difference between 2005 and 2006), the price increase is similar to the 7.6% price increase estimated in this analysis for petroleum and coal products, which is likely to include very similar industries and commodities. The largest difference between the Ho, Morgenstern, and Shih estimates and the estimates produced in this analysis is for the coal mining industry (0.6% versus 12%), which is most likely attributable to their removal of intra-industry transactions from their analysis.

For the non-manufacturing industries, the results presented in Figure 4 are somewhat comparable. Most of the price increases are very small in their analysis, and, even when doubled, are close to what is reported in this analysis. Ho, Morgenstern, and Shih estimate, however, a 16.6% increase for the electricity industry, compared with the 9.3% increase estimated here. The larger price increase, to some degree, could be attributable to their combination of state and local government electricity production with the electricity industry.

Effects on Composite PCE Commodity Prices

The values shown in Figure 4 are IO commodity price increases for domestically produced commodities expressed in producers' values. There are several additional calculations necessary, however, to convert the values in Figure 4 into estimates of commodity purchases made by households. The domestic price changes must be converted to composite price changes based on the proportions of commodities purchased that are domestically produced and imported; the basis of the price changes need to be converted from producers' prices to purchasers' prices; and the price changes need to be mapped from the IO commodity classifications, which are primarily based on the North American Industry Classification System (NAICS), to the Personal Consumption Expenditures (PCE) component of GDP.[29]

The first step is to calculate a vector of composite price changes based on a weighted average of domestic and imported commodity purchases, where prices for imported commodity purchases are held constant. The weights are calculated based on the proportion of domestic and imported

[29] Metcalf (1999) and Hassett, Mathur, and Metcalf (2007) go one step further and map the PCE price changes to the Uniform Commercial Code (UCC) system of commodity classification used by the Bureau of Labor Statistics in the collection of their Consumer Expenditure (CE) Survey data to analyze the relative burden of a carbon policy by household income deciles.

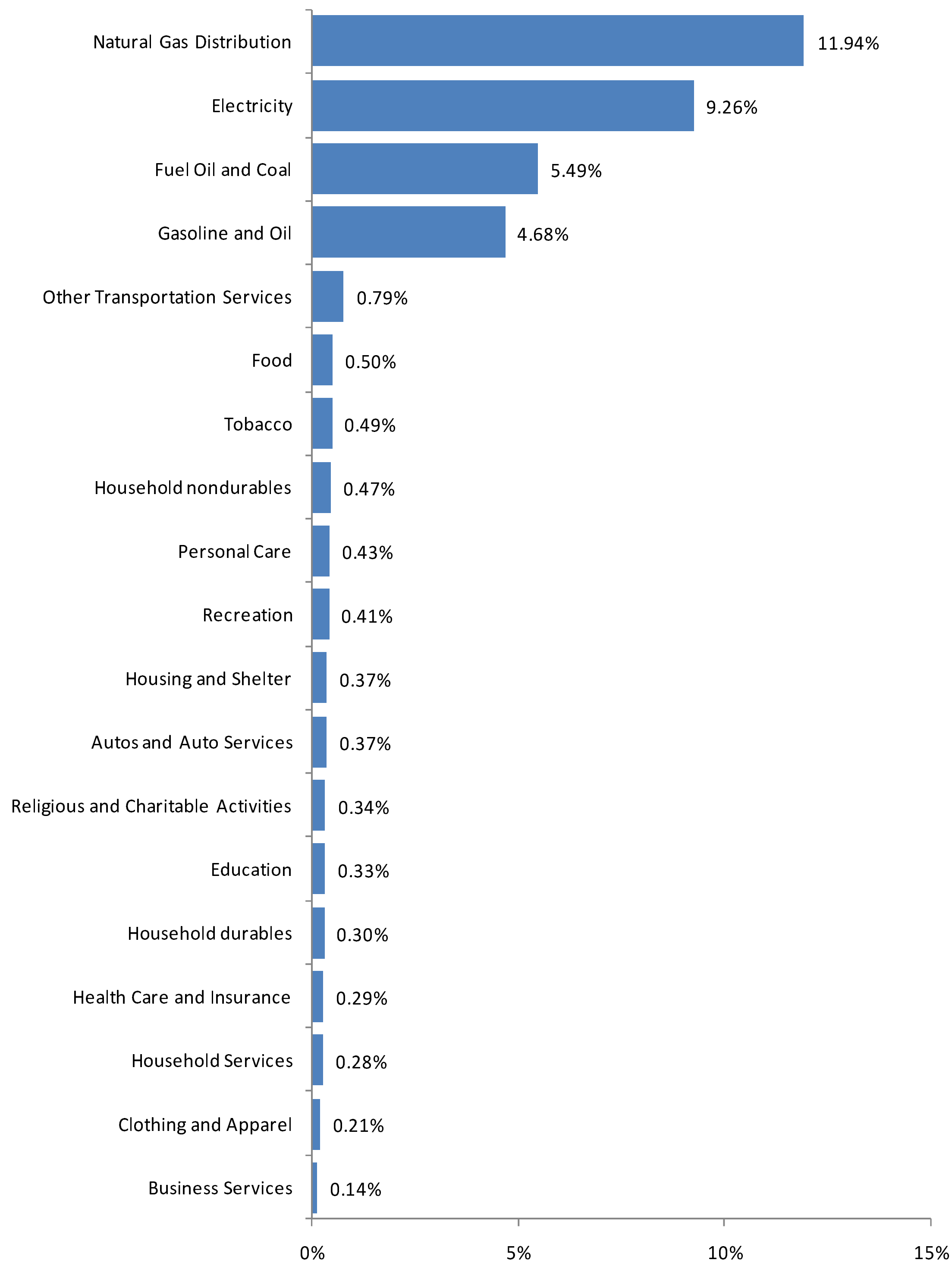
commodity purchases in the personal consumption expenditures (PCE) component of GDP, available in the imports matrix released by the BEA.

Fortunately, the BEA releases data with the import proportions of PCE and an IO/PCE bridge table that facilitates the conversion of price bases and the crosswalk between IO commodity classifications and PCE commodity classifications. The bridge table contains not only the commodity crosswalk between the IO commodity classes and the PCE commodity classes, but it also contains the trade and transportation margins for each commodity. By applying the estimated price increases for the commodities at the producers' price levels and separately for the trade and transportation margins, a new set of purchasers' prices can be calculated, and the price change in purchasers' prices can then be calculated relative to the purchasers' prices before the policy. Finally, the PCE price increases in purchasers' prices for domestic commodities are adjusted based on the proportion of each PCE category from domestic production. The proportion coming from imports is held fixed, and the weighted average of the domestic price change and the fixed imported prices produces the combined commodity price change in purchasers' prices. Figure 5 presents the combined (domestic/imported) price changes for 19 PCE categories.

Similar to the domestic IO price changes in producers' prices presented in Figure 4 the policy effect on combined commodity price changes in purchasers' prices is dominated by a few commodities: natural gas distribution, electricity, fuel oil and coal, and gasoline and oil. The most striking difference between the estimates in Figure 4 and Figure 5 is for gasoline and oil. Although the only input to the PCE category of gasoline and oil in the bridge table from IO categories is from petroleum and coal products, there is a significant difference in the price increases predicted (4.68% vs. 7.61%, respectively). This is entirely attributable to the difference in producers' prices and purchasers' prices. Although the producers' prices increased by 7.61%, the trade margins, which experience trivially small price increases in response to the carbon policy, account for approximately 39% of the purchasers' final price of gasoline (15% wholesale, 24% retail).

Figure 5.

Commodity Price Changes in 2006 Purchasers' Prices



There are several more IO inputs to the PCE commodity of fuel oil and coal, although a very small proportion is actually coal (approximately 0.5%). The bulk of the inputs to fuel oil and coal comes from the IO category for petroleum and coal products (approximately 97%). The IO commodity petroleum and coal products increases by 7.61%, but the fuel oil and coal PCE commodity increases by slightly less because of the other inputs to the commodity (which experienced lower price increases) and the significant trade margins between the producers' price and purchasers' price for petroleum and coal products (approximately 24%).

Using the 2002 benchmark data to simulate commodity price changes in response to a $15 tax (in 2005 dollars) per metric ton of CO_2 emissions in 2003, Hassett, Mathur, and Metcalf (2007) estimate a price increase of approximately 12% for both natural gas distribution and electricity, approximately 10% for home heating oil (which is likely just a different name for fuel oil and coal), and approximately 8% for gasoline and oil. A $15 tax per metric ton of CO_2 in 2005 is equivalent to a $14.13 tax in 2003, which, when applied to 5.8 billion metric tons of emissions, is expected to raise approximately $82.0 billion in revenues in 2003 in their analysis. That $82.0 billion in tax revenues represents 0.77% of GDP in 2003. The policy simulation performed in this analysis applies a $20 tax per metric ton of CO_2 emissions to 5.0 billion metric tons of emissions to raise $100 billion in tax revenues (or 0.75% of GDP). To convert the Hassett, Mathur, and Metcalf policy to account for the price inflation of the tax as well as the growth in the underlying economy so that it is approximately equivalent to the policy simulated here, their results can simply be scaled down by 97% (0.75/0.77). When that adjustment is made, their results are similar to the results presented in this analysis. Their 12% price increase for natural gas distribution and electricity becomes 11.6%, which is quite similar to the 12% price increase estimated for natural gas distribution in this analysis, but somewhat less similar to the 9.3% price increase estimated for electricity presented here. Their adjusted estimates for home heating oil (9.7%) and gasoline (7.7%), however, are both significantly higher than the estimated price changes found in this analysis (5.5% and 4.7%, respectively). The magnitude of a commodity price change is dependent, however, on the size of the tax relative to the base price of the commodity. Even though the results from the Hassett, Mathur, and Metcalf policy experiment have been adjusted to align with the policy experiment conducted here, the adjustment ignored differential price increases across commodities in the economy. If the prices for gasoline and home heating oil grew significantly faster than the overall economy, then the results presented here would be expected to be lower than the adjusted results from the Hassett, Mathur, and Metcalf paper. In fact, the average weekly price of gasoline increased by 65% between 2003 and 2006, but the overall size of the economy increased by only 20% over the same time period.

Effects on Sectors of the Economy

In addition to the effects on commodity prices, an IO model can be used to examine the effects of increasing carbon prices on different sectors of the economy by major component of output or by industry. The effect by sector of the economy depends on the level and mix of expenditures in each sector and the price change vector estimated by the IO model. This can be estimated by multiplying the price change vector with the final demand matrix available as a panel in the Use table:

$$b = p'E \tag{32}$$

where E is the final demand matrix (see Figure 3) with the columns containing the components of gross domestic product (GDP) such as personal consumption expenditures (PCE), private fixed investment, change in inventories, exports and imports, and government expenditures, p' is the transpose of the estimated price change vector; and b is a row vector containing the tax burden across the components of final demand.

Table 2 illustrates the distribution of costs across the components of GDP for the hypothetical policy under consideration. These calculations show that almost 70% of the carbon tax burden will fall on household expenditures, which is approximately equal to the proportion PCE makes up of GDP. The extent to which the distribution of the revenue raised by the carbon policy differs from the aggregate proportions of the final demand is based on the distribution of commodities that make up each component of final demand and the distribution of the price increases. The federal government, for example, was 7% of GDP, but less than 3% of the revenues from the carbon policy would come from the increased prices the government would have to pay.

Imports are not included in Table 2 because the policy being modeled does not include a border tax adjustment. The model, however, treats imported coal, oil, and natural gas as being domestically produced, and the revenues collected from the consumption of those imported commodities are captured in the other sectors. Under those model assumptions, the price of all other imported commodities remain fixed.

Table 2.

Revenue Collected under a Carbon Tax, by Final Uses

	$ Billion
Personal consumption expenditures	$69.4
Private fixed investment	$7.8
Change in private inventories	$0.9
Exports of goods and services	$9.5
Federal government	$2.7
State and local government	$9.5
Total Revenue Raised[a]	$99.7

a. Total is not exact because of rounding.

Those sectoral burden calculations serve as a useful internal consistency check on the IO model. The tax levied on the economy was designed to raise $100 billion, and approximately $100 billion is collected across the final use categories.[30] This equality holds because of the balancing requirement in the IO tables that the sum of all final demand equal the sum of all value added, each of which are equal gross domestic product.

There are several reasons, however, the ultimate distribution of the burden by sectors will likely be different than what is shown in Table 2. It is possible that the federal government could either exempt itself from the policy or hold on to the revenues it collects to keep itself whole—that is, to maintain the current level of services even under the additional costs it would face for higher carbon.[31] If that were the case, the costs would be higher in the other sectors to offset the lost revenue from the government.

Finally, the proportion of costs passed on to exported commodities is likely to be lower than the 9.5% shown in Table 2 because of exchange rate adjustments. In the absence of a border tax

[30] Approximately 0.3% of the revenue is lost because of the various adjustments, data inconsistencies, and rounding in the modeling process.
[31] Boyce and Riddle (2008) and Dinan and Rogers (2002) have similar estimates of government carbon intensities.

adjustment policy where exports are rebated the amount of the carbon tax, exchange rates would adjust for the higher cost of exports from the U.S.—that is, the dollar would depreciate relative to foreign currencies. Consumers and government, then, would ultimately bear the cost attributable to exports in that IO accounting framework in the form of higher import prices.

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9. Appendix: Data

The Bureau of Economic Analysis (BEA) releases large amounts of data on the U.S. economy, including a wide variety of input–output data. The selection of available input–output data to use for this analysis required consideration of various options, and, given the data selection, several adjustments were necessary. This appendix describes the considerations, sources, and adjustments made in preparing the data used in the analysis.

Benchmark vs. Annual Tables

Broadly, the BEA releases the IO Make–Use tables based on two separate data sources. The primary data for the benchmark tables come from the Economic Census, which is conducted every five years. The benchmark tables are also released every five years, but there is significant amount lag between the data collection and the release of the IO tables. The benchmark tables currently available are for 2002. The 2007 benchmark tables are not expected to be released until 2013.

The other set of IO tables the BEA releases are annual tables. The data for the annual tables are generally not as detailed as the data available in the benchmark tables, and are based on a previous benchmark table with aggregate updates based on several sources of annual survey data. The annual data, thus, have the most up-to-date aggregate output data, but the inter-industry transaction data are held relatively constant. The most recent annual data are for 2007, and are based on the 1997 benchmark tables with annual adjustments based on aggregate output data.

Secondary Output

The key improvement in the Make–Use system is the incorporation of secondary output in the data collection stage. Reinstituting a distinction between commodities and industries and the bookkeeping technique of logging each transaction in both the Make and the Use tables ensures that a proper balance of input and output in an economy is maintained. There are three types of secondary products identified in the data-collection process:

1. Reclassified products,
2. Redefined products (which are connected to reallocated products), and
3. Other secondary products.

Reclassifications occur when the BEA determines that output classified as a primary product in the Economic Census should more appropriately be regarded as a secondary product for IO purposes. In these instances, the commodity class to which the product belonged is changed. Reclassifications affect only the commodity totals and do not affect the industry totals. For example, the Economic Census considers both newspapers and newspaper advertising the primary products of the newspaper industry. For IO table analyses, however, newspaper advertising is regarded as an intermediate business input and, as such, should be moved to the advertising commodity. The newspaper industry output is unchanged (it still includes all the newspaper output and newspaper advertising), but the commodity outputs for newspapers and advertising are changed by equal, offsetting amounts.[32]

Redefinitions are similar to reclassifications, but occur at the industry level rather than at the commodity level. When the BEA determines that the production and input structure of a secondary product is substantially different from the production and input structure of the primary product in a given industry (and sufficiently similar to the input structure of a similar product in another industry), that secondary product is redefined from its original industry over to the industry that uses a similar production process for its primary output. After a product has been redefined and changed from a secondary product in one industry to a primary product in another, the inputs that were associated with that product are also moved from the original industry into the new industry. This movement of the inputs to the redefined products is known as reallocation.

In addition to reclassifications and redefinitions (and their associated reallocations) there remain some other secondary products that are neither reclassified nor redefined in the BEA's preparation of IO tables. The extent to which secondary output remains in the IO tables can easily be seen in the Make table. If all secondary output were reclassified or redefined, the Make table would be a perfectly diagonal matrix with zeros in all the off-diagonal cells. The significant number of non-zero cells in the off-diagonals indicate that these other secondary products constitute the largest share of the three types of secondary products identified.

[32] Advertising from all industries is similarly reclassified to the advertising commodity. Another common reclassification in the preparation of BEA's IO tables is the classification of all scrap receipts into the scrap commodity.

Although the reclassifications, redefinitions, and reallocations affect the total outputs for given industries and commodities, these regroupings do not change the total economy-wide value of industry or commodity output. Thus, industry and commodity output at the economy level are identical in the standard (pre-redefinition) IO tables and in the supplementary (post-redefinition) IO tables.

Standard vs. Supplementary Tables

The Make–Use tables are also released as either standard tables or supplementary tables. The difference between these two sets of tables is the amount of reclassification and redefinition of secondary products in the tables. The standard IO tables have only a basic level of data adjustments made to them. In addition to these basic data adjustments, the BEA performs and releases a set of supplementary tables that have more extensive adjustments to reduce the amount of secondary production present in the tables. The reclassifications and redefinitions are conducted based on BEA analyst expertise, but are largely based on the assumption that each commodity has a unique production function and that secondary products are moved to industries in which their production is the primary output. The supplementary tables are thus a step closer to the homogeneous output assumption of the standard Leontief IO model.

Producers' vs. Purchasers' Prices

The BEA releases the Use table in both producers' prices and purchaser's prices. The primary difference is in the handling of trade and transportation margins for the inter-industry transactions. Although the Use table is released in both producers' prices and purchasers' prices, the Make table is released only in producers' prices. Ultimately, however, model results expressed in purchasers' prices are more useful when analyzing the impact of a carbon policy. Because there is no Make table in purchasers' prices, and because the BEA releases a bridge table that identifies the trade and transportation margins for each commodity, the model uses the Make and Use tables at producers' prices to estimate the price change effects, but converts them to purchasers' prices as the final step in the analysis.

Levels of Detail

Three levels of detail are released in the benchmark data. At the detailed level, there are approximately 400 categories of commodities and industries. At the summary level, there are approximately 70 categories, and at the sector level there are fewer than 20 commodity and industry categories. The annual data, however, are not released at the detailed level. While the most recent benchmark data are categorized based on the 2002 North American Industry Classification System (NAICS), the annual data are based on the 1997 NAICS. The differences in the two NAICS are minor, but there are different numbers of categories produced at the summary level between the benchmark data and the annual data. Because the summary level data provide most of the detail required while keeping the size of the matrix manipulations tractable, the model uses summary-level data as the foundation.

Data Used in This Analysis

The input–output tables used in this analysis are the supplementary 2006 annual Make and Use tables at the summary level in producers' prices. The selection of this set of files as the base for the model is based on several occasionally competing considerations. Many analysts rely on benchmark tables because they are built from the most detailed data available. There is a trade-off, however, between detail and timeliness when choosing to use benchmark tables.

The most recent benchmark tables are for 2002. Because this analysis does not require the level of detail in the detailed benchmark IO tables, and because the benchmark data are currently eight years old, the model uses the more recent though less detailed annual data as the foundation for the analysis.

Adjustments

The selection of the annual tables at the summary level has a few shortcomings that need to be addressed. At the summary level, the utilities commodity is not disaggregated; it includes electricity, natural gas, and water. For an analysis of a carbon policy on commodity prices, it is crucial that these commodities be disaggregated. This analysis relies on the distribution of the utility commodities found in the 2002 benchmark data to split the 2006 utility amounts into their component parts.

Similarly, the mining commodity does not differentiate between coal mining and other mineral mining. Data from the 2002 detailed benchmark tables are again used to disaggregate the mining commodity so that coal is isolated from other mining. Both of these adjustments require row and column adjustments in both the Use table and the Make table. When this is done, however, the commodity and industry output totals in the Use table and the Make table are no longer identical for these newly disaggregated commodity classes. To account for this discrepancy, a RAS algorithm is applied to the Make table, using the row and column output vectors from the Use table as the control totals to adjust the intermediate cells in the Make table until the row and column output vectors are identical to those produced in the Use table.[33]

Unfortunately, even at the most detailed level of data collection, the oil and gas extraction commodity is combined. The oil and gas extraction commodity is the most upstream location for the carbon policy to be applied, and that is where the policy is applied in this model. The combined commodity is retained in underlying data, but the application of the policy is split based on some simplifying assumptions, as detailed below.

Data on Imported Commodities

The intermediate inputs and final use panels of the Use table released by the BEA include both domestically produced commodities and imported commodities. The imported commodities, however, are subtracted out in the national accounts with negative values that show up in the imports column of the final demand section of the Use table.

To allow analysts to isolate domestic demand and price effects in an input–output framework, the BEA releases a matrix that contains the imported commodities used in the intermediate and final demand sections of the Use table. Although the BEA does not have enough detail by industry to show the exact proportion of commodity inputs used in production abroad, it releases the imports matrix with these values, based on the assumption that all production technologies have identical domestic/import ratios. Data from the imports matrix are subtracted from the Use table to separately calculate domestic and imported production technical coefficients. Except for three carbon-intensive commodities (oil, natural gas, and coal), prices of all other imported commodities are held constant in this paper's analysis.

[33] A RAS algorithm is a well-known, iterative approach to adjusting interior quantities to adjust input–output tables to new or different output aggregates. RAS stands for the matrix algebra notation used in the calculation $A = rAs$, where r is the column vector of adjustment factors and s is the row vector of adjustment factors. For more details on the procedure, see Chapter IX in United Nations (1999).